Primary
& Early Years
Magazine 70

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another edition of our new-look and more compact Primary Magazine. This magazine has been serving primary practitioners for 69 editions with a varied collection of different articles related to maths education and mathematics professional development, which are accessible through the Primary Magazine Archive.

Contents

In each edition we have a selection of interesting and useful articles. New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study. This edition focuses on *fluency, reasoning and problem solving in Measurement in KS1*.

Where's the Maths in That? shares ideas for ensuring that mathematics is taught and experienced across the curriculum. In the coming months, this series of articles that will explore opportunities for mathematics and mathematical thinking within the new science programme of study. This month the theme is *Animals, Including Humans for Y2*.

Finally, Maths in the Staff Room provides simple plans for CPD meetings in your school to be led by a member of staff. These are short meetings that can be used exactly as indicated or adapted to meet the CPD needs of the school. In this edition we explore mental fluency.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

Maths Hubs

The first wave of teachers from Shanghai, who came as part of the England – China project within the Maths Hubs programme, have now returned to China, after spending most of November working in 22 different primary schools. The NCETM brought out a special report during the latter part of their visit explaining how the visit was organised and what their partner teachers in the host schools were learning from the experience. A second wave of Shanghai teachers is due to spend a similar period in a different set of schools in February/March next year.

Ofsted clarification on evaluation of maths teaching

Ofsted's National Lead for Mathematics, Jane Jones, has clarified the situation on how maths teaching is evaluated during inspections. In a guest blog on the NCETM website, she addresses questions raised by teachers following a blog on mastery teaching from the NCETM's Director, Charlie Stripp.

Performance Descriptors Consultation

The DfE has recently published a draft set of performance descriptors for the end of KS1 and KS2 against which attainment will be measured through statutory assessment. The consultation remains open until 18 December, so there is still time to respond.

National Numeracy Parent Toolkit

National Numeracy have launched their online parent toolkit to support parents in understanding mathematics in education settings. This easy-to-navigate website is something you may want to link to from your own school website, and to familiarise yourself with in order to help parents of pupils in your school support you and their child with their mathematical progress

NCETM National Curriculum support

Have you explored our National Curriculum Planning Tool yet? This interactive tool will support you in the following ways: your subject knowledge; making connections within and across the primary curriculum; suggest helpful papers, pupil activities, exemplification of expectations, and links to the suite of NCETM videos. There are also sections on the Bar Model, Teaching Fractions, Progression in Reasoning, and Developing a Scheme of Work - all accessible via buttons on the main National Curriculum information page.

Mathematics CPD

Don't forget that if you are looking for high quality providers of maths CPD in the next academic year, use our Professional Development Directory to find CPD Standard Holders (gold rosette) or Accredited Professional Development Leads (purple rosette).

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New National Curriculum in Focus

New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics

Fluency, Reasoning and Problem Solving in Measurement in KS1

While there is a great emphasis on arithmetic in the new curriculum, the remaining programmes of study still retain an important feature of a broad and balanced curriculum. In this section we will explore some of the changes in the new National Curriculum for upper KS2 in Measurement, suggest how to refresh subject knowledge for this area of the curriculum, and provide some suggested activities.

Previously known as Shape and Space, this strand is now referred to as Geometry and is a term consistent across all Key Stages (including KS3). The new programme of study requires the following for Y1 and Y2:

Y1

Pupils should be taught to:

- compare, describe and solve practical problems for:
  - lengths and heights [for example, long/short, longer/shorter, tall/short, double/half]
  - mass/weight [for example, heavy/light, heavier than, lighter than]
  - capacity and volume [for example, full/empty, more than, less than, half, half full, quarter]
  - time [for example, quicker, slower, earlier, later]
- measure and begin to record the following:
  - lengths and heights
  - mass/weight
  - capacity and volume
  - time (hours, minutes, seconds)
- recognise and know the value of different denominations of coins and notes
- sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]
- recognise and use language relating to dates, including days of the week, weeks, months and years
- tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.

Y2

Pupils should be taught to:

- choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using >, < and =
- recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value

- find different combinations of coins that equal the same amounts of money
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change
- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times
- know the number of minutes in an hour and the number of hours in a day.

Subject Knowledge

Firstly, KS1 teachers must be confident in their own subject knowledge; not just for KS1 but also for KS2 in order to understand how the subject progresses from EYFS, and to ensure that the foundations that are laid in KS1 enable a seamless journey through the measurement curriculum. Building misconceptions early may cause difficulties later. The Self-evaluation Tools for measurement in KS1 and KS2 are a useful way to monitor and develop teacher subject knowledge.

Haylock and Cockburn (1989) identify four fundamental mathematical ideas that are involved in the process of measuring: comparison, ordering and transitivity, conservation or measures and the idea of a unit.

Comparison – a purpose of making a measurement is to compare two or more attributes that can be measured. Initially children will make direct comparison and develop the language of comparative and superlative adjectives such as longer or longest. No measuring unit is required. Haylock and Cockburn suggest that young learners need to use the language of comparatives in such a way that they develop the idea that every comparison statement has an alternative equivalent form. E.g. "the branch is *longer than* the stick" **and** "the stick is *shorter than* the branch". i.e. a>b = b<a. When young learners have grasped the use of this language and discovered that direct comparison is not always possible, children are ready to move on to using units of measures.

Transitivity – the principle of transitivity is for pupils to identify a relationship between three or more measurements. For example if box A is heavier than box B and box B is heavier than box C then box A is also heavier than box C. ie. A>B and B>C, then A>C and also using the alternative equivalent form B<A and C<B, then C<A. Knowing this enables the measurements to be ordered according to the attribute being measured. Transitivity also applies to measurements of equal amounts. i.e. A=B and B=C therefore A=C. Although apparently obvious to us as adults, children will need to have a grasp of this idea before they can understand why a standard unit can be used to measure, i.e. using different cm rulers will still give the same measurement.

Conservation – Piaget's conservation tests for length, capacity and mass are well known. This video clip is a helpful reminder of some of the typical misconceptions when children are learning about measures.

Unit – Once young learners have grasped the use of direct comparison to take measurements, they typically explore non-standard units to develop the idea that a commonly understood unit is needed. Haylock and Cockburn argue that this process is important before introducing the abstract 'cm' or 'Kg' etc. which may be somewhat 'mysterious' to children. However children will need to arrive at an understanding for why a 'unit' of measure has mysteriously been invented through comparison of measurements using arbitrary units of measures. i.e. comparing the length of a desk using handspans and discovering that using different people's hands gives different results.

Finally, understanding measurements depends on an understanding of **approximation** and **accuracy**. In most cases any measurement is an approximation to a certain degree of accuracy. Measuring the value of money is an example where measurement is discrete. Otherwise length can be measured in kilometres, metres, millimetres but even measuring in mm will not give an exact measurement, only to the nearest mm. Pupils are likely to come across this problem when first measuring using non-standard units.

You may also find the Maths4Life Measurement booklet helpful as a self-study unit for understanding measures.

The NRICH article Money Problems? helps unpick why some children find learning about money so difficult.

Activities for *Fluency, Reasoning, and Problem Solving* in Measurement in KS1

In order for pupils to be fluent in the properties of shapes they will need to become increasingly familiar with and confident in using accurate vocabulary. Below is a list of suggested key vocabulary to introduce across Y1 and Y2.

> Length: *length, height, long, short, longer, shorter, tall, short, double, half, wide, wider, narrow, low, lower, high, higher, narrower, deep, deeper, shallow, shallower, ruler, tape measure, metre stick, metre (m), centimetre (cm)*
>
> Mass: *heavy, heavier, light, lighter, weigh, weight/ mass, scales, balance, Kilogram (kg), gram (g)*
>
> Capacity (and volume): *full, empty, more than, less than, half-full, half-empty, container, measuring jug litres (l), millimetres (ml)*
>
> Money: *more than, less than, coins, notes, pence (p), pound (£), copper, silver, change, value, heads, tails, total, amount, pay, sell, buy*
>
> Time: *quick, quicker, slow, slower, early, earlier, late, later, days of the week, months of the year, o'clock, half-past, quarter-past, clock, digital clock, timer, seconds, minutes, hours, hands, today, yesterday, this morning, midday, midnight, tomorrow*
>
> Temperature: *hot, hotter, cold, colder, warm, warmer, thermometer, Celsius (°C)*
>
> Other general vocabulary: *measure, compare, estimate, order*

Using the above words, you could play a 'Just a Minute' type game so that the children become familiar with the vocabulary related to measurement. Write a selection of words on pieces of card. Children play in pairs or small teams of two or three. Take turns to describe the word without saying it and the other then have to work out as many of the word on the cards within a fixed time.

Length

Provide your children with problems where they identify the need to carry out the act of comparing or measuring in order to solve it. E.g.

- Stack these books on the bookshelf in order of height.
- How many people can I hug? How many people can two of us hug?
- Who has the biggest feet in the class? (How many ways could we solve this?)
- Here are two royal crowns in a locked case. Whoever it fits will be prince and princess for the day and will be able to release the crowns from their cases. The crown is 35cm around the base. Who will be the prince and princess today?
- Who can cut the longest strip of card from a kitchen roll tube?

Encourage your children to solve logic problems involving the transitivity of lengths.

E.g. Mum has a green, red and blue ribbon. The green ribbon is longer than the blue ribbon. The red ribbon is shorter than the blue ribbon. Which is the shortest ribbon? Encourage the children to draw representations of the problem to help them, then leading to the use of the <, > and = sign to represent the problem.

When children have become confident at answering these questions, ask them to write their own either from inequality statements or from a given diagram.

Keep rulers that have the ends broken off. Ask pupils to work out how they can still measure with them even though the 0 marker is missing.

There are plenty of opportunities to use measuring lengths in games/ P.E. lessons. Can You Do it Too? from NRICH involves throwing a beanbag and measuring how far it has been thrown. Who can throw the furthest? Or find out who can jump the highest over a hurdle?

Mass

Providing opportunities for children to understand how to use balance scales to compare how heavy things are will deepen their understanding and foster their reasoning skills about mass. Invite children to find different objects that will balance each other on the scales. Ask children to split a piece of plasticine so that it balances, then ask them to reshape the plasticine in each pan. What do they notice? (helping them to grasp the conservation of mass). Another activity to challenge their understanding of the conservation

of mass is to fill identical pots full with different things such as water, sand, salt, flour, etc. Ask the children to investigate how heavy they are and put the pots in order, asking them to explain why they think the pots do not all weigh the same even though they are the same pots and each full of something.

Encourage your children to solve logic problems involving the transitivity of weights.

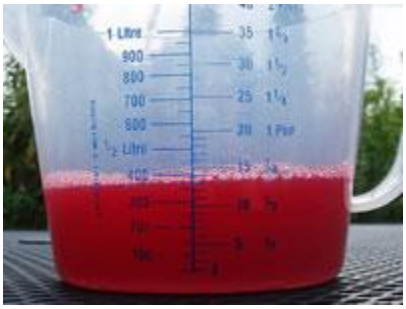
E.g. Mum has three parcels one for Gran, one for dad and one for Aunty Sue. Aunty Sue's parcel is heavier than dad's parcel. Dad's parcel is lighter than Gran's parcel. Which is the heaviest? Encourage the children to draw balance scales to represent the problems to help them, then leading to the use of the <, > and = sign to represent the problem.

Ask children to reason about images of scales such as this one:

Ask children to say what they see. Use what they see to work out which is the lightest/ heaviest box. Re-represent the problem using the inequality signs. Provide examples of contextual problems to wrap around this image and invite pupils to create their own. Some pupils might be challenged to solve similar problems with four objects to compare.

When children have become confident at answering these types of questions ask them to write their own from inequality statements.

Capacity



Provide children with opportunities to challenge their grasp of the conservation of capacity. E.g. fill three different shapes containers with different amounts of water. Ask the children to first sort the containers of water from shallowest to deepest. Ask them to investigate whether this is the same order for the amount of liquid in each container.

Compare the Cups from NRICH uses a selection of cups as a focus for a discussion.

Provide real-life contexts to measure capacity. E.g. how could we find out how many millilitres/cups of water are drunk in our class each day?

The logic problems that have been suggested above for length and mass are equally applicable for capacity as well.

Money

In KS1 children need to gain a grasp of the equivalent values of different denominations of coins. Offer 'would you rather' type choices to use as a starting point for a discussion leading towards the rest of the lesson. E.g. *Would you rather have a purse with 10 2p coins or 3 5p coins? Why?* The NRICH activity Fair Exchange helps pupils to understand the exchange value and equivalence of money/coins.

Encourage pupils to solve problems that require them to think deeply. For example Money Bags from NRICH involves using pennies (1p coins) in unknown bags such that a combination of all or some of the bags can be used to find all totals from 1p to 15p. This can be extended to consider what would be the combinations if 2p's or 5p's were the coins in each bag.

Use this poster (free registration required) from pfeg (the Personal Finance Education Group) to help develop children's money sense. Barclays have also developed some activities around developing awareness of money, providing ideas such as role play areas that can be used to help build on children's understanding of the value of money, recognising coins and making consumer choices.

Why not hold a money week in school to focus on pupils' learning of money matters which can also involve parents? This case study from pfeg will help you with ideas.

Time

One of the most effective ways of teaching the children to tell the time is to allow opportunities to occur during the course of a school day. Show the daily timetable with clocks to represent the time that certain things will happen so that children can match the class clock to the timetable. Making references to how many more minutes until something is going to happen will help children to understand the passing of time as well as having a context in which to understand time.

This video clip of children using stop-watches to time children around an assault course is a lovely example of how pupils can be engaged in some meaningful learning from the earliest age.

The Stop the Clock activity from NRICH is a fun strategy game to try to get from 6 o'clock to midday by taking it in turns to choose either an hour or ½ an hour.

Further links:

- NCETM National Curriculum Y1 Measurement
- NCETM National Curriculum Y2 Measurement
- National Stem Centre eLibrary Y1 & 2 Measurement
- BEAM Money and Measures
- NRICH Measurement KS1
- NRICH Time KS1
- pfeg Financial Planning Framework
- pfeg.

References

Haylock, D. and Cockburn, A. (1989) Understanding Early Years Mathematics, London: Paul Chapman Publishing

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Where's the Maths in That? – Maths across the curriculum

*In this section of this Primary Magazine we explore how mathematics can be embedded into other subjects in the context of the new curriculum. The subject in this new series is **science** and over the next few months we will explore the different themes for the KS1 and KS2 science programmes of study and how maths can be embedded in and enhance understanding of scientific ideas.*

In this edition we look at the theme of **Animals, Including Humans** for Y2 and how a scheme of work for this might incorporate mathematical skills.

The statutory requirements for **Animals, Including Humans** in the Y6 programme of study are:

Pupils should be taught to:

- notice that animals, including humans, have offspring which grow into adults
- find out about and describe the basic needs of animals, including humans, for survival (water, food and air)
- describe the importance for humans of exercise, eating the right amounts of different types of food, and hygiene.

Ask children to compare themselves with a new-born baby. If one of the class has a new-born sibling you could invite the parents in with their baby. Use these questions to focus discussions:

What can you do now that a new-born baby hasn't learnt to do?
What can your teacher do that you can't yet?

Provide a set of cards each with some things that babies, toddlers, six-year olds and adults can do. Ask the children to sort these or order them.

Ask the children for homework to find out how old they were when they learned to say their first word; crawl; take a first step. Collect whole class data to explore when is it most common for babies/children to learn to…? Consider how to organise the information so that it is quick and easy to see the answer to the questions. E.g. using a block graph or pictogram or…?

Building on the previous activities, discuss with the children how they have grown, physically – what has led to this change since you were born? – i.e. eating, exercise.

The children could keep a food diary such as this or this one for a week and then compare to find out what the most popular meals/foods are. They could compare whose diets are most balanced by considering how to present the information in a way that is meaningful.

Use this resource from the Great Grub Club to monitor the whole class to find out if pupils are drinking enough at school? This could be used to measure in cups or in mls (see this month's National Curriculum in Focus).

Children could use this fitness diary (again from the Great Grub Club) to record the activities that they carry out and ask children to record to the nearest 5 minutes/15 minutes how much physical activity they do each day. Ask the children to add up the amounts using a geared clock as a calculation tool to find the total amount of physical activity in day/week. Use the Train with Pedal sheet to monitor what happens to your body by counting heart beats in a minute before and after an activity for a given period of time. There are other activities to support this theme on the Great Grub Club website.

Ask groups of children to make a smoothie by measuring out/weighing quantities of fruit, and recording their recipes.

Develop a theme around looking after your pet…start by making a bar chart of pets that children have, including the number of children that have no pets. Set up a pet food shop role play area reflecting the pets that are owned by children in the class. Find out the typical prices of different types of foods for pets. The children could use a till to practise giving change. Children can also weigh out bird feed etc. Base word problems around what needs to be bought for the different animals, how much will it cost for food for one week?

Suggested Links:

National Stem Centre eLibrary: Y2 Animals Including Humans.

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Maths in the Staff Room – Short Professional Development Meetings

This section provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school

Developing Mental Fluency

Meeting Aims

- Establish a common understanding of what it mean to work mentally with fluency

Timing

- 1.5 hours

Resources

- Developing Number Fluency - What, Why and How, Lynne McClure, NRICH
- video clip: Countdown Amazing Calculation

1. Setting the scene: Experiencing what it means to work fluently and mentally

Share the aim of the professional development meeting.

You may find it helpful to have read this article from NRICH, or ask your colleagues to read it beforehand.

Ask teachers to discuss in small groups what they currently understand 'mental fluency' to mean? Share ideas on a flip chart. Leave the list up to return to later.

Watch this clip from the TV quiz *Countdown*. Is this an example of fluency? Why?

Reveal this problem and ask colleagues to solve this problem mentally:

- $\square + 19 = 33 + 9$

Take examples of strategies. Some will have added the 33 and 9 and subtracted the 19. Others may notice that 19 is 10 more than 9 so the missing box will have to be 10 less than 33. The latter is a more sophisticated strategy and that we would want pupils to use to demonstrate fluency i.e. be noticing **additive** relationships in problems.

Reveal the problem below and ask colleagues to solve it then share their strategies. It is more likely that teachers will now look for a relationship either side of the equals sign but of course this time the relationships are **multiplicative**, i.e. the unknown is found by scaling up a known number:

- $14 \times \square = 28 \times 3$

Using additive or multiplicative relationships will enable a calculation to be performed more **efficiently**. In these examples noticing these relationships would mean that fewer calculations are needed.

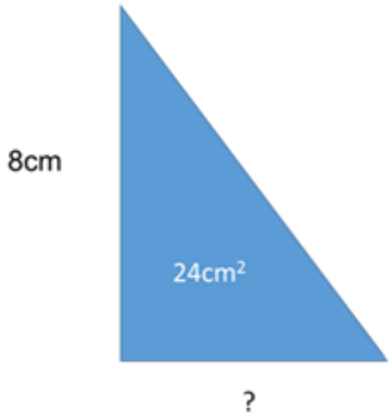
Provide the following problem and ask the teachers to write down as many related facts as they can in one minute:

7 x 4 = 28

Now ask some of the teachers to provide a related fact and ask them to explain how they used the original fact to derive the new fact.

Explain that we want children to be able to work **flexibly** when solving problems mentally, and therefore encouraging them to use related facts is one way that this might be done.

Reveal this problem:



Ask colleagues to suggest two ways to work out the missing side of a right angle triangle with height 8cm and area 24cm^2. One way may be to work backwards with the formula for the area of a triangle, knowing that 8 x 3 is 24 so 3 is half the length of the base. Or combine a similar triangle with the first triangle to make a rectangle with area 48cm^2 and then derive that the missing side is 6 since 8 x 6 is 48. This illustrates how we might encourage pupils to work **flexibly** by asking for more than one strategy and then comparing these.

In all of the above the problems have been solved with mental fluency but also with deep conceptual understanding of the mathematics involved.

Share this quotation from Russell, 2000, who defines fluency as the ability to

> *"…provide correct answers quickly … use facts and computation strategies they know to efficiently determine answers that they do not know"*

This quotation also highlights the importance of **accuracy** in calculating mentally and fluently.

Summarise the session by referring to the three key elements of what it means to work mentally and fluently: efficiently, flexibly and accurately.

Refer back to the list created by the teachers at the beginning and compare – are any/all of these elements included in the list?

2. Activities to foster mental fluency (45 mins)

Choose from this selection of NRICH activities to try out:

- Find the Difference
- Roll These Dice
- Twenty Divided Into Six
- Strike it Out: watch this clip from NRICH and see if the rules of the game can be worked out. Then try and play the game
- Totality: watch the video clip with the sound turned off and see if the teachers can work out the rules of the game. Play the game, adapting own rules/ parameters
- Cops and Robbers
- Other number fluency activities.

3. Reflection

Reflect on the activities that have been tried out and discuss which of the elements of fluency (efficiency, flexibility and accuracy) the activities develop. Discuss how mental fluency might look in your school by writing collectively a statement about what you hope to achieve in mental fluency by the end of each Key Stage.

e.g. *By the end of KS1 all pupils will be able to work with addition and subtraction facts to 20 and multiplication and division facts for the 2, 5 and 10 x tables in an efficient, flexible and accurate manner.*

If there's time, provide this *Countdown* set of numbers for fun. Give the teachers the set of 6 numbers numbers first and then the target figure. Who can find the target? [a solution is (50 + 3) x (5 x 3) – 25 – 4 = 766].