The holiday is nearly here - at last! In between the end of term reports, exams, festive lunches and the endless bleating of "do we have to have a proper lesson, can't we just watch a DVD?" (and that's just your colleagues, ho-ho), we hope that finding Issue 117 tucked behind the door of an advent calendar, or with your name on it in the Secret Santa box, helps you get to the 19 December finish line!

As this time of year is traditionally one of reflection and forward planning, we've focused two of the articles on decisions that are upcoming, rather than curriculum content. As always, let us know what you think, by email to info@ncetm.org.uk or on Twitter, @NCETMsecondary.

Speaking of advent calendars: here are three that we like.

Contents

Heads Up
Here you will find a check-list of some of the recent, or still current, mathematical events featured in the news, by the media or on the internet: if you want a "heads up" on what to read, watch or do in the next couple of weeks or so, it's here. This month we've included a guest blog from Jane Jones – Ofsted's National Lead for Mathematics, a celebration of the Cockcroft report, the *Your Life* project, the film *The Imitation Game*, and the work of Alexander Grothendieck.

Building Bridges
This month, rather than the usual discussion of a secondary mathematics topic, we share some thoughts about choosing a GCSE specification for your current Year 9s.

Sixth Sense
As this is often the time when Year 11s start to make choices about sixth form study, we've decided to bring together some sources of advice and guidance for them, which you too may find interesting and useful.

From the Library
Want to draw on maths research in your teaching but don't have time to hunker down in the library? Don't worry, we've hunkered for you: in this issue you can be inspired by an article considering pupils' understanding of probability.

It Stands to Reason
Developing students' reasoning is a key aim of the new KS3 and 4 Programmes of Study, and this monthly feature shares ideas how to do so. In this issue we think about developing reasoning about probability.

Eyes Down
A picture to give you an idea: "eyes down" for inspiration.

Image credit
Page header by Robert Fries (adapted), some rights reserved

Heads Up

Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.

On 1 December there was a definite "hold the front page" moment: the DfE has announced that, having listened to ALCAB's advice, the new A-level specifications will come into effect in September 2017 not 2016. You can read the NCETM's response here.

Exemplifications of new GCSE levels 8, 5 and 2 were published at the end of November.

On 28 November we said "zai jian" to the 29 teachers from Shanghai who have been sharing their expertise in 22 primary schools right across England, in half the new Maths Hubs. You might like to read the NCETM review of the exchange so far. If you want to find out more when the next set of teachers visit in late February, or about the intended secondary exchange next year, get in touch with your local Maths Hub Lead School.

Have you read the guest blog on the NCETM website from Ofsted's National Lead for Mathematics, Jane Jones? In it she explains what Ofsted inspectors will be looking for when evaluating teaching for mastery.

There has been a lot of discussion recently about the quality and use of maths textbooks. You can read the core arguments, including an article by NCETM Director Charlie Stripp, on the TES website.

The Cockcroft Report Mathematics Counts was published in 1982. In particular paragraph 243 had a huge influence on mathematics teaching in the 1980s and states that:

> *Mathematics teaching at all levels should include opportunities for exposition by the teacher; discussion between teacher and pupils and between pupils themselves; appropriate practical work; consolidation and practice of fundamental skills and routines; problem solving, including the application of mathematics to everyday situations; investigational work.*

To celebrate that important moment in the teaching of our subject, the ATM has brought out a special 'Cockcroft' edition of special edition of Mathematics Teaching. It's available as a free downloadable resource for a limited time.

Your Life is a three-year campaign aimed to increase take up of A level maths and physics. You might like to encourage your pupils to enter the Formula 100 competition. Or you might like to show the video What could Your Life be like in a parallel universe?

National Centre
for Excellence in the
Teaching of Mathematics

On 16 January 2015, the London Mathematical Society (LMS) is launching Mathematics: Unlocking Worlds as part of its 150th Anniversary Celebrations. The event will be live-streamed, and is suitable for a wide audience from secondary school and college students; the talks will be at a similar level to the Royal institution Christmas Lectures.

Have you seen *The Imitation Game*? It features the work of the mathematician and computer scientist Alan Turing in breaking the Enigma code in the Second World War. You might like to watch the trailer to whet your appetite, or read the review from *Plus* magazine.

And, our last word for 2014, why not have a listen to BBC Radio 4's programme Last Word, first broadcast on 21 November, which included an interview with Dr Eugenia Cheng, Senior Lecturer in the School of Mathematics and Statistics at the University of Sheffield about the mathematician Alexander Grothendieck, who died on 13 November aged 86.

Image credit
Page header by NS Newsflash (adapted), some rights reserved

Building Bridges

The Roman god Janus and a Year 9 teacher have quite a lot in common at this time of year: both are looking back over the year that's been, and also are looking ahead to the opportunities and possibilities of the future. Whether Janus had sufficient length of foresight to see the current changes to the GCSE specifications is not recorded by Virgil or Livy, but we're sure that Year 9 teachers are very well aware of the decision that they have to make, and soon.

KS4 is, obviously, a bridge between KS3 and 5, but therefore it is also a bridge between thinking and doing mathematics largely in the concrete to largely in the abstract: GCSE study is the cognitive bridge from Cuisenaire rods to the Argand diagram. We therefore feel that this is the right place in the magazine to bring together some information and advice around making the best choice for your pupils. We hope it's helpful; let us know @NCETMsecondary.

All the new GCSE specifications have been approved and published, along with specimen assessment materials: links to these were included in Issue 115. The exam boards have said that they will be producing further specimen materials, but with different timescales. They are also all offering information and training sessions. Many of the Maths Hubs have a workgroup focused on helping schools prepare for the new GCSEs, so if you'd like to find out more, or be involved directly, contact your local Maths Hub Lead School.

At the NCETM, we are committed to the principle of teaching for mastery: you can read the latest blog about this by Charlie Stripp, the NCETM's Director, and also a very supportive response from Jane Jones, HMI and Ofsted National Lead for Mathematics. Teaching for mastery is entirely consistent with the aims and ethos of the new National Curriculum, namely that at all stages, **ALL** pupils

- become **FLUENT** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;
- **REASON MATHEMATICALLY** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;
- can **SOLVE PROBLEMS** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

with the expectation that

the majority of pupils will move through the programmes of study at **broadly the same pace**. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered **rich and sophisticated problems** before any acceleration through new content. Those who are not sufficiently fluent should consolidate their understanding, including through **additional practice**, before moving on.

(the emphases are our own).

Please, therefore, scrutinise carefully the GCSE specification(s) and specimen assessment task(s) that are on your shortlist, and consider:

- How will you teach the content: linear or spiral, or a blend?
- How will you assess your pupils' knowledge, now that the current National Curriculum level descriptors are obsolete?
- How will you develop their mathematical fluency and reasoning and problem solving skills: "little and often" or "Fun Friday", or a blend?
- How will you assess their reasoning and problem-solving skills?
- How will you challenge your high attainers: what rich and sophisticated problems are offered by the exam board(s)?
- How will you support your low attainers: how will the exam board(s) help you provide effective additional practice?

What resources and support from the exam boards do you and your colleagues need to meet the National Curriculum aims and exemplify its ethos? What do your pupils need? How will the resources offered by your shortlisted exam board(s)

- give all your pupils intelligent practice?
- develop their procedural fluency?
- and, develop their conceptual understanding?

And then, how will you assess your pupils'

- procedural fluency,
- conceptual understanding
- and, their mastery?

Many of us are asking ourselves these and similar questions. Let's share our answers and pool our thinking, either @NCETMsecondary or in the NCETM Secondary Forum.

Image credit
Page header by PauliCarmody (adapted), some rights reserved

National Centre
for Excellence in the
Teaching of Mathematics

Sixth Sense

Deck the halls with boughs of holly, 'tis the season for … GCSE mocks – in many schools they're happening now (as the tired pupils limp towards the end of a long term) or they're scheduled for early January, so that they hang over Christmas and New Year worse than, say, a hangover. One consequence of the mocks, usually an unintended one, is that the pupils use them to steer their decisions about further study post-16: a low grade now, despite all the caveats around it, often leads to a hard-to-reverse decision not to take that subject further in Year 12. It therefore seems an appropriate time for our monthly sixth form-focused feature to help and support you as you encourage your Year 11 pupils to aim to study Maths post-16.

In doing so, you are not alone! There are a lot of resources available to teachers and pupils, mainly because the leading Maths organisations, understandably, are keen to promote further study of the subject, but also because the government has made increasing the study of Maths post-16, especially at level 3, one of its priorities, and has allocated funding to achieving this. Good places to look include:

- The Your Life website. This is a "three-year campaign to ensure young adults in the UK have the maths and science skills needed to succeed in the current competitive global economy. It looks to inspire young people to study Maths and Physics as a gateway into wide-ranging careers whilst also triggering employers to recruit and retain this talent". There is a particular focus on encouraging girls to study these two subjects, but the resources are not gender-specific.
- On the Further Maths Support Programme website there is lots of advice directly for pupils, as well as resources for teachers. On the page Encouraging Girls to Take Mathematics there is a link to a very good presentation to show to parents/carers (of both boys and girls). Although lots of the material is designed to encourage pupils to study Further Maths, many of the arguments apply also to studying single Maths.
- Websites such as Best4Me and Maths Careers are full of good case studies to answer the inevitable "why Maths?" question.
- A number of research papers can be accessed through the NCETM website. The 2010 paper Widening Participation in Mathematics is an excellent digest of recent research and activity.

Your current Year 11 pupils may have the opportunity to choose to study Core Maths in Year 12. This is a new qualification being taught for the first time this year; so far about 200 schools and colleges are offering it, and more will do so next year. The "target audience" is pupils with grade C or above at GCSE who don't want to study AS Maths but do want to deepen their mathematical knowledge and develop their skills, perhaps to complement the other subjects they are studying: Core Maths is an ideal partner to any Humanity or Social Science, for example. If you're unfamiliar with Core Maths, the best place to get information is the ideally-named Core Maths Support Programme website, which is a microsite within the NCETM site.

Improving participation in level 3 Maths post-16 is a major priority of each of the new Maths Hubs. Your local Maths Hub Lead School is currently putting together a Work Group to lead research and innovation into this challenge, and so do get in touch if you'd like to find out more, or be involved directly.

We all know the value of peer-learning, so we've asked Claire Baldwin to share her experience with us. Claire, who now is one of the FMSP Central Coordinators, was Curriculum Area Leader for Mathematics and Sciences, and later Assistant Principal with responsibility for Curriculum, at the outstanding Rochdale Sixth Form College from 2010 to 2014. Claire was particularly involved with raising participation there.

"Our philosophy from the moment the College opened in September 2010 was one of 'compassionate rigour'. We cultivated close and meaningful relationships with local partner schools so that we had a real sense of where our students came from, their experiences and their perspectives. In the summer term we offered several highly popular 'Taster Days' where local Year 10 students could sample lessons in subjects they were interested in studying at A level. We also offered 'Mathematics Masterclasses' for groups of around 50 A*/A grade students from individual local schools, supported by current A2 Maths and Further Maths students who had previously studied at the same 11-16 school: this showed Year 11 students that the study of Mathematics post-16 is accessible and achievable.

"Every applicant to the College was interviewed by the Principal or a member of the senior team, so we knew that they were getting a consistent message and clear advice about their subject choices. Open Days were friendly, participatory events, with visitors invited to sample learning resources, ask questions and meet current students. Often the students gave a more glowing and sincere recommendation than any member of staff ever could!

"In the Mathematics department, we aimed to make the transition to A level as smooth as possible. Finding out students' mathematical backgrounds was key. Had they taken their GCSE in Year 10 or 11? Had they completed additional qualifications such as the Level 2 Certificate in Further Maths? This insight, along with fortnightly formal assessments, allowed careful tracking of progress and a tailored programme of targeted intervention activities, supported by regular liaison with each student's personal progress tutor as well as monitoring conversations with the student. Peer mentoring – the pairing of students exceeding their target grade with students who were below target – was also a successful strategy.

"Crucially, the culture was one of 'you can do it too', with aspirations to achieve embedded in every lesson".

Image credit
Page header by Richard Gray (adapted), some rights reserved

From the Library
Shh! No Talking!

Our regular feature highlighting an article or research paper that will, we hope, have a helpful bearing on your teaching of mathematics

In this issue we consider Children's understandings of probability: a literature review by Peter Bryant and Terezinha Nunes.

The paper first identifies four "cognitive demands" that pupils need to understand in order to make sense of probability in mathematics, and then explores how these areas are taught in school. They are:

- **Understanding randomness:** to understand the nature and the consequences of randomness, and the use of randomness in our everyday lives.
- **Working out the sample space:** to recognise that the first and essential step in solving any probability problem is to work out all the possible events and sequences of events that could happen. The set of all the possible events is called 'the sample space' and working out the sample space is not just a necessary part of the calculation of the probabilities of particular event, but also an essential element in understanding the nature of probability.
- **Comparing and quantifying probabilities:** probabilities are quantities based on proportions, and one has to calculate these proportions to make most (but not all) comparisons of the probabilities of two or more events. These proportions can be expressed as decimals, as fractions or as ratios.
- **Understanding correlation (or relationships between events):** an association between two kinds of event could happen randomly or, alternatively, could represent a genuine relationship. To discover whether there is a non-random relation or not, we have to attend to the relation between confirming and disconfirming evidence and check whether the frequency of confirming cases could have happened by chance.

You can read a longer and more detailed version of this report, which goes into more detail, is also available to download.

We think that, having read this paper, you will have a deeper understanding of why some pupils find probability difficult, and also have some ideas how to respond to this in order to help your pupils develop more secure conceptual understanding and procedural fluency. Let us know what you think.

And if your pupils find this xkcd cartoon as funny as we do, then you know you've taught them well!
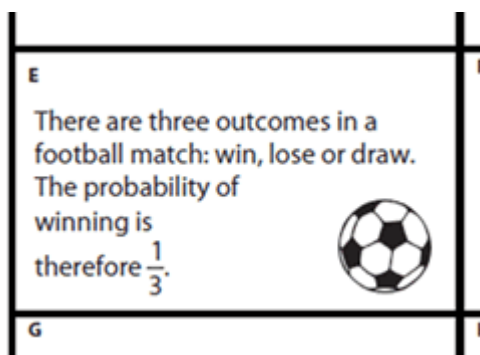
Image credit
Page header by Abhi Sharma (adapted), some rights reserved

National Centre for Excellence in the Teaching of Mathematics

It Stands to Reason

In this regular feature, an element of the mathematics curriculum is chosen and we collate for you some teaching ideas and resources that we think will help your pupils develop their reasoning skills. If you'd like to suggest a future topic, please do so to info@ncetm.org.uk or @NCETMsecondary.

When a relatively small football club is scheduled to play one of the giants of the game, for instance when little Crawley Town played Manchester United in the FA Cup a few years back, we are provided with just the situation to create a bit of cognitive conflict for some of the pupils we teach. What would your pupils say about this statement: is it true or false or can we not be sure?

E

There are three outcomes in a football match: win, lose or draw. The probability of winning is therefore $\frac{1}{3}$.

G

This statement comes from the resource Evaluating Probability Statements that consists of ten statements that pupils are asked to consider. Other statements are linked to the lottery, situations with dice, coin throwing and other situations – ripe for misconception! – associated with classical probability. To introduce pupils to the idea of sampling, the problem Counting Fish gives a real life context. The teachers' notes suggest a way of simulating the situation using cubes to support pupils develop their understanding.

You may have already looked at Placing Numbers from the NCETM microsite What Makes A Good Resource.

5×7

35

The relatively simple activity of placing products of pairs of numbers on a grid may not immediately suggest a probability resource, however, the task can be extended to consider the sample space of outcomes to inform the placing of the products – a more complex task altogether!

Pupils may be familiar with the situation of buying lottery tickets in the UK. This is a rich source of mathematics, particularly with the support of the Lottery Simulator from NRICH. The Number Tumbler allows you to pick up to 6 numbers from up to 49 numbers and simulate up to 1000 draws which should allow you to build up to the actual UK lottery draw scenario (picking 6 from 49) but start as small scale as you like. You may like to explore the Key Stage 2 problem Winning the Lottery for further ideas for Year 7 and 8 pupils.

Further resources include:

- The NRICH article Probability Through Problems: a new approach to teaching probability
- Some real-life contexts are outlined in Conditional Probability Is Important for All Students!
- Your pupils could use their knowledge to solve this mystery
- The Probability Computer Games include some Flash resources to use in the classroom. Working through these resources introduces the idea of tree diagrams in a logical sequence.
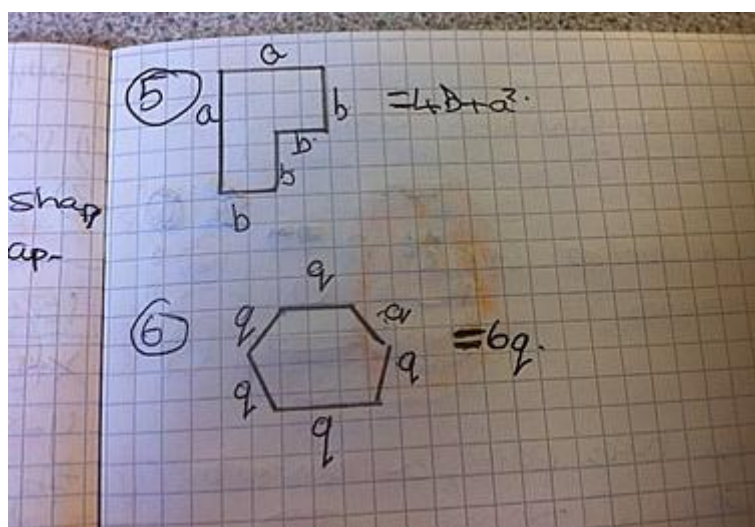
Image credit
Page header by Juanedc (adapted), some rights reserved

Eyes Down

Our monthly picture that you could use with your pupils, or your department, or just by yourself, to make you think about something in a different way

This picture is a snapshot from a Year 7 pupil's exercise book and shows a common mistake in using algebra notation. The pupil was asked to find the perimeter of the figures (not drawn to scale!). It is intriguing that the pupil can write $6q$ (not q^6) and yet writes a^2 instead of $2a$.

How would you explore this error with the pupil, and also draw on it for the benefit of the whole class?

- First you could ask the pupil to explain how the answers were reached, without saying which is correct: articulating the reasoning will be instructive, and may lead to self-correction
- It may be helpful to write the terms out in 'long hand' which would give $a + a + b + b + b + b$ and then discuss combining the a terms and the b terms.
- You could have a set of numerical expressions such as 3+3, 3x3, 2 x 3 and 3^2, and ask pupils to match them up. Similar algebraic expressions, eg $a + a$, a x a, $2a$, a^2 would then form the next set of cards to match.
- You may like to consider the area of the figure, using a diagram

Let us know @NCETMsecondary what you would do.

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk, with a note of where and when it was taken, and any comments on it you may have.