MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Fractions

3.1 Preparing for fractions: the part–whole relationship

Teacher guide | Year 3

Teaching point 1:

Any element of a whole is a part; if a whole is defined, then a part of this whole can be defined.

Teaching point 2:

A whole can be divided into equal parts or unequal parts.

Teaching point 3:

The relative size of parts can be compared.

Teaching point 4:

If one of the equal parts and the number of equal parts are known, these can be used to construct the whole.

Overview of learning

In this segment children will:

- develop understanding that a whole can be made up of many parts and that many parts can make a whole
- be introduced to the concept of equal and unequal parts
- compare the size of parts within different wholes and compare the relative size of a part to different wholes
- explore the relationship between the part, the whole and the number of parts, moving both from the whole to the part and from the part to the whole.

The focus of this first segment of Spine 3 is on identifying a whole and part of a whole in different contexts, as well as exploring some aspects of these, for example whether the parts within the whole are equal or unequal, and the relative size of a part and a whole. Children will already be very familiar with the language of parts and wholes from *Spine 1: Number, Addition and Subtraction*, where the term 'part–part–whole' is used to express additive relationships (for example, if 3 + 2 = 5 then 2 + 3 = 5, 5 − 2 = 3 and 5 − 3 = 2). In this spine, the term *'part–whole'* is used to express a proportional relationship between a whole and part of the whole (for example, where the whole is divided into two equal parts and each part is one-half of the whole).

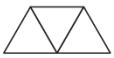
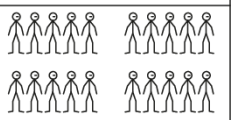
The word *'fraction'*, however, is not used within the segment at all by the children and at no point do they write using fractional notation, e.g. $\frac{1}{4}$, or verbalise the names of any fractions (for example *'one-quarter'*). This may seem to be moving very slowly, but spending time looking at the relative sizes of parts and wholes gets to the crux of the proportional aspect of a fraction. Fractions are essentially multiplicative comparisons between a part (expressed by the numerator) and a whole (expressed by the denominator). Later on, children will need to understand that it is the sizes of these relative to each other that are critical in determining the value of a fraction, e.g. $\frac{7}{11} < \frac{5}{6}$, even though both the numerator and denominator in the first fraction are larger numbers than in the second. Seven-elevenths is smaller than five-sixths because seven is a smaller part/proportion of eleven than five is of six. Likewise, in the map activity in *Teaching point 1*, Manchester is almost certainly a smaller part/proportion of the UK than a Year 3 classroom is of a school building, even though Manchester is, of course, much bigger than a Year 3 classroom.

Throughout this spine, concepts are shown to children in a variety of contexts. Three models are consistently considered alongside each other: area (presented as different shapes), length (and linear journeys), and sets of objects (which leads into fractions of quantities). Other models are touched on at times, including volume and time. The linear model is particularly key, because in later segments it will be important for children to understand that fractions are numbers (e.g. one-third) as well as operators on a set (e.g. one-third of the children). The latter of these is often overemphasised at the expense of the former. Representing fractions on a linear model will support children with positioning fractions on a number line later on, and with understanding that fractions are numbers (which children sometimes don't realise – they just think they are 'part of' something).

This segment builds on the concept of fractions from the very beginning, and although children will have some prior knowledge of fractions from Year 2, their knowledge is now built-up comprehensively using the concept of a part and a whole. The language of parts and whole will be drawn on heavily in

future segments, so it is important that the attention of all of the children is focused on this, even if some children appear to have moved beyond it.

Throughout this segment (and indeed the whole spine), it is important to embed the concept that it is possible to move from a part to the whole as well as from a whole to its parts, and this segment explores both of these. Moving from a part to a whole will also help children's understanding of the inverse, for example, when they are given a fractional amount and are required to calculate the whole from this amount. This includes their understanding of how fractions relate to both division and multiplication (for example, *'If I know that one-fifth of a bag of sweets is three sweets, then I can multiply by five to calculate the number of sweets in the whole bag. Inversely, if I know that the whole is fifteen, I can divide by five to calculate one-fifth of the bag.'*). At the end of the segment, this diagram is used to draw together the relationship between the part, the whole and the number of equal parts:

Part	Number of equal parts	Whole
	3	
	5	
	4	

In future segments, we see that we can add an additional column: the fraction that each equal part forms of the whole.

In order for children to gain a deep understanding of the concepts, the examples given are necessarily very visual. They include a range of carefully constructed images and incorporate work with some concrete manipulatives, in particular the cutting and folding of paper.

One final point is the extensive use of the word 'whole'. Children have of course met this word before in a mathematical context. However, it is worth reiterating to them that word whole, spelt with a 'w', refers to all of something, to avoid any of the children creating a mental image of a hole in the ground.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Any element of a whole is a part; if a whole is defined, then a part of this whole can be defined.

Steps in learning

	Guidance	Representations
1:1	Begin this segment by exploring the concept that if a whole is defined, then a part of this whole can be identified. Demonstrate this by showing children a map of Europe and saying '*If Europe is the whole, then the United Kingdom is part of the whole.*' Give further examples, using other countries as the part of the whole, and then ask the children to come up with their own examples using the stem sentence: ***'If Europe is the whole, then ___ is part of the whole.'*** Ensure that all children use this full sentence with their examples. Doing so will help them to understand the relationship between the part and the whole, rather than thinking about each country as a whole in its own right.	 *'If Europe is the whole, then the United Kingdom is part of the whole.'*
1:2	When children can confidently identify parts of this prescribed whole correctly, redefine the whole. Tell the children we now have a different whole and ask them to identify parts of this new whole using an appropriate stem sentence, such as: ***'If the United Kingdom is the whole, then ___ is part of the whole.'*** Choosing a new whole (such as the United Kingdom) that was previously a part will help children to understand that the same thing can be either a part or a whole, depending on the context. This does not need to be explicitly explained to the children at this stage, but having been exposed to the concept will help them in later steps of learning.	

	Continue redefining the whole, 'zooming in' on the location of your school (for example, *'If London is the whole…'*, *'If Croydon is the whole…'* or *'If Croydon Primary School is the whole…'*), asking the children to identify parts of each whole.	 *'If the United Kingdom is the whole, then London is part of the whole.'*
1:3	Continue 'zooming in', defining your school as the whole and identifying appropriate parts. Be aware that there are different ways of defining the school: one is as a collection of the people in it (*'If Croydon Primary School is the whole, then Year 3 is part of the whole.'*) and another is as the buildings which make it up (*'If Ashburnham School is the whole, then the Year 3 classroom is part of the whole.'*). Take this opportunity to discuss the need to carefully define both the whole and the part, and the idea that the part is a subset of the whole; if the part is the people in Year 3, then the whole must be the people who make up the school.	

1:4

Now broaden your exploration of the part–whole relationship to include other examples, including both measurement and cardinal (quantity-value) contexts.

As mentioned in the *Overview of learning*, it is important to use linear models at this early stage, because in later segments children will need to understand fractions as numbers that can be positioned on a number line. Prepare children for this concept with models such as a journey between various friends' houses, asking children to identify different parts.

Use the stem sentence from the previous steps: '***If ___ is the whole, then ___ is part of the whole.***'

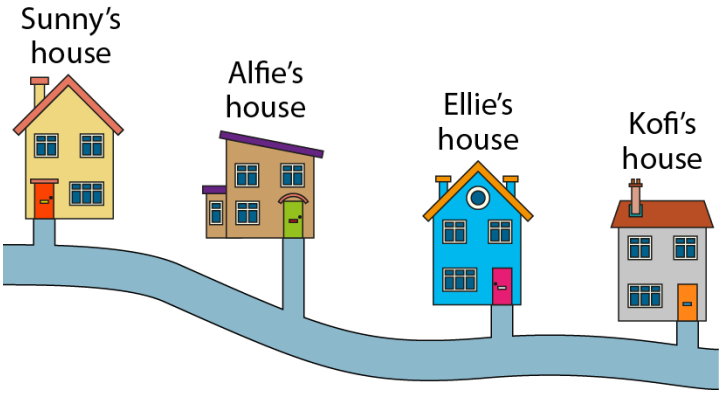
Ensure a clear definition of the whole in each context, as exemplified opposite.

Draw out the idea that a part can be the journey from one house to the next, but could also be a longer journey, and not necessarily starting from Sunny's house.

As well as area and length, it is important to consider models in which the whole is a set or quantity. The individual items within a set may have characteristics that allow parts to be identified in a number of different ways (in the example shown opposite, there are adult sheep and lambs, black sheep and white sheep, and sheep facing left and sheep facing right).

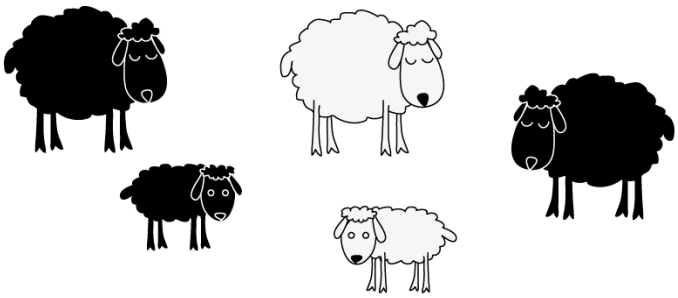
Take time to encourage children to look for parts and wholes in many contexts (for example, children playing on different equipment in the playground, different items of food making up a picnic, and different parts of the human body). This will help children understand that multiple objects (that may be the same or different) can be defined as a part of the whole.

Part–whole relationship – measurement context:



- *'If the journey from Sunny's house to Kofi's house is the whole, then the journey from Sunny's house to Alfie's house is part of the whole.'*
- *'If the journey from Sunny's house to Kofi's house is the whole, then the journey from Sunny's house to Ellie's house is part of the whole.'*
- *'If the journey from Sunny's house to Kofi's house is the whole, then the journey from Alfie's house to Ellie's house is part of the whole.'*

Part–whole relationships – cardinal context:

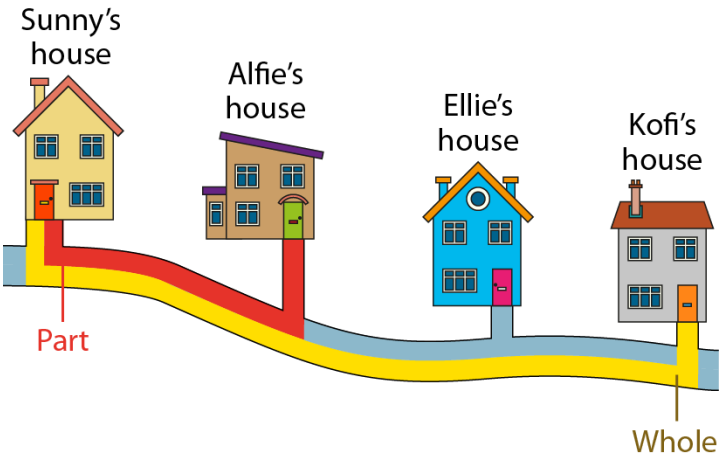
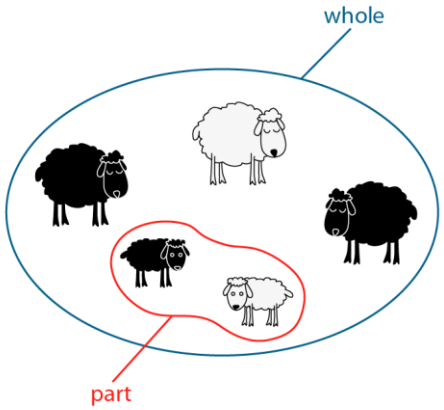


'If the group of sheep is the whole, then the black sheep are part of the whole.'

Part–whole relationship – cardinal context (time):

Mon	Tue	Wed	Thu	Fri	Sat	Sun

'If the week is the whole, then Tuesday is part of the whole.'

	You could also include part of a quantity that can't be seen, for example time; days in the week or lessons in the school day: • ***'If the week is the whole, then ___ is part of the whole.'*** • ***'If the school day is the whole, then ___ is part of the whole.'***	
1:5	When children have been exposed to lots of different contexts, encourage them to generalise about the relative size of a part compared to the whole. They should understand that a whole can be made of two or more parts and that the parts are always smaller than the whole. To ensure this generalisation is made by all children, show them familiar images with the whole and a part labelled. Draw attention to the fact that in each image the part is smaller than the whole. After considering several images, agree on the generalisation: ***'A part is always smaller than the whole.'*** Note that at this stage the relative size of the parts within the same whole is not the focus. Children will be given the opportunity to explore equal and unequal parts in *Teaching point 2*.	Part–whole relationships – relative size: • Measurement context  • *'If the journey from Sunny's house to Kofi's house is the whole, then the journey from Sunny's house to Alfie's house is part of the whole.'* • *'The part is smaller than the whole.'* • Cardinal context  • *'If the group of sheep is the whole, then the lambs are part of the whole.'* • *'The part is smaller than the whole.'*

1:6

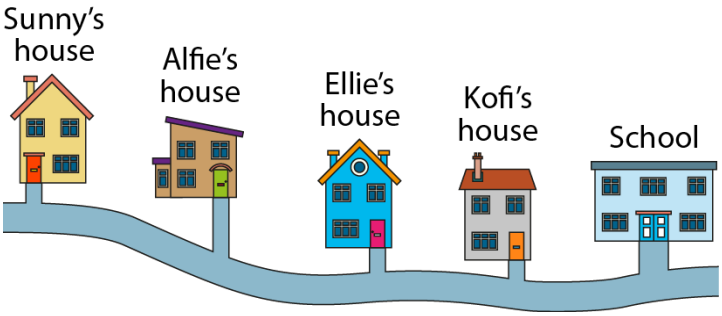
When children can confidently define a whole and identify parts of this whole within different contexts, show them further images in which a whole in one context can become a part of a different whole. For example, return to the map of Europe and remind children that the whole can be redefined by briefly revisiting the activity in step *1:2* (*'If Europe is the whole, then the UK is part of the whole; if the UK is the whole, then London is part of the whole.'*). Challenge their thinking by asking them to agree or disagree with statements such as:

- *'The United Kingdom can only be the whole; it cannot be a part.'*
- *'The United Kingdom can not only be the whole but can also be a part.'*

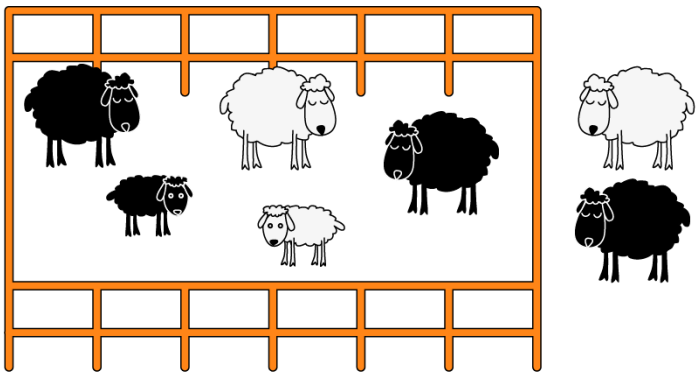
Conversations around this concept should hinge on the fact that it is critical to define the whole in all instances; a part can only be called a part in relation to a whole.

Include some examples where an image shows an extended whole, making the previous whole a part of a bigger whole. For example, extend the picture of a road used in step *1:4* so that the journey between the friends' houses is just part of the journey to school. Use the stem sentence '***If ___ is the whole, then ___ is part of the whole'*** to establish that the original whole is now part of the new whole, and also that a part of the original whole is now part of the new whole.

It is crucial that all children understand that there are many different ways to define a whole and only when this is done is it possible to define a part. Reinforce this, both by extending the whole and then by 'zooming in', for example, using the original whole again, and applying different criteria.



- *'If the journey from Sunny's house to Kofi's house is the whole, then the journey from Sunny's house to Alfie's house is part of the whole.'*
- *'If the journey from Sunny's house to school is the whole, then the journey from Sunny's house to Kofi's house is part of the whole.'*
- *'If the journey from Sunny's house to school is the whole, then the journey from Sunny's house to Alfie's house is part of the whole.'*



- *'If all the sheep are the whole, then the sheep in the pen are part of the whole.'*
- *'If the sheep in the pen are the whole, then the white sheep in the pen are part of the whole.'*
- *'If the black sheep are the whole, then the black lamb is part of the whole.'*

1:7

Incorrect examples may have been provided by children during steps of learning up to this point that are not accurate, for example, *'If the UK is the whole, then France is a part of the whole.'* It is essential to challenge these misconceptions when they arise, by each time referring back to what has been defined as the whole.

It is important for all children to be exposed to non-examples of the part–whole concept, to ensure that they fully understand the concept of parts within a whole. Look at a collection of statements, and discuss whether each is true or false, and how any false ones could be adapted to make them true. You may choose to use examples such as:

- *'If the UK is the whole, then Italy is part of the whole.'*
- *'If the school is the whole, then the classroom is a part of the whole.'*
- *'If the school is the whole, then the children in Year 3 are a part of the whole.'*
- *'If the children in Year 3 are the whole, then the school is a part of the whole.'*
- *'If the black sheep are the whole, then the lambs are part of the whole.'*
- *'If my face is the whole, then my toe is a part of the whole.'*

In this final face/toe example, children may believe that because an object is small it is always a part of something else. Again, confront this misconception by returning to the definition or an image of the whole.

Challenge the children to produce their own 'non-examples'. For example:

- *'If London is the whole then ___ is **not** part of the whole.'*
- *'If Year 3 is the whole, then ___ is **not** part of the whole.'*

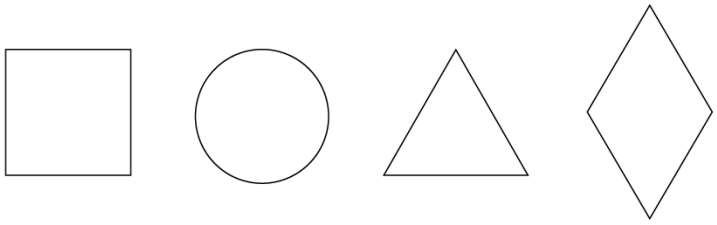
'If the UK is the whole, then Italy is part of the whole.'

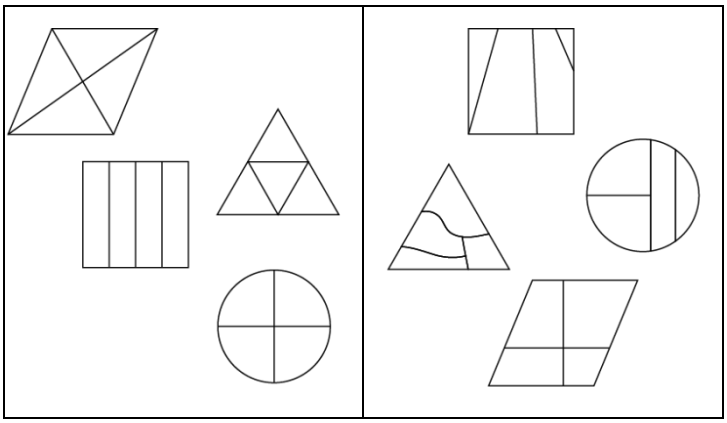
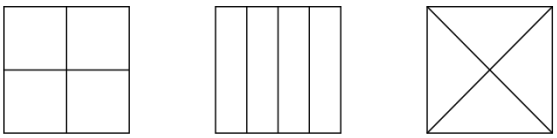
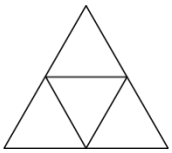
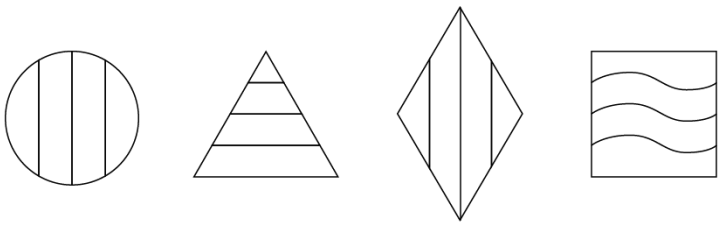
	Look for opportunities beyond the maths lesson to identify parts and wholes. The children should be able to find them everywhere: parts of their journeys around the school building, parts of the school day or of individual lessons, parts of the playground and parts of their dinners are just a few contexts they will come across on a daily basis at school.	

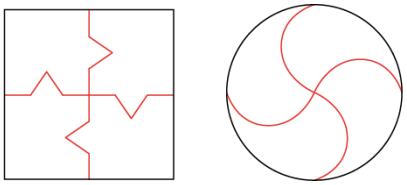
Teaching point 2:

A whole can be divided into equal parts or unequal parts.

Steps in learning

	Guidance	Representations
2:1	This teaching point explores the relative size of parts of the same whole and introduces the language of *'equal parts'* and *'unequal parts'*. Understanding this concept will be crucial when children learn about fraction notation in segment *3.2 Unit fractions: identifying, representing and comparing*. When multiplication was introduced in Year 2, children spent time identifying equal and unequal groups before meeting the concept of multiplication; being able to recognise equal groups is of course central to multiplication. Time spent here on equal groups will help link fractions, multiplication and division later on. Distribute a range of shapes cut out from paper (they should be relatively large – taking up most of an A4 sheet is ideal). Shapes could include a square, a circle, an equilateral triangle and a (non-square) rhombus. Ask children to cut their shapes into a number of parts (up to five), colour each part and then stick the parts back together to form the original whole. This provides the opportunity to focus on parts (through cutting the whole into parts) and then the whole again (through bringing the parts together again to form the whole). Draw attention to the fact that: *'A whole is made of many parts; many parts make one whole.'* Repeat this with the class. Next, ask the children to work in small groups and group the shapes in a way of their choosing. Prompt discussion	

	with questions such as: • *'What is the same within each group?'* • *'What is different between the groups?'* Note that in the next step you should draw particular attention to sorting the shapes into equal and unequal parts.	
2:2	Choose a group that have sorted the shapes according to whether they are divided into equal parts or unequal parts, and stick their shapes somewhere for the class to see. If children do not naturally come up with this criterion for grouping the shapes, provide your own groupings and ask the prompt question *'Why have these shapes been grouped in this way?'*. Once children have grasped the idea of equal parts and unequal parts, look at individual shapes and agree in turn if each is divided into equal parts or unequal parts, making sure the children are confident with this language. Choose an interesting range of shapes to discuss. This might include: • squares divided into four equal parts in different ways • an equilateral triangle divided into equal parts (it is unlikely that children will have produced this, so have one prepared in advance) • shapes divided into parts of equal width but where the parts are not of equal size. At this stage, focus on equal parts that are the same shape (congruent). If children show non-congruent parts (parts that are different shapes) which may have the same area, this should not be explored in detail until segment *3.2. Unit fractions: identifying, representing and comparing*. Acknowledge the similarity to any children that notice it, and explain you will return to this in future lessons.	 Squares divided into four equal parts in different ways:  An equilateral triangle divided into equal parts:  Shapes divided into parts of equal width but where the parts are not of equal size: 

	Children do not need to be able to use the word 'congruent' at this stage, but should understand the concept of the parts looking the same.	
2:3	After discussion, give children the opportunity to explore ways to divide given shapes into a different number of equal and unequal parts. For example, give them several copies of the same shape and ask them to draw on them to divide them into equal and unequal parts in different ways. They should then label them appropriately. When children write the number of equal parts of a whole as a fraction, it will be important for them to be able to identify the number of equal parts that the whole has been divided into. Encourage children to start doing this at this stage. They could use the following stem sentence to help them: '***The whole has been divided into _____ equal/unequal parts.***' Deepen children's understanding using a dòng nǎo jīn problem: '*Do you agree or disagree with the following statement? It is only possible to divide a whole into equal parts using straight lines.*'. Give children the opportunity to explore this statement. You could give them some example shapes and challenge them to divide them into equal parts using curved lines. Alternatively, ask them to predict whether the parts of a whole that have been separated using curved lines are equal or unequal. They could make these predictions and then cut them out to see if they are correct or incorrect.	Some examples of shapes divided into equal parts without the use of single straight lines: 

2:4

As in *Teaching point 1*, move on from an area model to a linear model so that children are exposed to different contexts in which wholes are divided into equal and unequal parts.

Give the children strips of paper (these can be of a variety of lengths), and ask them to fold the strips in different ways to make either equal or unequal parts and mark the folds with pen. In some cases, children may need to measure the length of the parts to check if they are of equal length or not. They can use the following stem sentence to summarise the number of parts they have made and whether they are equal or unequal: ***'I have folded my whole length of paper into ___ equal/ unequal parts.'***

Ask several children to come to the front with their paper strips and ask the rest of the class questions such as:

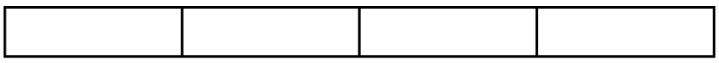
- *'Who has folded their strip into eight equal parts?'*
- *'Who has folded their strip into two unequal parts?'*

The children can also ask each other questions like this.

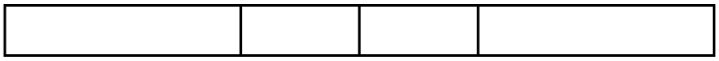
For a more abstract exploration of this concept, show children a line with divisions marked and ask them if the parts are of equal size. Discuss how we can check whether the parts are of equal size, then give the lengths of each part to allow a definitive answer to be established.

Once children are confident with identifying equal and unequal parts, present a dòng nǎo jīn problem like the one shown opposite.

Folding paper:



- *'I have folded my whole length of paper into four equal parts.'*



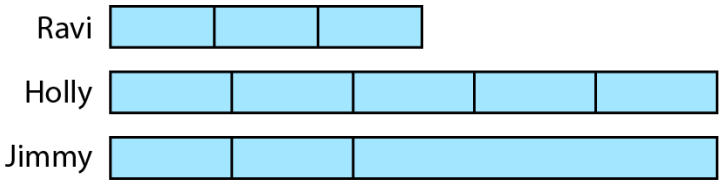
- *'I have folded my whole length of paper into four unequal parts.'*

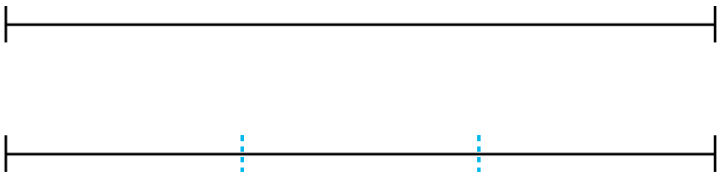
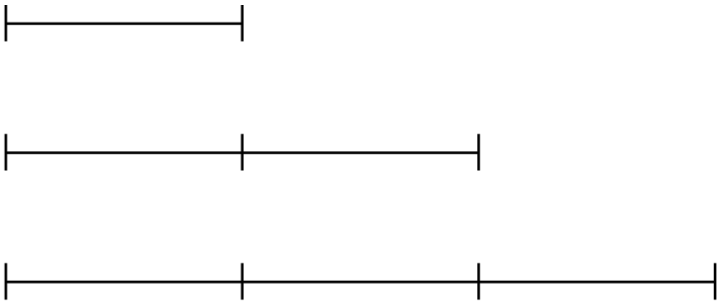
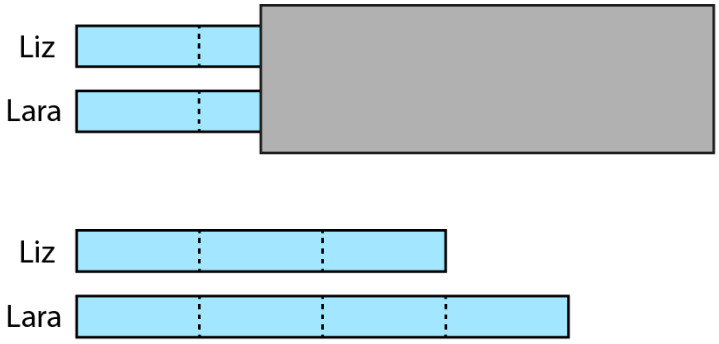
Lines with divisions marked:

'Are the parts of equal size?'

Dòng nǎo jīn:

'Three children have folded their paper strips in different ways. Which strip is the odd one out? Can you find a way to make each of the strips the odd one out?'



2:5	Now ask children to construct a line divided into three equal parts, and discuss the different ways in which this can be done. One way is to draw a line and then divide this into three parts (the length of this line should be a multiple of 3 cm so that decimal lengths are not required for each part; if appropriate, provide a suitable line for children). A second way is to draw one part, measure it and then draw two more parts of the same length. Do not expect many (if any) children to come up with this strategy as they are more used to dividing a whole into its parts than making a whole from its constituent parts. This is the first time that children have met this critically important point: that if we know the size of each part, and the number of parts, we can replicate these parts to make a whole. This is a concept that children are very familiar with in multiplication, but work on fractions tends to focus on moving from the whole to the part. Knowing the number of parts in a whole makes it equally possible to move from a known part to the whole. Early exposure to this concept will support children later on, with understanding how fractions link to both multiplication and division. This will be covered in greater depth in *Teaching point 4*. You can explore this concept further using a dòng nǎo jīn problem like the one shown opposite. Allow children to discuss their ideas before revealing the answer.	Dividing a given line into three:  Measuring one part and drawing two more:  Dòng nǎo jīn: *'Liz has folded her paper strip into three equal parts. Lara has folded her paper strip into four equal parts. Part of their strips are hidden. Whose paper strip is longer?'* 

2:6

After area and length, move onto a third model: sets of objects. Children have already looked at equal and unequal groups of objects in *Spine 2: Multiplication and Division, segment 2.2 Structures: multiplication representing equal groups*, as identifying equal groups is, of course, essential to multiplication and division too. Exposing children to the same images that they encountered in segment *2.2*, will allow them to make connections to previous learning.

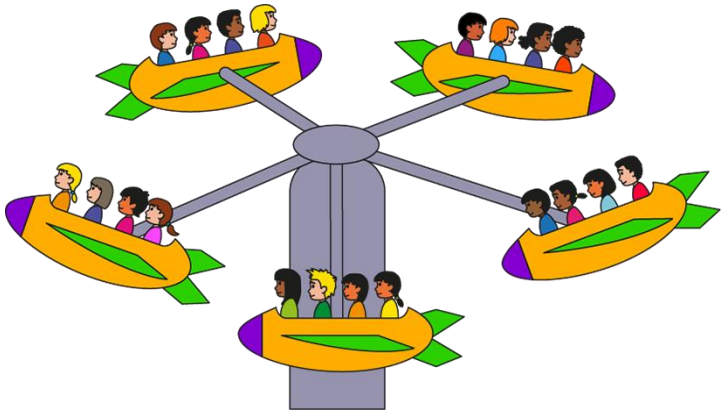
Look at pictures similar to the ones opposite and discuss whether the sets are divided into equal parts or not. This may be the first time that children are exposed to situations where the parts do not look the same, but the quantity within each part is equal. For example, children may not feel that the groups of four people are 'equal', but encourage them to focus on whether the number of people in each part is the same, using similar stem sentences to those used in segment *2.2*:

- ***'The parts are equal. I know this because the number of ____ in each part is the same.'***
- ***'The parts are unequal. I know this because the number of ____ in each part is not the same.'***

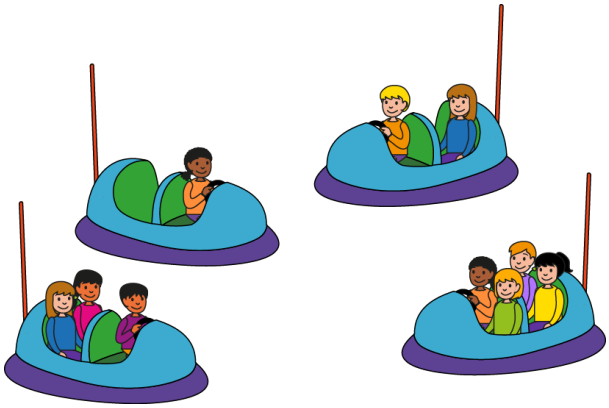
Include sets in which some but not all of the parts are equal in size. Children should understand we can only say a whole is divided into equal parts if all parts of the whole are of equal size. Discussing how a whole set is composed of equal and unequal parts will support children later on in identifying contexts which have both multiplicative and additive elements.

To promote depth of understanding, use a dòng nǎo jīn question that presents a set of objects divided into

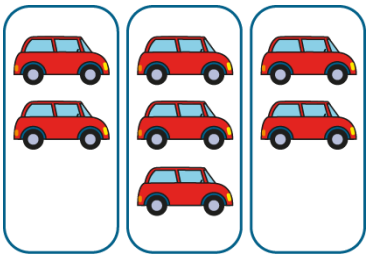
Equal and unequal parts:



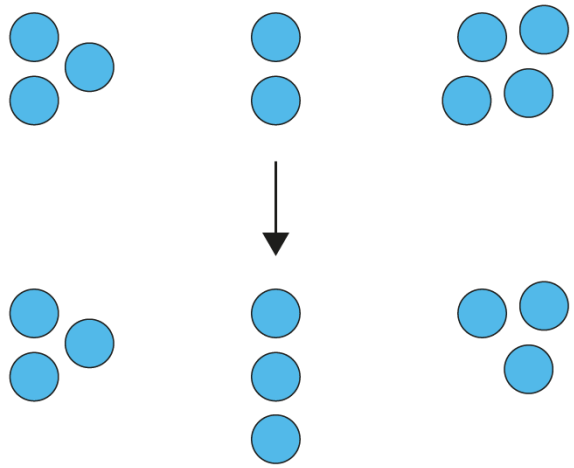
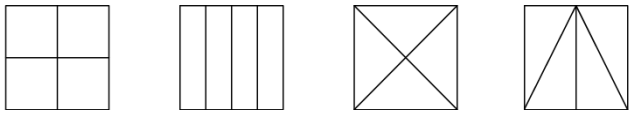
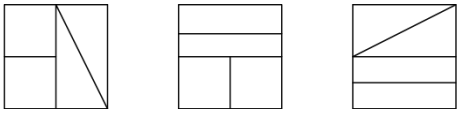
- *'The parts are equal. I know this because the number of people in each part is the same.'*

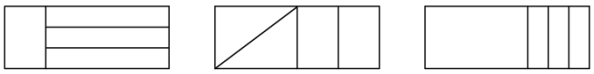


- *'The parts are unequal. I know this because the number of people in each part is not the same.'*



- *'The parts are unequal. I know this because the number of cars in each part is not the same.'*

	unequal parts. Ask children whether it is possible to rearrange the objects so that the parts become equal. Look for opportunities outside the maths lesson to highlight equal and unequal parts to the children, and encourage them to find these for themselves, for example getting into groups in PE, putting pencils into pots or cutting up fruit for a snack.	Dòng nǎo jīn: *'Can we rearrange the objects so the parts become equal?'* 
2:7	When children understand that groups or parts can be equal, even though they may appear to be different, expose them to more examples that challenge the idea that equal parts must look the same. Revisit the task from step *2:2*, in which children were presented with shapes divided into four parts and they had to determine whether the parts were equal or unequal. Show children four squares, each divided into four congruent parts and ask them to discuss whether they have been divided into equal or unequal parts. All children should be able to explain that all the square have been divided into equal parts. Now get children to focus on one part from each of the shapes, asking: • *'What is the same about each part?'* • *What is different?'* Children will most likely describe the parts as different, since they do not look the same. Help them understand that, because all of the wholes have	Squares divided into four equal parts in different ways: *'Has each square been divided into equal or unequal parts?'*  One part of each whole: • *'What is the same about each part?'* • *'What is different?'*  Possible combinations of parts: 

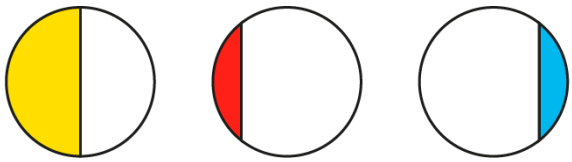
	been divided into four equal parts (do not refer to them as quarters at this stage) and because all of the wholes are the same size, all of the parts represent the same amount of the whole. If a child refers to each piece as a quarter of a whole, acknowledge that this is correct and that another way to say this is that it is one of four equal parts of the whole. Introduce the following generalisation: ***'Equal-sized parts do not have to look the same.'*** Children are more likely to understand this concept if they are given the opportunity to find different ways to combine the parts together to make the same original whole. Give children copies of the whole squares with the different parts drawn on them and ask them to cut out the parts and combine the parts of the different wholes together in different ways. Keep asking the children to justify how they know the whole is still divided into equal parts. Use the generalisation above in your own language, and prompt the children to use it as well. Deepen children's understanding of this concept further by using a dòng nǎo jīn question that exposes them to an unfamiliar situation with different shapes that they have not previously cut out and manipulated. Encourage children to reason about whether the shapes shown opposite have been divided into equal or unequal parts, and challenge them to try to convince their peers if necessary. For example, *'The parts of the third shape can not be equal because all the parts are the same width and one part is longer than the other three'*. If children describe the parts using their fractional names (for example, one-half, one-quarter), acknowledge that these	Dòng nǎo jīn: *'Has each of these shapes been divided into equal or unequal parts?'* 

	names are correct, but encourage children to focus on whether the parts are of equal size or not; the names of fractions will be introduced in segment *3.2 Unit fractions: identifying, representing and comparing*.	

Teaching point 3:

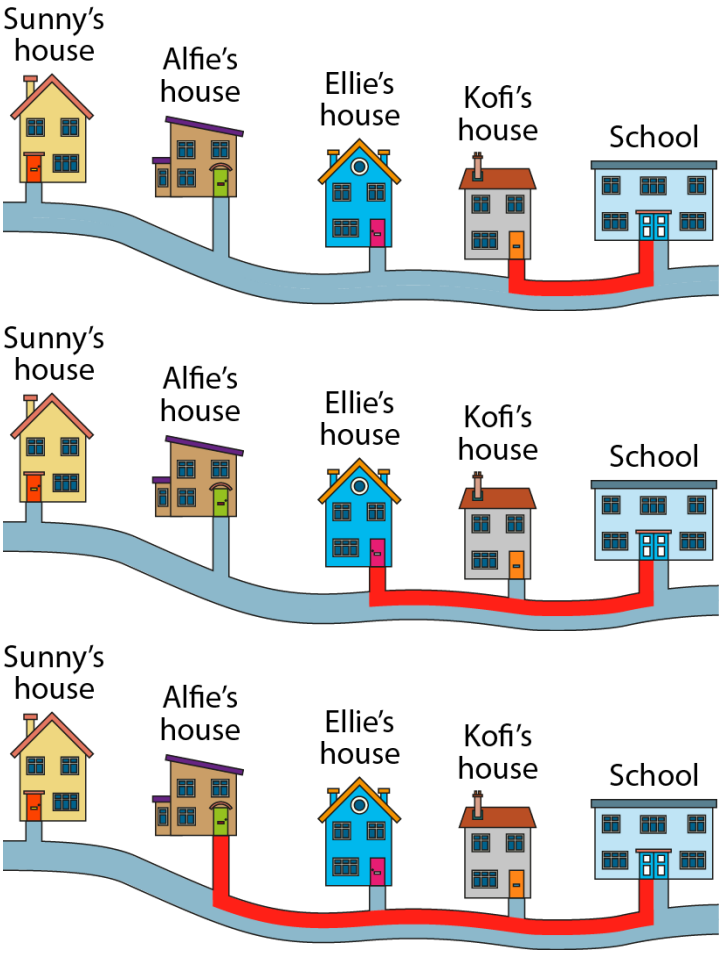
The relative size of parts can be compared.

Steps in learning

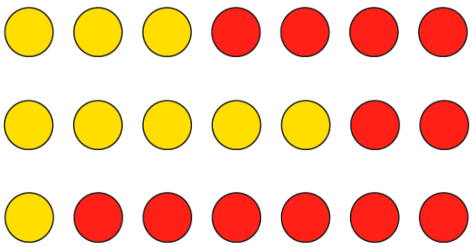
	Guidance	Representations
3:1	When children can confidently explain whether the parts within a whole are equal or unequal, they can begin to compare the parts of different wholes. Begin with wholes that are the same shape and size. Show children three wholes with different-sized parts shaded and ask: • *'What is the same?'* • *'What is different?'* Children should see that the wholes are the same size and quickly realise that we can compare the different highlighted parts. Using different colours for the shaded parts will make it easier for children to verbalise similarities and differences. Ask the children to make comparisons as follows: • *'The yellow part is a larger part of the whole circle than the red part.'* • *'The red part and the blue part are equal-sized parts of the whole circle.'* Children should begin to relate the size of each part to the amount (or fraction) of the whole that is shaded. For example, the yellow part is larger than the red part because more of the circle is shaded. Don't expect children to use the word 'fraction', as this has not yet been introduced to them.	• *'What is the same?'* • *'What is different?'*  • *'The yellow part is a larger part of the whole circle than the red part.'* • *'The red part and the blue part are equal-sized parts of the whole circle.'*

3:2	Give children the opportunity to explore this concept within different contexts. Returning to some familiar examples, such as the map of Europe and the linear journey, will help to embed this concept more fully and allow the children to generalise across contexts. Note the variation in the examples given opposite. The part in focus is not always at the beginning of the whole. Discuss observations such as the following: • *'Poland is a bigger part of Europe than Portugal is.'* • *'In the first set of counters, the yellow counters make up a smaller part of the whole than in the second set.'* When children can confidently make such observations, introduce the generalisation: ***'Different parts of the same-sized whole can be directly compared based on their size.'***	Different-sized parts within equal wholes: • Area context  

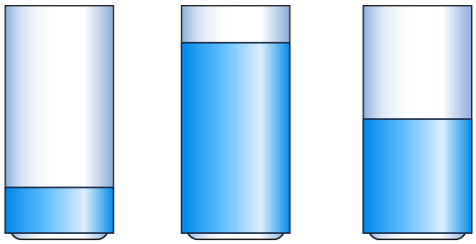
- Length context

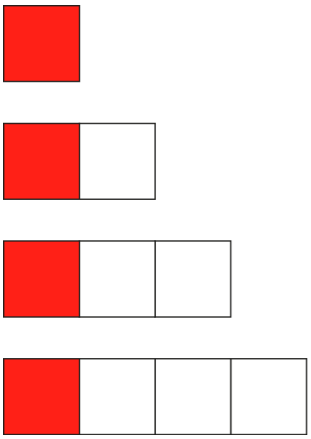


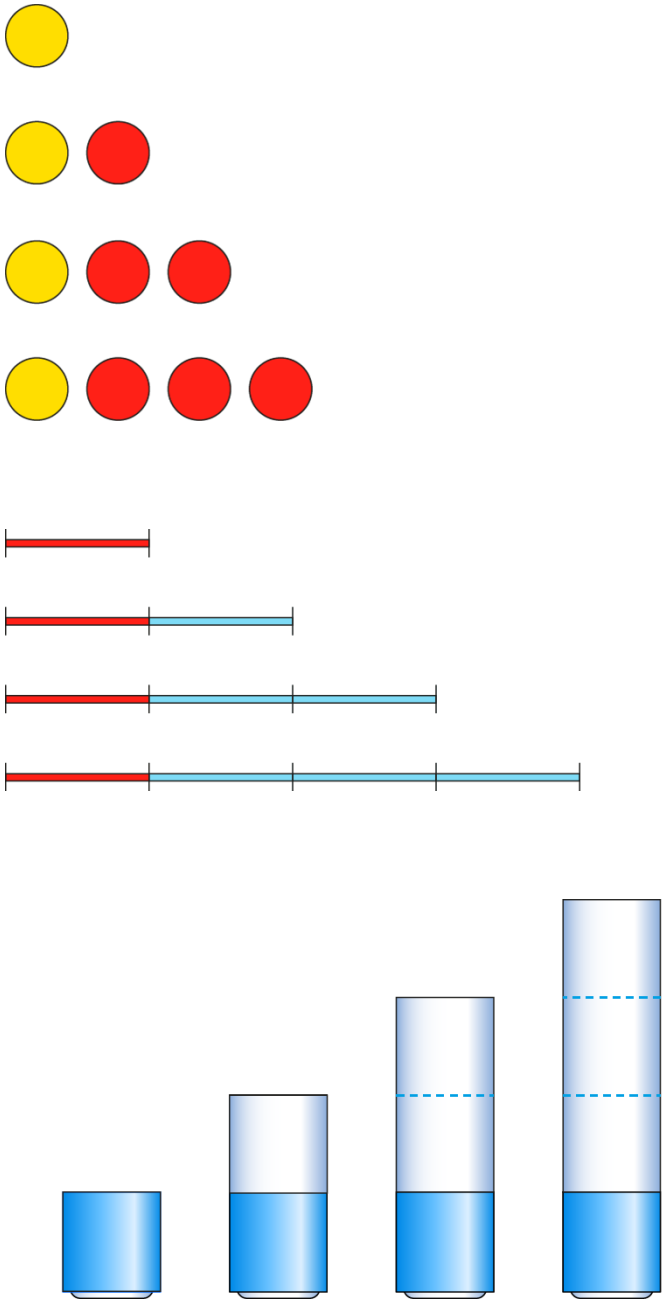
- Sets context

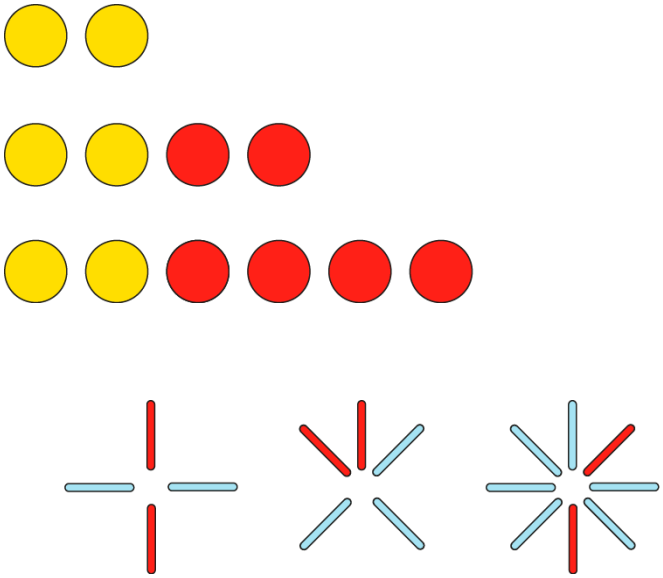
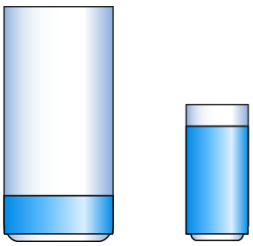
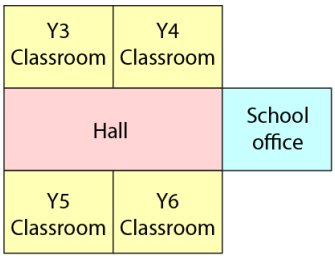


- Quantity context



3:3	Next, compare equal-sized parts of different-sized wholes. Show children images positioned vertically above each other, as shown opposite, to allow them to easily see that the parts are the same size, but that the whole is becoming larger in each situation. Ask: • *'What is the same?'* • *'What is different?'* as you reveal each image in turn. Children should see that in each image one part has been shaded, that the parts are of equal size and that the wholes are becoming progressively larger. A difficult but important point, is that as the whole increases in size and the size of the shaded part remains the same, the amount (or fraction) of the whole that is shaded is becoming smaller each time. Again, don't expect children to use the word 'fraction', but encourage them to compare the size of the part of the whole that is shaded to the (increasing) size of the whole. Help them come to this realisation by giving them two different opinions and asking them who they agree with: • *'Basil says "One part is shaded in each image. This means the same amount of each whole is shaded".'* • *'Jess says "Each whole is one part larger than the previous whole. Only one part of each whole is shaded. This means that each time a smaller amount of the whole is shaded".'* • *'Who is correct?'*	Equal-sized parts of different-sized wholes: • *'What is the same?'* • *'What is different?'* 

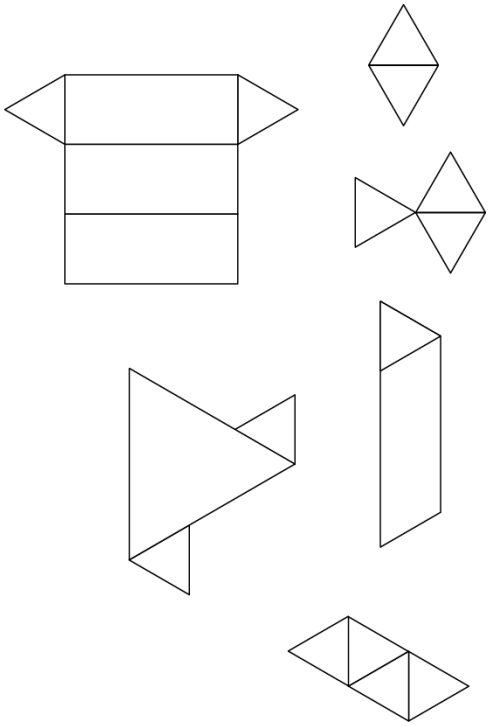
3:4	Continue to compare equal-sized parts of different-sized wholes, using a range of different contexts. For example, an increasing number of coloured counters, an increasing length of string, and progressively taller glasses containing the same amount of liquid. Again, ask: • *'What is the same?'* • *'What is different?'* and encourage children to compare the size of the identified part with the size of the whole. Try to move beyond quantitative language (such as *'First the yellow was one of one part. Then it was one of two equal parts. Then it was one of three equal parts.'*) to more relational language (statements like *'Each time the yellow counter becomes a smaller part of the whole.'*). As children become more confident with the concept, introduce the generalisation: ***'As the whole increases in size and the size of the selected part remains the same, each part becomes smaller in relation to the whole.'*** This will develop children's understanding of fractions where the part is being viewed as a proportional relationship to the whole. This is a very different idea from an additive relationship. In an additive relationship, three is always three whether it is part of five or seven, whereas the value of a fractional part will change if the size of the whole increases.	Equal-sized parts of different-sized wholes – different contexts: • *'What is the same?'* • *'What is different?'* 

3:5	Introduce further variation by showing children images where more than one part of the whole is highlighted each time, but this number remains the same as the whole increases in size. The location of the consistent parts could also change in each representation, to add variation and challenge if appropriate. In all cases, reinforce the generalisation: ***'As the whole increases in size and the size of the selected part remains the same, each part becomes smaller in relation to the whole.'***	Equal-sized parts of different-sized wholes – further variation: • *'What is the same?'* • *'What is different?'* 
3:6	Finally, show the children pairs of images where either a large part of the whole or a small part of the whole is highlighted, varying both the size of the part and the size of the whole. Use various familiar contexts, such as liquid in glasses (for example, a nearly empty large glass and a nearly full small glass) and maps (for example, a school plan with the classrooms – most of the rooms – shaded, and a map of Europe with Italy – a relatively small area – shaded). Using informal language, ask the children to make judgements about whether a large or small part of the whole is identified in each case. The key issue here is to ensure children consider both the part and the whole in each image, and make a judgement about *the relative size of these to each other*. As mentioned in the *Overview of learning*, the size of the part in relation to the whole is critical in determining the value of a fraction, so it is important that children consider this relationship at an early stage.	 • *'The large glass is nearly empty. A smaller part of the glass has water in it.'* • *'The small glass is nearly full. A larger part of the glass has water in it.'*   • *'The classrooms make up quite a large part of the whole school building.'* • *'Italy is quite a small part of the whole (Europe).'*

Teaching point 4:

If one of the equal parts and the number of equal parts are known, these can be used to construct the whole.

Steps in learning

	Guidance	Representations
4:1	In step *2:5* children encountered the idea that, given a part and the number of parts, we can construct the whole. This concept is explored explicitly in this teaching point. Children need to understand that the size of the part, the number of parts, and whether the parts are of equal size, must be identified before this is possible. Understanding this concept lays the foundations for understanding the inverse relationship. Begin by showing children a shape and asking them to draw the whole that this part is from. The children are all likely to produce different wholes. Ask: *'What information do we need so that we all draw the same whole?'* Children will probably realise that they need to be told how many parts there are, but may not identify the need to know that all parts are the same shape and size. Show them representations that contain different combinations of parts to help them understand these important pieces of information.	*'What is the whole?'*  one part • *'The wholes might be…'* 
4:2	Using the same part as in the previous step, tell the children that the whole has been divided into three equal parts (now providing these critical pieces of information that were lacking before). Again, ask them to draw the whole that this part is from. You could also ask them to make the whole using a practical resource, like pattern blocks or paper cut-outs of pattern-block shapes. Since the given part is an equilateral triangle, and the whole is made of three equal parts, there is only one possible whole.	• *'What is the whole?'*  one part of a whole made out of three equal parts

Now present children with a rhombus. Again, tell them that this is one part of a whole that has been divided into three equal parts. This time the wholes that the children create will be different in appearance, but they will take up the same space (children will come to learn that they have the same area, but haven't met this term yet). Discuss this by showing examples of children's work or the examples given opposite, asking:

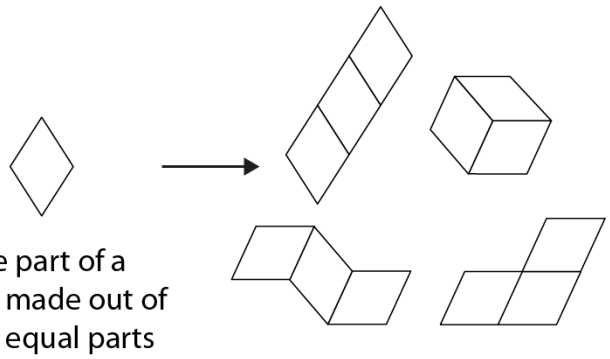
- *'What is the same?'*
- *'What is different?'*

Note that using a shape such as a rhombus as the part, produces wholes that don't necessarily look like 'completed', or recognisable, wholes. Ask children to explain why the shape they have drawn or created does represent the whole; that is, '*If the whole has been divided into three equal parts, then three of those parts make one whole.*'

To further explore the idea that a whole may not look like a 'complete' shape, use dòng nǎo jīn problems like the one shown opposite. Encourage children to explain why each shape is correct or incorrect:

- Max – incorrect, wrong number of parts
- Beth – correct, right number and shape of parts
- Ellen – correct, right number and shape of parts
- Florent – incorrect, right number of parts, wrong shaped parts.

- *'What is the whole?'*



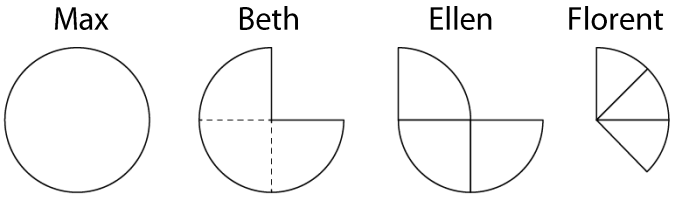
one part of a whole made out of three equal parts

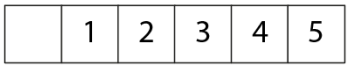
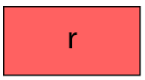
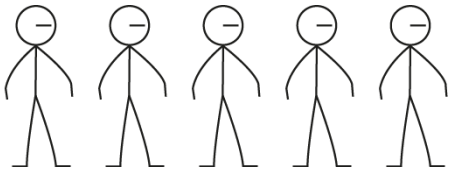
- *'What is the same?'*
- *'What is different?'*

Dòng nǎo jīn:

'Four children have been given a part of a shape and have been asked to draw the whole shape. Who has drawn a correct shape? Why?'

one part of a whole made out of three equal parts



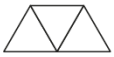
4:3	Give children further practice in constructing wholes with different numbers of equal parts. When they are asked to make a whole with a larger number of parts, children may miscount the number of parts they have used or think that the number of parts the whole has been divided into means drawing this many additional parts. To counter this misconception, ask questions such as *'Can you show me the five equal parts?'*.	*'What is the whole?'* one of five parts ✘  ✔ 
4:4	When children are confident drawing a whole shape given one part, give them the opportunity to explore the concept of creating wholes from a known part in different contexts, including linear contexts and sets or quantities. Continue to ask them to explain how they know their whole is correct, using the language of equal parts within a whole.	Constructing a whole – linear context: • *'I have cut a ribbon into five equal parts. Here is one part of my ribbon. How long was my ribbon to start with? Draw the whole ribbon.'*  • *'Here is a red Cuisenaire® rod. It is one of four equal parts of another rod. Which rod is that?'*  Constructing a whole – cardinal context: *'The class has been divided into four equal teams. Here is one of the teams.'*  *'How many children are in the class? Draw the whole class.'*

4:5

Finally, summarise these contexts, and any others you have used, in a table. Throughout this segment we have divided a whole into a given number of equal parts. Here we have also seen that, given one part and the number of equal parts, we can construct the whole.

Discuss the table with the class, showing that we can move both from part to whole and from whole to part. The following sentences will support the children in describing this two-way relationship. For example, for the triangle you might say:

- *'This is the whole. The whole is divided into three equal parts. This is one of the parts.'* (Moving from the right-hand column, via the middle column, to the left-hand column.)
- *'This is a part. There are three of them. This is the whole.'* (Moving from the left-hand column, via the middle column, to the right-hand column.)

Part	Number of equal parts	Whole
	3	
	5	
	4