MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.7 Addition and subtraction: strategies within 10

Teacher guide | Year 1

Teaching point 1:

Addition is commutative: when the order of the addends is changed, the sum remains the same.

Teaching point 2:

Ten can be partitioned into pairs of numbers that sum to ten. Recall of these pairs of numbers supports calculation.

Teaching point 3:

Adding one gives one more; subtracting one gives one less.

Teaching point 4:

Consecutive numbers have a difference of one; we can use this to solve subtraction equations where the subtrahend is one less than the minuend.

Teaching point 5:

Adding two to an odd number gives the next odd number; adding two to an even number gives the next even number. Subtracting two from an odd number gives the previous odd number; subtracting two from an even number gives the previous even number.

Teaching point 6:

Consecutive odd / consecutive even numbers have a difference of two; we can use this to solve subtraction equations where the subtrahend is two less than the minuend.

Teaching point 7:

When zero is added to a number, the number remains unchanged; when zero is subtracted from a number, the number remains unchanged.

Teaching point 8:

Subtracting a number from itself gives a difference of zero.

Teaching point 9:

Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Memorised doubles/halves can be used to calculate near-doubles/halves.

Teaching point 10:

Addition and subtraction facts for the pairs five and three, and six and three, can be related to known facts and strategies.

Overview of learning

In this segment children will:

- explore structures underlying addition and subtraction facts within ten
- build fluency with these facts
- make connections between real-life contexts, involving quantities to ten, and the expressions and equations which can be used to represent them.

This segment focuses on developing fluency in addition and subtraction facts within ten, and relating them to everyday situations around us. Just as multiplication tables are the basis of all other multiplicative calculation, the defined set of facts shown in the addition grid below are the building blocks for all additive calculation, both mental and written. Strategies for addition/subtraction within ten are covered in this segment; calculations which bridge ten will be discussed in later segments.

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

The highlighting in the grid is intended to draw attention to facts which can be derived using the same dominant strategy, however many of the facts can be derived using multiple strategies. Only an addition fact grid is shown (a subtraction fact grid is not included), as children should be encouraged to use known addition facts to quickly derive subtraction facts. A part–part–whole representation (cherry or bar model) can be very useful in securing these related facts.

Although children will be developing fluency in partitioning all numbers to ten, using the strategies in this segment allows children not to have to rely solely on memorisation, as well as providing them with a much deeper understanding of the number facts and how the numbers to ten relate to one another. The aim is for all children to either have memorised, or be able to quickly derive, each fact, such that they no longer rely on counting-based approaches for calculation. However, additional practice outside the main maths lesson will be required for children to become fully fluent in the facts. Accessing the rest of the segments in the spine is dependent on fluency in these facts, so assess all children closely to identify any who require additional provision.

Varied practice has been suggested incrementally, as each new point is introduced, but further practice combining each new point with all previous points is recommended. Similarly, at the end of the segment, practice should cover *all* of the facts.

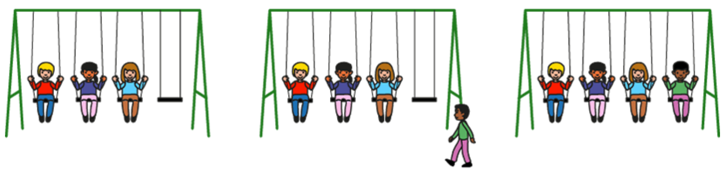
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks

Teaching point 1:

Addition is commutative: when the order of the addends is changed, the sum remains the same.

Steps in learning

	Guidance	Representations
1:1	In segment *1.5 Additive structures: introduction to aggregation and partitioning*, children practised writing aggregation expressions with the two addends written in either order (e.g. $2 + 4$ and $4 + 2$). In this segment, we formalise the commutative law of addition. Describe an aggregation context to the children, for example *'There are two adult cats and four kittens.'* As before (segment *1.5*), ask them to write an equation to show how many cats there are altogether. Discuss how we can write the addends in either order, but we still have six cats altogether. Present a range of contexts, remembering to encourage children to vary the position of the equals sign, for example $6 = 2 + 4$ versus $2 + 4 = 6$. Vary the position of the equals sign *between* contexts but not *within* a context. For example, including $6 = 4 + 2$ and $6 = 2 + 4$ when presenting the cat context could draw attention away from the change in order of the addends, but in a new context *('There are three red hats and two blue hats.')* you could place the equals sign at the start ($5 = 3 + 2$ and $5 = 2 + 3$). Work towards the generalised statement: ***'If we change the order of the addends, the sum remains the same.'***	*'Write an equation to show how many cats there are altogether.'*  $2 + 4 = 6$ $4 + 2 = 6$ $2 + 4 = 4 + 2$ *'Two plus four is equal to four plus two.'*
1:2	Now look at the augmentation structure, using concrete and pictorial examples. The predefined order of the ***'first…, then…, now…'*** structure can	Concrete: *'At first there are two children in the book corner. Then five more children go to the book corner. Now there are seven children in the book corner.'*

	make it more difficult to see that commutativity applies here; the same sum is given by each of these sentences:
	• *'At first there is one child on the swings and then three more arrive.'*
	• *'At first there are three children on the swings and then one more arrives.'*
	Work through the following:
	• Show children the pictures for the first story and tell the story together. Then write the corresponding equation:
	$1 + 3 = 4$
	• Now ask children to think about what the sum would be if the children arrived in the other order. Again, show the pictures and tell the story, before writing the equation for this situation:
	$3 + 1 = 4$
	• Compare the two sets of pictures and equations:
	  ◦ Write
	  $1 + 3 = 3 + 1$
	  ◦ Say, *'One plus three is equal to three plus one.'*
	Continue to use the generalised statement from step *1:1*.
		$7 = 2 + 5$
		
		'At first there are five children in the book corner. Then two more children go to the book corner. Now there are seven children in the book corner.'
		$7 = 5 + 2$
		
		$2 + 5 = 5 + 2$
		'Two plus five is equal to five plus two.'
		
		Pictorial:
		First Then Now
		
		1 +3 4
		$1 + 3 = 4$
		First Then Now
		
		3 +1 4
		$3 + 1 = 4$
		$1 + 3 = 3 + 1$
		'One plus three is equal to three plus one.'

1:3	In the aggregation and augmentation contexts described in steps *1:1* and *1:2*, the individual objects, or 'ones', within the set are visible; this helps children to see the equivalence of the two expressions. Now present measures contexts where the 'ones' can't be seen as easily, for example:	

- increasing the length of a fence
- adding a quantity of money to a purse.

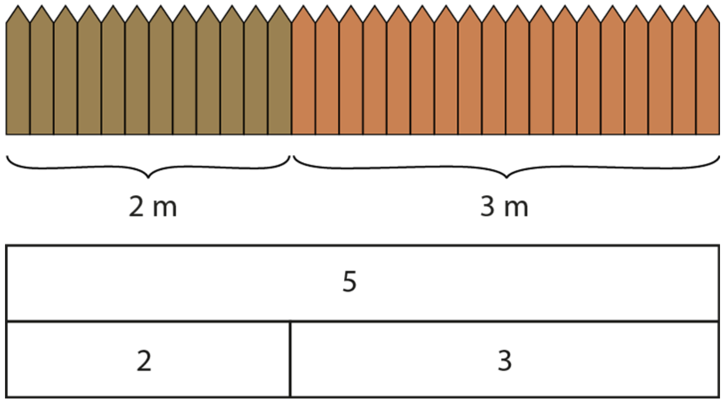
Some children may be able to confidently calculate and say, for example, *'The fences will be the same length because two plus three equals five and three plus two equals five.'* This is true, but specific to this situation; encourage children to generalise by asking what they have noticed about the addends, and what they know so far about changing the order of the addends.

Again, encourage use of the generalised statement from step *1:1*, and make sure children are able to read and write equations of the form *4 + 5 = 5 + 4*.

Augmentation measures context:

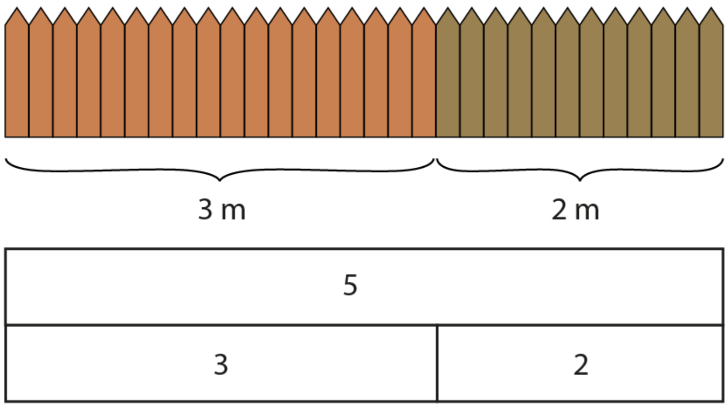
'Which fence ends up the longest?'

- *'This fence was two metres long; then the caretaker made it three metres longer.'*



2 + 3 = 5

- *'This fence was three metres long; then the caretaker made it two metres longer.'*



3 + 2 = 5

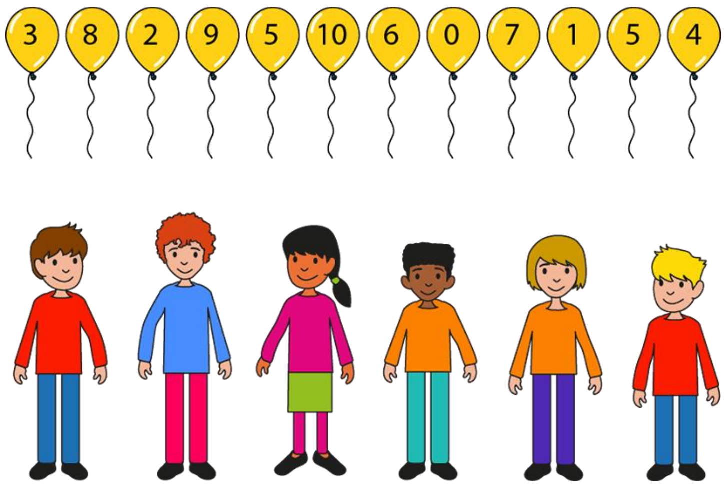
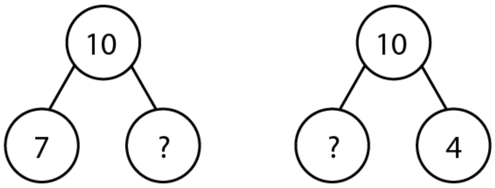
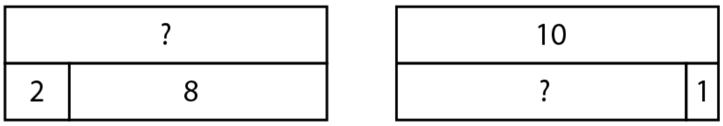
		Aggregation measures context: *'Lola has four pounds in her purse and another five pounds. Finn has five pounds in his purse and another four pounds.* *Who has more money?'* Lola  4 + 5 = 9 Finn  5 + 4 = 9
1:4	Once children have explored the different kinds of contexts described in steps *1:1* to *1:3*, discuss the fact that the generalised statement is *always* true and introduce the term 'commutative'. **Commutative law of addition: *'We can change the order of the addends and the sum remains the same.'*** Emphasise that whatever the value of the addends, however big or small, we can change their order and the sum remains the same. Say the generalised statement together until children are able to say it on their own.	
1:5	Review some of the equations of the form $a + b = b + a$ that children have already seen in this teaching point. Emphasise that although these have more than one number on both sides, they are still equations. Provide children with practice completing equations, looking out for the following type of error in children's work:	*'Fill in the missing numbers.'* $1 + 6 = \square + 1$ $4 + 5 = 5 + \square$ $6 + \square = 3 + 6$

	1 + 6 = [7] + 1 ✗ This occurs when children are interpreting '=' to mean 'the answer is'. It is important to address this misconception clearly. Use balances to show that both sides of the equation need to be equivalent. You could end with true or false questions as shown opposite.	*'True or false?'* 2 + 1 = 1 + 2 3 + 5 = 8 + 3 3 + 4 = 3 + 4 4 + 5 = 4 + 4 2 + 2 = 2 + 2

Teaching point 2:

Ten can be partitioned into pairs of numbers that sum to ten. Recall of these pairs of numbers supports calculation.

Steps in learning

	Guidance	Representations
2:1	From their work in segments *1.3 Composition of numbers: 0–5* and *1.4 Composition of numbers: 6–10*, as well as additional fluency work outside of the main maths lesson, children will be developing fluency in identifying pairs of numbers which make ten. However, children don't always recognise these pairs in written addition or subtraction contexts. For example, a child may quickly be able to answer *'three'* when asked *'What do I add to seven to make ten?'*, or complete a part–part–whole diagram, but may not immediately recognise that *7 + 3* is equal to ten; they resort to other strategies to try to 'calculate' the answer, without drawing on the known number fact. This teaching point reinforces the link between known number facts and addition/subtraction problems. Start by asking children to find pairs of numbers that make ten, referring back to *Teaching point 4* of segment *1.4*. Use classroom games to build fluency, and practise representing the pairs with part–part–whole diagrams (cherry or bar model). Identify children who aren't yet fluent and: • provide additional practice until fluency is achieved • display the part–part–whole diagrams for ten on the classroom wall for them to refer to. Children can look out for their own errors by drawing on the pattern	*'Give each child a pair of balloons which sum to ten.'*  *'Fill in the missing numbers.'*  

<table>
<tr><td></td><td>explored in segment 1.4: ten is an even number, so it can be partitioned into either two even numbers or two odd numbers. Use the generalised statement: 'Even numbers can be partitioned into two odd parts or two even parts.'</td><td></td></tr>
<tr><td>2:2</td><td>Once children are confident in partitioning ten, and finding missing parts, move on to teaching them to quickly identify pairs of numbers that sum to ten.
Begin by playing a game of 'Snap', using two sets of 0 to 10 number cards: children say 'Snap!' when pairs that sum to ten are revealed.
Then move on to working with written addition expressions; children can sort expressions into those that sum to ten and those that don't sum to ten.
Throughout, do not ask children to calculate the sum for expressions which don't equal ten; the purpose of the step is for children to be able to identify, at a glance, whether two numbers have a sum of ten or not.

To provide further challenge, use dòng nǎo jīn questions in which children must use their knowledge of number bonds to ten to decide whether two given numbers have a sum greater or less than ten, for example:

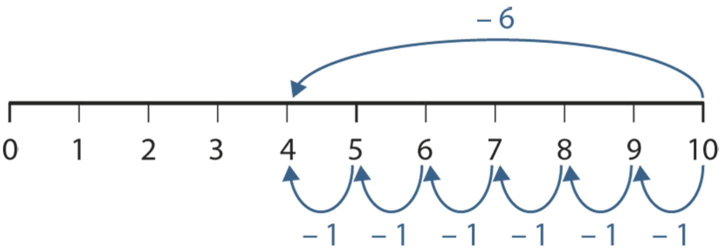
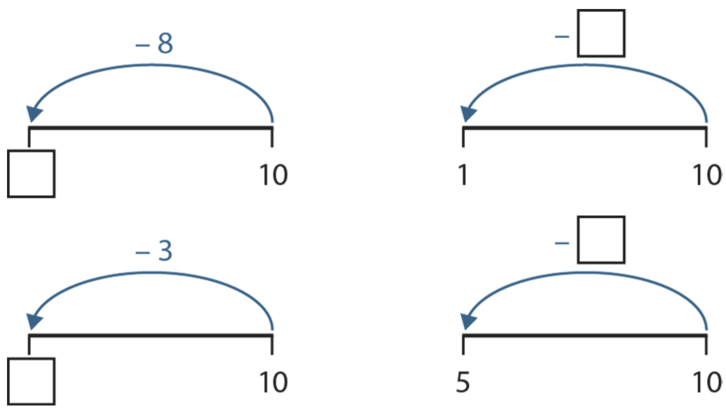
$7 + 3 = 10$

so $7 + 4 > 10$

Draw attention to the fact that we can answer the question without having to calculate the sum.</td><td>'Sort the expressions.'

$8 + 2$
$7 + 6$
$1 + 0$
$4 + 4$
$5 + 5$
$0 + 10$

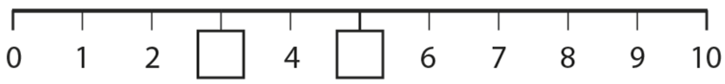
<table><tr><th>Sum to ten</th><th>Do not sum to ten</th></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>2:3</td><td>Now move on to subtraction from ten; children can again draw on their partitioning knowledge from segment 1.4 Composition of numbers: 6–10.</td><td></td></tr>
</table>

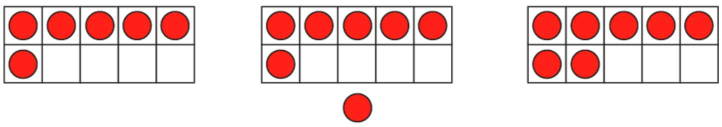
	Include both partitioning and reduction contexts, using the part–part–whole model, pictorial representations and other representations, such as the number line. Emphasise that we don't need to 'count back' to subtract from ten – we can just use our knowledge of pairs that make ten to perform the subtraction in one step.	Missing difference problem with number line:  10 – 6 = ☐
2:4	Provide missing difference and missing subtrahend problems for practice.	*'Fill in the missing numbers.'* 10 – 4 = ☐ 10 – ☐ = 6 10 – 5 = ☐ 10 – ☐ = 7 10 – 6 = ☐ 10 – ☐ = 8 10 – 7 = ☐ 10 – ☐ = 9 10 – 1 = ☐ 10 – ☐ = 2 10 – 8 = ☐ 10 – ☐ = 10 10 – 3 = ☐ 10 – ☐ = 4 10 – 0 = ☐ 10 – ☐ = 0 

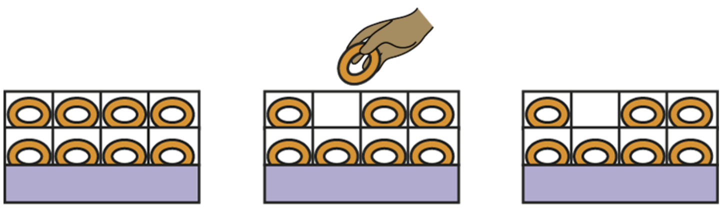
Teaching point 3:

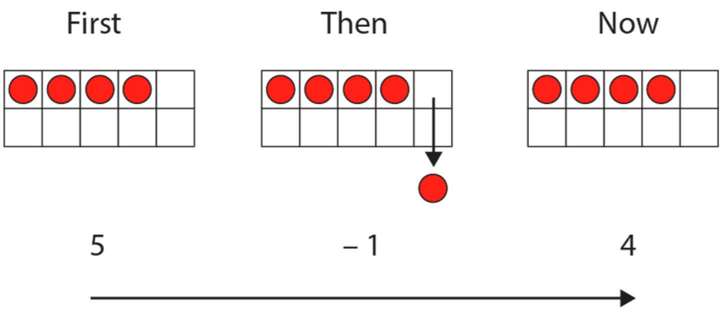
Adding one gives one more; subtracting one gives one less.

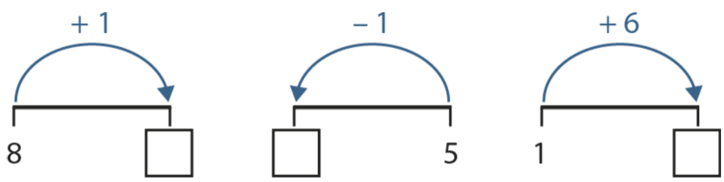
Steps in learning

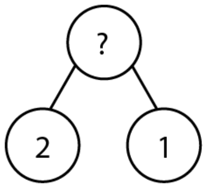
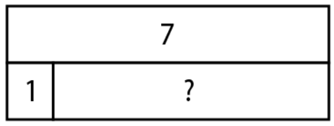
	Guidance	Representations
3:1	This teaching point builds on previous work on 'one more' and 'one less' in segments *1.3 Composition of numbers: 0–5* and *1.4 Composition of numbers: 6–10*. Children will progress to formalising this as addition or subtraction of one. Begin by reviewing counting in ones forwards and backwards between zero and ten, tapping a number line or as the children count. Then review 'one more' and 'one less'. Start by asking children, for example, *'What is one more than four?'* and *'What is one less than four?'*, before presenting missing numbers on the number line and missing number sequences. Use the stem sentences: • ***'One more than ___ is ___.'*** • ***'One less than ___ is ___.'*** Practise until children are fluent in completing such problems. Initially children may need to 'silently count' to work out what 'one more' is; for example, to find one more than five, they count *'One, two, three, four, five, six.'* In order to add or subtract one confidently (steps *3:2* and *3:3*), children will need to move beyond this, identifying one more or one less for the numbers to ten *without* counting.	*'Fill in the missing numbers.'*  one more 7 → ☐ one less ☐ ← 7 • *'One more than seven is eight.'* • *'One less than seven is six.'*
3:2	Now progress to writing and solving equations with an addend of one. Begin with augmentation contexts, since these more readily relate to 'one more' and 'one less'. Use both concrete/pictorial representations	Augmentation context – pictorial representation: *'First there were three flowers in the vase. Then one more flower was added to the vase. Now there are four flowers in the vase.'*

	(such as the flowers story opposite) and generalised representations (for example, counters on tens frames). Ask the children to write the equation as they tell each story, and draw attention to the fact that the sum is always one more than the starting addend. Then start to include aggregation contexts, for example, *'There are three footballs and one basketball. There are four balls altogether.'* Work towards use of the generalised statement: ***'Adding one gives one more.'***	First Then Now  3 + 1 4 3 + 1 = 4 Augmentation context – generalised representation: *'First we had six. Then we added one. Now we have seven.'* First Then Now  6 + 1 7 6 + 1 = 7
3:3	Now move on to equations where the first addend is one. Show a pair of equations with the second sum missing, as in the example shown opposite. Challenge children to solve the second equation by using their knowledge from *Teaching point 1*. Ask, *'What happens when we change the order of the addends?'* Prompt children to use the language from *Teaching point 1: 'If we change the order of the addends, the sum remains the same.'* Once the children have 'predicted' what the sum will be, demonstrate using an augmentation or aggregation context that this is indeed correct.	3 + 1 = 4 so: 1 + 3 = ☐
3:4	Now that the structure has been 'unpicked', and a generalisation made, present the series of missing number	

	problems shown opposite. By now children should have the necessary skills to complete the number sentences with only the generalised statement as a scaffold (rather than needing support in the form of concrete or pictorial representations). Generalised statement: ***'Adding one gives one more.'*** Note that, although the equations including zero follow the same rules, children can find these more challenging; putting these at the end of the sequence will support children in solving them.	*'Fill in the missing numbers.'* 9 + 1 = ☐ 1 + 9 = ☐ 8 + 1 = ☐ 1 + 8 = ☐ 7 + 1 = ☐ 1 + 7 = ☐ 6 + 1 = ☐ 1 + 6 = ☐ 5 + 1 = ☐ 1 + 5 = ☐ 4 + 1 = ☐ 1 + 4 = ☐ 3 + 1 = ☐ 1 + 3 = ☐ 2 + 1 = ☐ 1 + 2 = ☐ 1 + 1 = ☐ 1 + 1 = ☐ 0 + 1 = ☐ 1 + 0 = ☐
3:5	Once children are secure in adding one, move on to subtracting one, taking a similar approach to that used in step *3:2*. Beginning with reduction contexts with concrete/pictorial or generalised representations, ask children to write each equation as they tell the story. Draw attention to the fact that the difference is always one less than the minuend. Then start to include partitioning contexts, for example, *'There are eight children. One child has a balloon. Seven children do not have a balloon.'* Work towards use of the generalised statement: ***'Subtracting one gives one less.'***	Reduction context – pictorial representation: *'First there were eight doughnuts. Then one was eaten. Now there are seven doughnuts.'* First Then Now  8 – 1 7 → 8 – 1 = 7

		Reduction context – generalised representation: *'First we had five. Then we subtracted one. Now we have four.'*  5 – 1 = 4
3:6	Once you have generalised the pattern, use this to solve the series of equations, as shown opposite.	*'Fill in the missing numbers.'* 10 – 1 = ☐ 5 – 1 = ☐ 9 – 1 = ☐ 4 – 1 = ☐ 8 – 1 = ☐ 3 – 1 = ☐ 7 – 1 = ☐ 2 – 1 = ☐ 6 – 1 = ☐ 1 – 1 = ☐

3:7	Provide varied practice, including both addition and subtraction of one (missing sum or difference) as well as missing addend/subtrahend/minuend questions.
	Present problems in the form of:
	• missing number problems (number sentences and number lines)
	• part–part–whole diagrams
	• real-world contexts, for example:
	  ◦ *'I find one conker in the playground. I pick up six more on the way home. How many do I have now?'*
	  ◦ *'I challenge myself to swim four widths of the swimming pool. I have swum one width. How many more widths do I need to swim?'*
		'Fill in the missing numbers.'
		3 + 1 = ☐ ☐ = 1 + 1 9 – 1 = ☐
		1 + 9 = ☐ ☐ = 1 + 5 1 – 1 = ☐
		4 + ☐ = 5 ☐ + 1 = 8 7 – ☐ = 6
		

	Show children how to draw the equation as a part–part–whole model to see the structure of the problem and which element is missing.	Part–part–whole diagrams:  

Teaching point 4:

Consecutive numbers have a difference of one; we can use this to solve subtraction equations where the subtrahend is one less than the minuend.

Steps in learning

Guidance | **Representations**

4:1

Guidance

In this teaching point we explore subtraction facts which have a difference of one (e.g. *8 – 7 = 1*). These facts are part of the same family as those covered in *Teaching point 3*, but structurally are a little different in that they can't be solved by finding 'one more' or 'one less'. For example:

3 + 1	'one more'
1 + 3	
4 – 1	'one less'
4 – 3	'difference of one'

The aim is for children to be able to quickly recognise the relationship between the minuend and subtrahend in such problems and realise that the difference must be one, rather than resorting to counting-based strategies (for example, counting back four from the number five to solve *5 – 4*).

Start by presenting a reduction story, writing out the equation and solving it as you tell the story. Discuss the numbers and draw attention to 'all except one' object leaving.

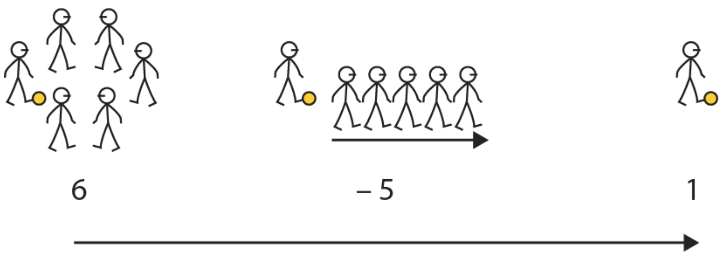
Explore similar 'difference of one' contexts, always making a clear link between 'all except one' leaving, and the subtrahend being one less than the minuend. Work towards use of the generalised statement: ***'Consecutive numbers have a difference of one.'***

Representations

Reduction context – pictorial representation:

'First there were six children playing. Then five children went home. How many children are playing now?'

First Then Now



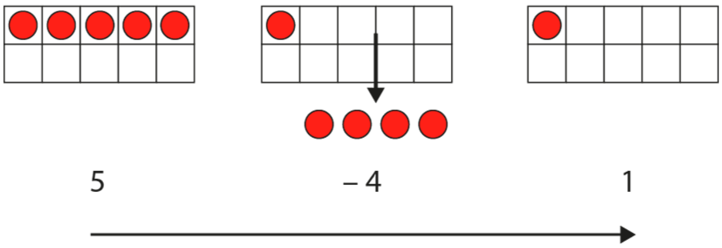
6 – 5 = 1

'First there were six children playing. Then all except one went home. Five is one less than six so there must be one child playing now.'

Reduction context – generalised representation:

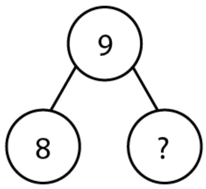
'First we had five. Then we subtracted four. How many do we have now?'

First Then Now



5 – 4 = 1

'Four is one less than five, so there must be one left.'

4:2	Now present a partitioning story. As in segment *1.5 Additive structures: introduction to aggregation and partitioning*, draw out and complete a part–part–whole model for the situation described, and write and solve the equation. Continue to draw attention to the minuend and subtrahend being consecutive numbers, and the difference being one.	*'There are nine children. Eight of them are reading. How many of them are not reading?'*   9 – 8 = ☐
4:3	Once you have generalised the pattern, use this to solve the series of equations shown opposite.	*'Fill in the missing numbers.'* 10 – 9 = ☐ 5 – 4 = ☐ 9 – 8 = ☐ 4 – 3 = ☐ 8 – 7 = ☐ 3 – 2 = ☐ 7 – 6 = ☐ 2 – 1 = ☐ 6 – 5 = ☐ 1 – 0 = ☐
4:4	Provide children with practice in identifying expressions with a difference of one from a range of expressions. Ask children to justify their choices, using the language of the generalised statement: • *'They are consecutive numbers, so they have a difference of one.'* • *'They are not consecutive numbers, so they do not have a difference of one.'*	*'Sort the expressions to show which represent a difference of one.'* 4 – 3 8 – 7 1 – 0 4 – 1 6 – 4 9 – 7

<table>
<tr><td></td><td></td><td>

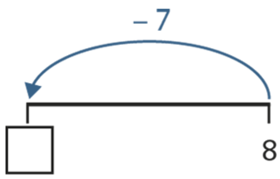
Difference of one	Not a difference of one

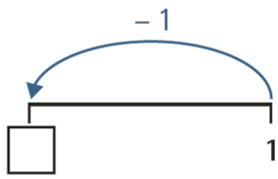
</td></tr>
<tr><td>**4:5**</td><td>Provide varied practice including both subtracting one and difference of one.

Present problems in the form of:
• missing number problems (number sentences and number lines)
• part–part–whole diagrams
• real-world contexts, for example:
  ◦ *'My coat has six buttons. I have done up five buttons. How many more buttons do I still need to do up?'*
  ◦ *'I had ten conkers. I gave nine away. How many do I have left?'*</td><td>Missing number problems:

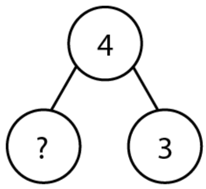
5 – 1 = □   7 – 6 = □

□ = 9 – 1   □ = 4 – 3





Part–part–whole diagrams:



</td></tr>
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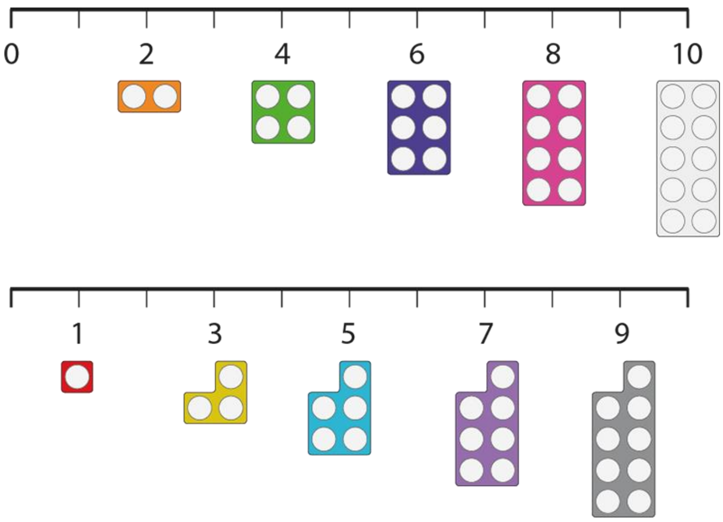
Teaching point 5:

Adding two to an odd number gives the next odd number; adding two to an even number gives the next even number. Subtracting two from an odd number gives the previous odd number; subtracting two from an even number gives the previous even number.

Steps in learning

	Guidance	Representations
5:1	The sequence of steps in this teaching point is exactly the same as for *Teaching point 3*, except now we are using children's knowledge of previous and next *odd and even* numbers, rather than previous and next numbers. The guidance here outlines key differences, but, to avoid repetition, doesn't exemplify the full guidance; please refer back to *Teaching point 3* as appropriate. For more information on even and odd numbers see segment *1.4 Composition of numbers: 6–10, Teaching point 3*. In a similar way to step *3:1*, practise counting forwards and backwards in odd and even numbers until children are fluent in this pattern. To add and subtract two without resorting to counting strategies, children need to be able to confidently identify the next or previous odd or even number. Provide practice, in the form of missing number problems where children must identify previous/next odd/even numbers, until children are fluent; initially they can use odd/even number lines for support, but should progress to solving problems without this scaffold.	'Skip counting' – number lines with base-ten number boards:  Missing number problems – consecutive odd/even numbers: previous odd number ← 7 → next odd number ☐ ← 7 → ☐ previous even number ← 4 → next even number ☐ ← 4 → ☐
5:2	Following the guidance in step *3:2*, present augmentation and aggregation contexts for the addition of two and write the corresponding	

equations. Draw attention to the fact that:

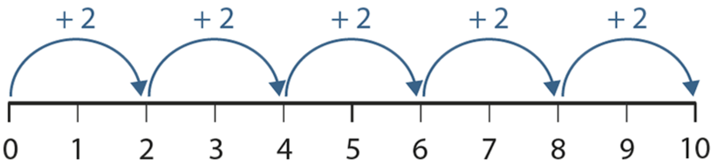
- when two is added to an odd number, the sum is the next odd number
- when two is added to an even number the sum is the next even number.

You can use a number line or tens frames (used 'twos-wise') to exemplify this pattern.

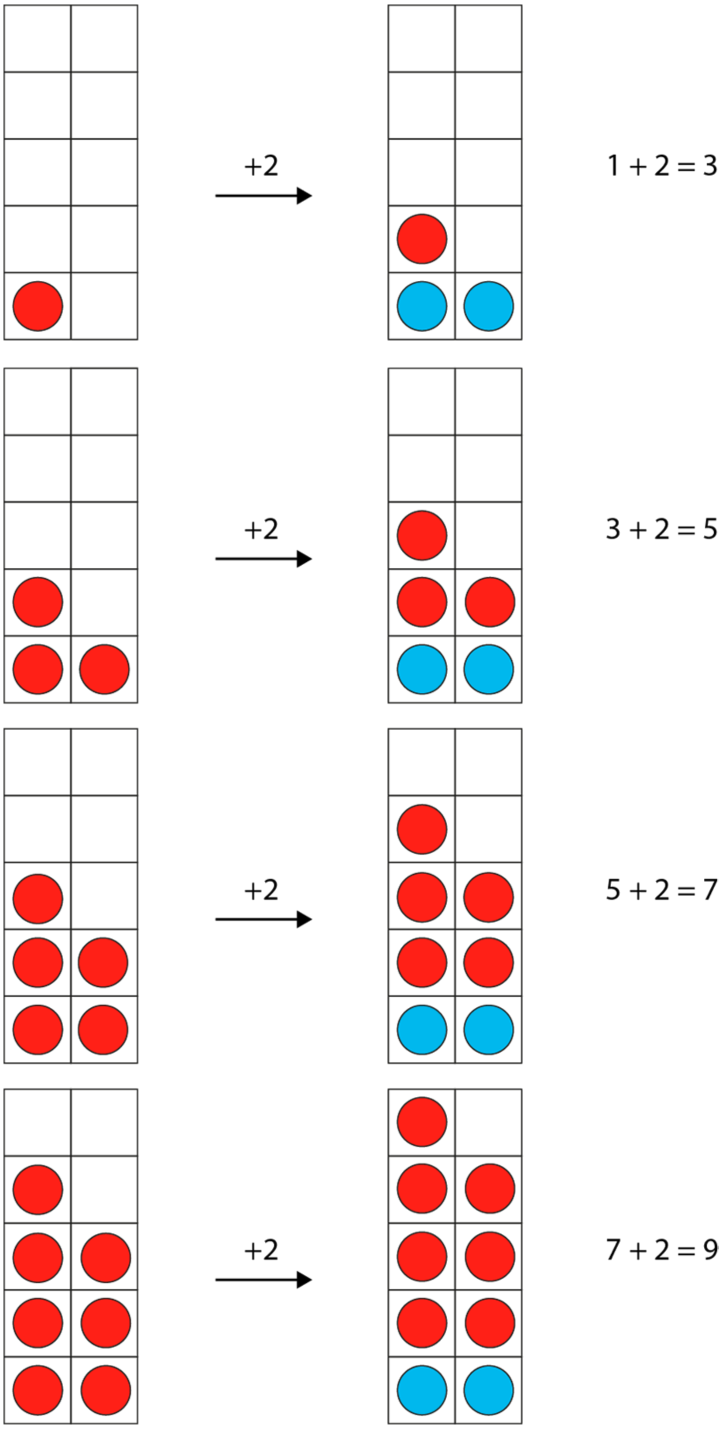
Summarise that to calculate two more, we don't need to count on (for example, *'seven, eight, nine'*) but can instead use our knowledge of odd and even numbers. Work towards use of the generalised statements:

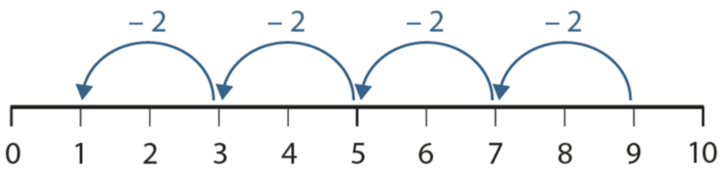
- ***'Adding two to an odd number gives the next odd number.'***
- ***'Adding two to an even number gives the next even number.'***

Adding two to even numbers – number line:



Adding two to odd numbers – tens frame:

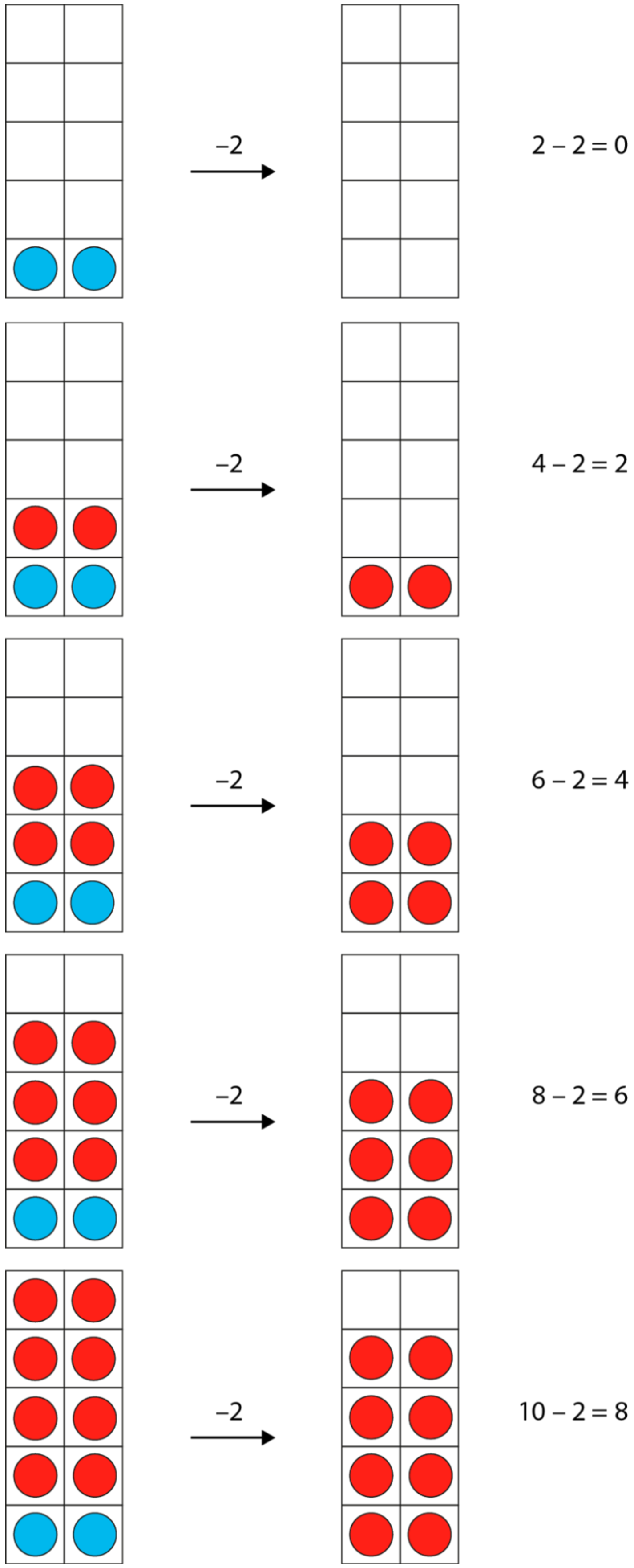


5:3	As in step *3:3*, check that children can apply their knowledge of commutativity, for example: • $7 + 2 = 9$, so $2 + 7 = 9$ • $4 + 2 = 6$, so $2 + 4 = 6$	
5:4	Now that the structure has been 'unpicked', and a generalisation made, present the full series of missing number problems shown opposite. Draw attention to the fact that, within this set, there are two equations (and their commutatives) that we can already solve using previous strategies: • Eight and two should be immediately recognised as a pair which sums to ten, so $8 + 2$ and $2 + 8$ do not need to be solved by identifying the 'next even number after eight' (see *Teaching point 2*). • $1 + 2$ and $2 + 1$ were explored in *Teaching point 3* and can be solved by finding 'one more than two'.	*'Fill in the missing numbers.'* $1 + 2 = \square$ $2 + 1 = \square$ $3 + 2 = \square$ $2 + 3 = \square$ $5 + 2 = \square$ $2 + 5 = \square$ $7 + 2 = \square$ $2 + 7 = \square$ $9 + 2 = \square$ $2 + 9 = \square$ $0 + 2 = \square$ $2 + 0 = \square$ $2 + 2 = \square$ $2 + 2 = \square$ $4 + 2 = \square$ $2 + 4 = \square$ $6 + 2 = \square$ $2 + 6 = \square$ $8 + 2 = \square$ $2 + 8 = \square$
5:5	Now move onto subtraction of two, following the same progression as in step *3:5*. Using both reduction and partitioning contexts, draw attention to the fact that the difference between consecutive odd/even numbers is always two. Work towards the generalised statements: • ***'Subtracting two from an odd number gives the previous odd number.'***	Subtracting two from odd numbers – number line: 

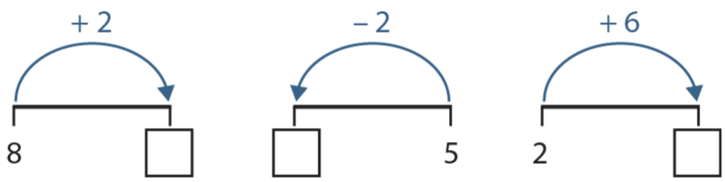
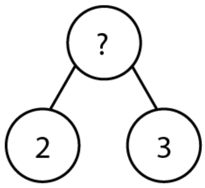
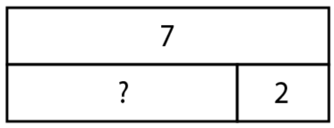
- ***‘Subtracting two from an even number gives the previous even number.’***

As in step *5:2*, you can use the number line and tens frame to highlight this pattern.

Subtracting two from even numbers – tens frame:



5:6	Once you have generalised the pattern, use this to solve the series of equations shown opposite.	*'Fill in the missing numbers.'* $10 - 2 = \square$ $9 - 2 = \square$ $8 - 2 = \square$ $7 - 2 = \square$ $6 - 2 = \square$ $5 - 2 = \square$ $4 - 2 = \square$ $3 - 2 = \square$ $2 - 2 = \square$

5:7	Provide varied practice, including both addition and subtraction of two, in the form of:
	• missing number problems (number sentences and number lines)
	• part–part–whole diagrams
	• real-world contexts, for example:
	  ◦ *'Charlie has been on holiday for two days. He has five more days of holiday to go. How long is Charlie's holiday altogether?'*
	  ◦ *'Ellie has a seven metre length of ribbon. She cuts off two metres to give to her friend. What length of ribbon does Ellie have now?'*
		Missing number problems:
		$3 + 2 = \square$ $\square = 6 + 2$ $9 - 2 = \square$
		$2 + 9 = \square$ $\square = 2 + 5$ $2 - 2 = \square$
		
		Part–part–whole diagrams:
		
		

Teaching point 6:

Consecutive odd / consecutive even numbers have a difference of two; we can use this to solve subtraction equations where the subtrahend is two less than the minuend.

Steps in learning

	Guidance	Representations
6:1	The sequence of steps in this teaching point is exactly the same as for *Teaching point 4*, but now we explore a difference of *two* (e.g. *7 – 5*), rather than a difference of *one*. The guidance here outlines key differences, but, to avoid repetition, doesn't exemplify the full guidance; please refer back to *Teaching point 4* as appropriate. Again, these facts are similar to those in *Teaching point 5*, but are structurally slightly different: [see table below] The table summarises the patterns which will allow children to solve these equations without needing to use counting strategies. As in step *4:1*, start with a reduction context, writing the equation as you tell the story and drawing attention to the 'all except two leave' structure. Include contexts with both odd and even starting numbers.	
6:2	Now present partitioning contexts with a difference of two. Represent these pictorially, with part–part–whole diagrams and with equations, as for step *4:2*. Work towards use of the generalised statements:	

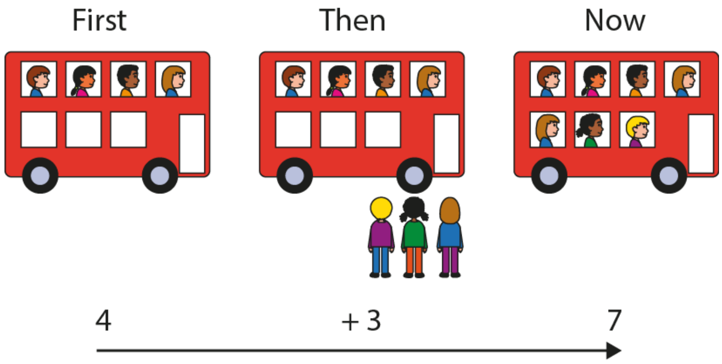
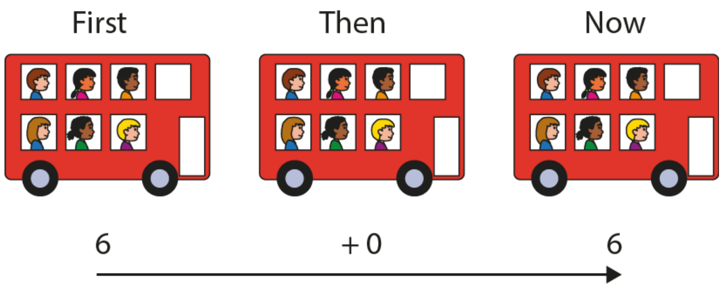
5 + 2	'next odd number'
2 + 5	
7 – 2	'previous odd number'
7 – 5	'difference of two'

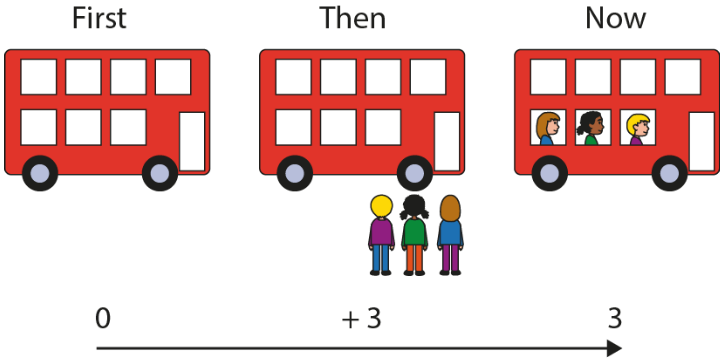
	• ***'Consecutive odd numbers have a difference of two.'*** • ***'Consecutive even numbers have a difference of two.'***	
6:3	Once you have generalised the pattern, use this to solve the series of equations as shown opposite.	*'Fill in the missing numbers.'* 10 – 8 = ☐ 9 – 7 = ☐ 8 – 6 = ☐ 7 – 5 = ☐ 6 – 4 = ☐ 5 – 3 = ☐ 4 – 2 = ☐ 3 – 1 = ☐ 2 – 0 = ☐
6:4	As in step *4:4*, present equations for children to sort according to whether they have a difference of two or not.	
6:5	Complete the teaching point by providing varied subtraction practice, as in step *4:5*, with problems involving both subtraction of two and difference of two.	

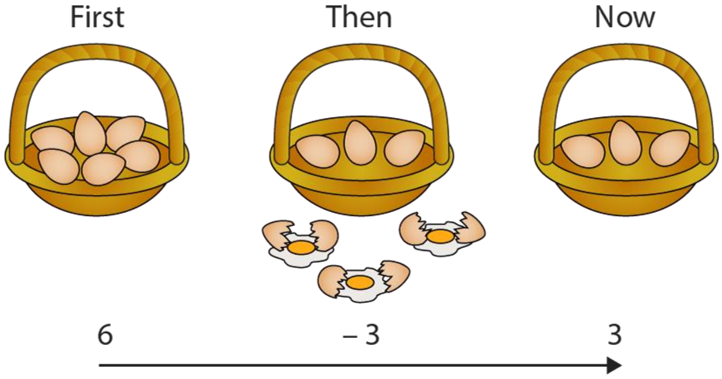
Teaching point 7:

When zero is added to a number, the number remains unchanged; when zero is subtracted from a number, the number remains unchanged.

Steps in learning

	Guidance	Representations
7:1	Children will already have explored addition and subtraction contexts where an addend or the subtrahend is zero in segments *1.5 Additive structures: introduction to aggregation and partitioning* and *1.6 Additive structures: introduction to augmentation and reduction*. Here we reinforce this group of number facts. Start by presenting a familiar augmentation context, with non-zero addends, and ask children to write an equation to represent it. Then build on the same context, looking at situations where one of the addends is zero. Explore cases where the first addend is zero, and those where the second addend is zero, and examine the resulting equations, for example: 6 + 0 = 6 0 + 3 = 3 Using a variety of augmentation and aggregation contexts, work towards use of the generalised statement: ***'When zero is added to a number, the number remains unchanged.'*** Sometimes children think that adding zero gives a sum of zero. Address this misconception explicitly, for example: *'Pippa says:* 'Five plus zero is zero. Zero means nothing so if I add zero I don't have anything.' *Explain why she is wrong.'*	Augmentation context with non-zero addends: *'First, there were four children on the bus. Then three more children got on. Now, there are seven children on the bus.'*  4 + 3 = 7 Augmentation contexts with one addend equal to zero: *'Write an equation to show how many children are on the bus.'*  6 + 0 = 6

		'Write an equation to show how many children are on the bus.'  0 + 3 = 3
7:2	As in previous teaching points, work through all of the relevant facts as shown opposite, using the generalised statement as a scaffold.	*'Fill in the missing numbers.'* 10 + 0 = ☐ 0 + 10 = ☐ 9 + 0 = ☐ 0 + 9 = ☐ 8 + 0 = ☐ 0 + 8 = ☐ 7 + 0 = ☐ 0 + 7 = ☐ 6 + 0 = ☐ 0 + 6 = ☐ 5 + 0 = ☐ 0 + 5 = ☐ 4 + 0 = ☐ 0 + 4 = ☐ 3 + 0 = ☐ 0 + 3 = ☐ 2 + 0 = ☐ 0 + 2 = ☐ 1 + 0 = ☐ 0 + 1 = ☐ 0 + 0 = ☐
7:3	Now move on to subtraction of zero. Look first at a familiar reduction context, with a non-zero subtrahend, and ask children to write an equation to represent it. Then build on the same context, looking at situations where the	

	subtrahend is zero. Discuss with the children that if we subtract zero, there is no change to our starting quantity and the total remains the same, for example, '*Harry has a basket of six eggs. He doesn't drop any. He* still has *six eggs left.*' Using a variety of partitioning and reduction contexts, work towards use of the generalised statement: ***'When zero is subtracted from a number, the number remains unchanged.'*** Similarly to before, address the common misconception that subtracting zero gives zero, for example: '*Sara says:* 'Seven minus zero is zero. If I take away zero it is all gone, so the answer is zero.' *Explain why she is wrong.*'	Reduction context with non-zero subtrahend: *'Harry has a basket of six eggs. He drops three eggs. Write an equation to show how many eggs he has left.'*  6 – 3 = 3
7:4	Work through all of the relevant facts as shown opposite, using the generalised sentence as a scaffold.	*'Fill in the missing numbers.'* 10 – 0 = ☐ 4 – 0 = ☐ 9 – 0 = ☐ 3 – 0 = ☐ 8 – 0 = ☐ 2 – 0 = ☐ 7 – 0 = ☐ 1 – 0 = ☐ 6 – 0 = ☐ 0 – 0 = ☐ 5 – 0 = ☐
7:5	Provide varied practice, including both addition and subtraction of zero, in the form of: • missing number problems (number sentences) • part–part–whole diagrams • real-world contexts.	

Teaching point 8:

Subtracting a number from itself gives a difference of zero.

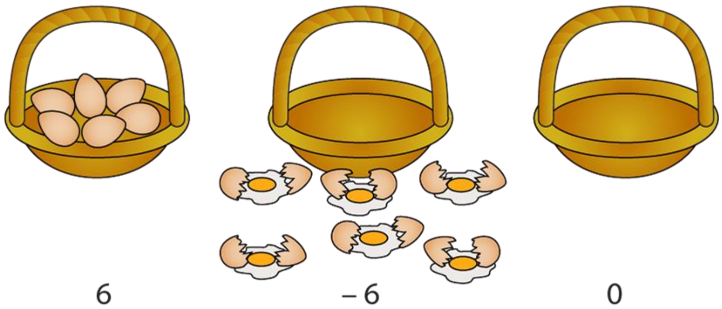
Steps in learning

	Guidance	Representations
8:1	Now explore subtraction contexts with a difference of zero (e.g. *6 – 6 = 0*). This teaching point has similarities to *Teaching points 4 and 6*, and we explore the difference in structure between, for example, *6 – 0 = 6* and *6 – 6 = 0*:	Reduction context – difference of zero: *'Harry has a basket of six eggs. He drops six eggs. Write an equation to show how many eggs he has left.'*

6 + 0	'no more'
0 + 6	'no more'
6 – 0	'no less'
6 – 6	'difference of zero'

Again, begin with a reduction context then, using a variety of partitioning and reduction contexts, work towards use of the generalised statement: ***'Subtracting a number from itself gives a difference of zero.'***

First Then Now



6 – 6 0

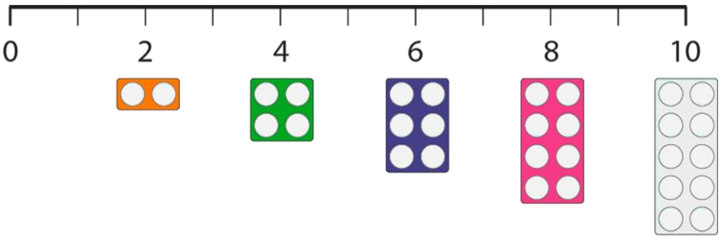
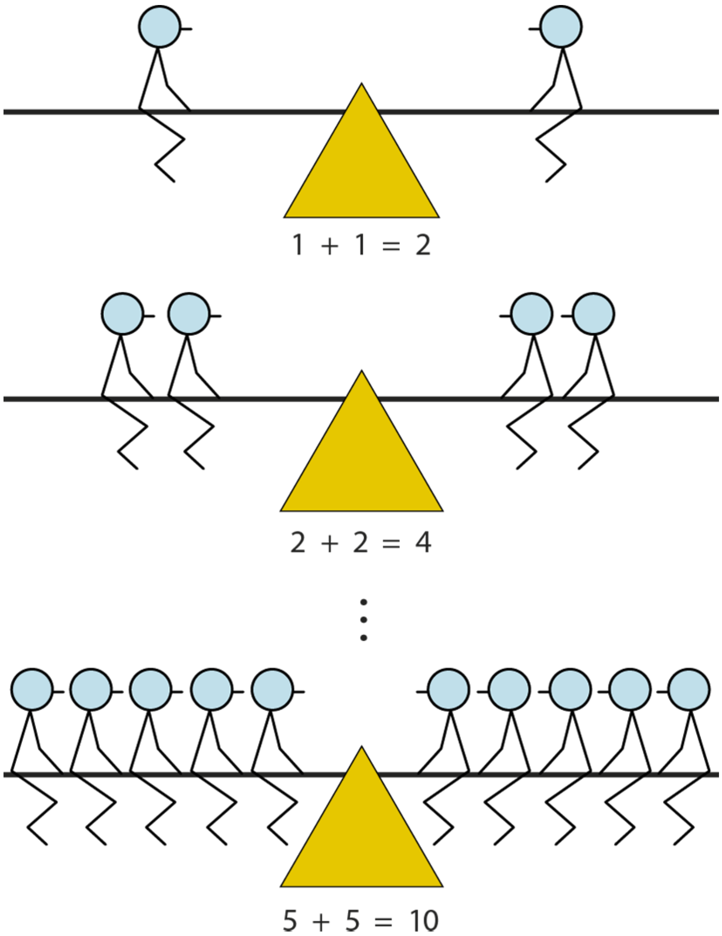
6 – 6 = 0

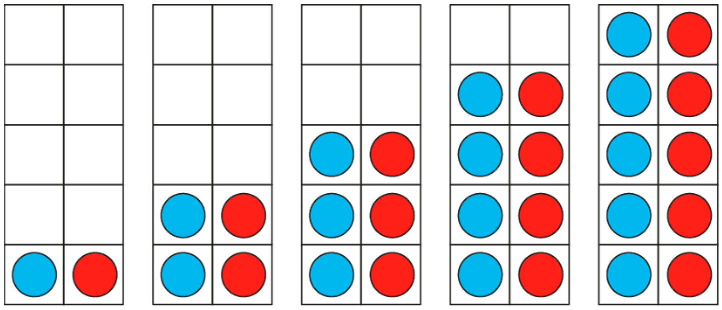
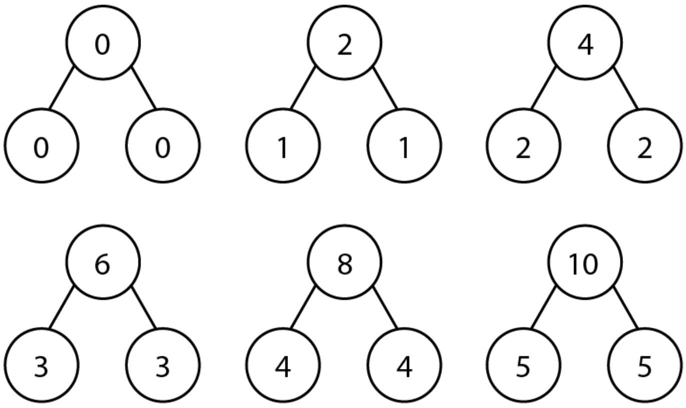
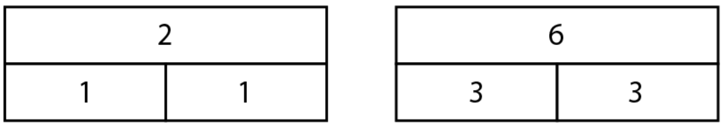
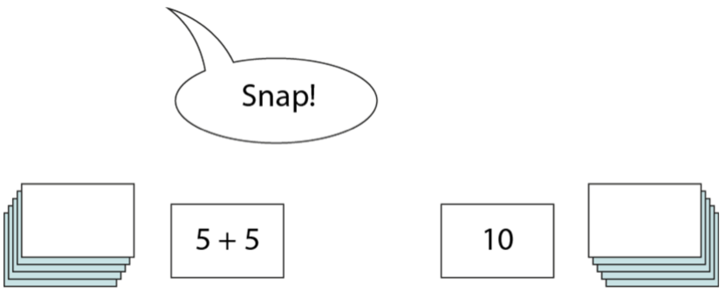
	Guidance	Representations
8:2	Work through all of the relevant facts using the generalised statement as a scaffold.	*'Fill in the missing numbers.'* 10 – 10 = ☐ 4 – 4 = ☐ 9 – 9 = ☐ 3 – 3 = ☐ 8 – 8 = ☐ 2 – 2 = ☐ 7 – 7 = ☐ 1 – 1 = ☐ 6 – 6 = ☐ 0 – 0 = ☐ 5 – 5 = ☐
8:3	Finally provide varied subtraction practice, as in steps *4:5* and *6:5*, with problems involving both subtraction of zero and difference of zero.	

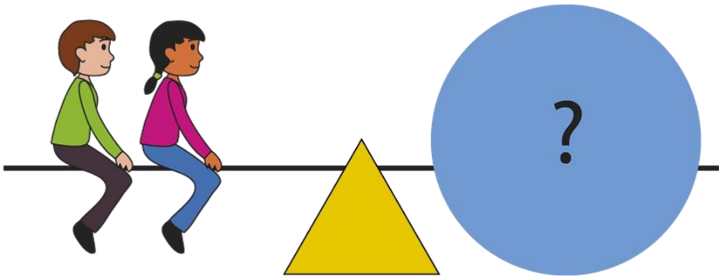
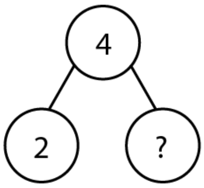
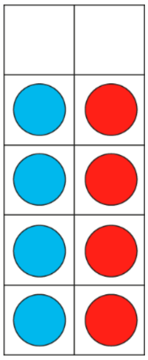
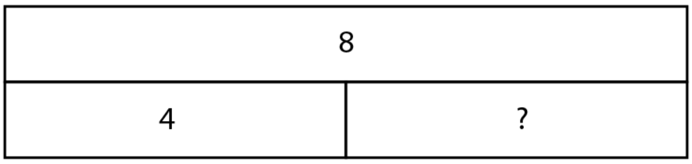
Teaching point 9:

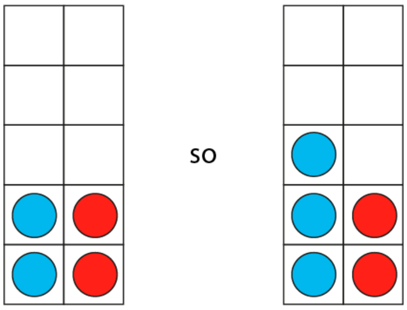
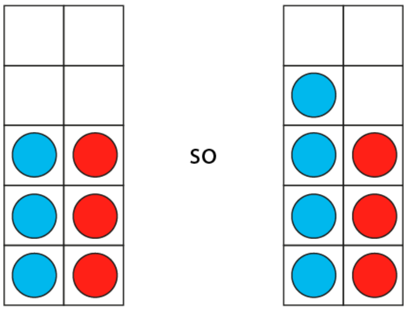
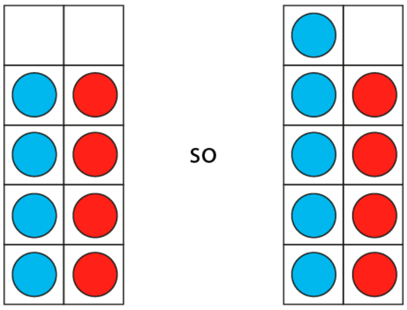
Doubling a whole number always gives an even number and can be used to add two equal addends; halving is the inverse of doubling and can be used to subtract a number from its double. Memorised doubles/halves can be used to calculate near-doubles/halves.

Steps in learning

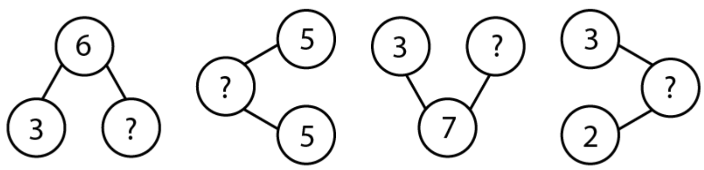
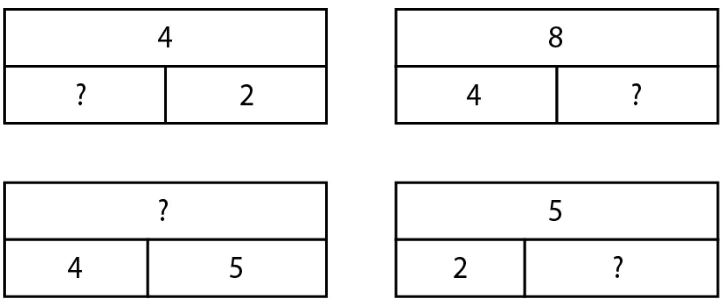
	Guidance	Representations
9:1	Begin by reviewing counting in even numbers to ten, forwards and backwards. Children have practised this before and should be fluent by now.	
9:2	Now use pictures of seesaws as shown (or other balanced items), writing an equation for each. Ask children, *'What do all of the seesaws have in common?'* and prompt for the answers: • *'Each seesaw has the same number of children on both sides.'* • *'The total number of children on each seesaw is an even number.'*	
9:3	Introduce the word 'double': • *'When both addends are the same, we are doubling.'* • *'If we have three plus three, we can say we are doubling three.'*	

	Use tens-frames 'twos-wise' to bring out the doubling structure; emphasise that doubling always gives us an even number as each 'one' has a double to pair up with. You can also draw part–part–whole diagrams, drawing attention to the fact that the two parts are the same and the whole is always an even number. Work towards use of the generalised statement: ***'Doubling a whole number always gives an even number.'***	Doubling the numbers one to five – tens frames:  1 + 1 = 2 2 + 2 = 4 3 + 3 = 6 4 + 4 = 8 5 + 5 = 10 Doubling the numbers one to five – part–part–whole diagrams:  
9:4	The doubles are one set of addition facts which children simply need to memorise outright. Display the doubles part–part–whole diagrams on the classroom wall for children to refer to as they learn them off-by-heart. For practice, play classroom games such as 'Snap', as shown opposite. All children should regularly repeat the facts; encourage them to use both addition language (for example, *'three plus three is equal to six'*) and doubling language (for example, *'double three is six'*).	 • *'Five plus five is equal to ten.'* • *'Double five is ten.'*

9:5	Once children are fluent in doubling the numbers zero to five use the same models as in steps 9:2–*9:4* to illustrate and develop understanding of halving. Using the seesaw image, you could cover up one half of the seesaw and ask *'How many children are on the other side of the seesaw?'* Then move on to representing these as subtraction problems where the subtrahend is half of the minuend (e.g. *8 – 4 = 4*). Using real-world contexts and generalised representations, including part–part–whole models, work towards use of the generalised statement: ***'Halving is the inverse of doubling.'***	Real-world context: *'There are four children on the seesaw. Two children are on one side. How many children are on the other side?'*  $2 + \square = 4$ $4 - 2 = \square$  Generalised representation:  $4 + \square = 8$ $8 - 4 = \square$ 

9:6	Once children are fluent in doubling and halving, introduce two new facts that can be learnt using near-doubles: *3 + 4* and *4 + 5* (and their commutatives). Note that *2 + 3* (and therefore *3 + 2*) has already been covered in 'adding two' (*Teaching point 5*). However, children may find it easier to relate this fact to 'near double two' rather than 'the next odd number after three', so this fact will also be revisited here. Although *0 + 1* and *1 + 2* (and their commutatives) are also near-doubles, they aren't included here as both are probably more easily solved using the 'one more'/'one less' structure. Use tens frames 'twos-wise' to demonstrate how each of the addition facts (*2 + 3*, *3 + 4* and *4 + 5*) can be derived from the corresponding double.	 2 + 2 = 4 3 + 2 = 5 *'Two plus two is equal to four, so* three *plus two is equal to* five.*'*  3 + 3 = 6 4 + 3 = 7 *'Three plus three is equal to six, so* four *plus three is equal to* seven.*'*  4 + 4 = 8 5 + 4 = 9 *'Four plus four is equal to eight, so* five *plus four is equal to* nine.*'*
9:7	Use intelligent practice, drawing attention to: • how each near-double can be derived from both the previous and next double	

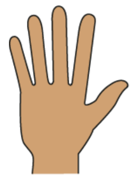
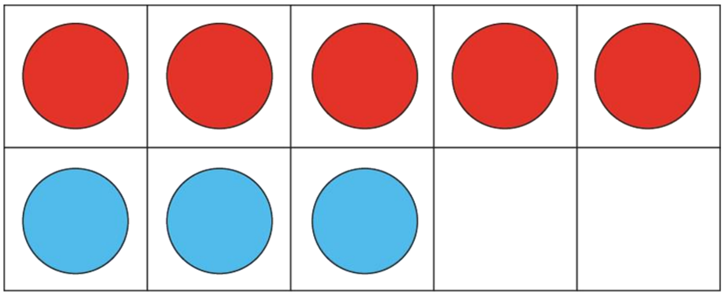
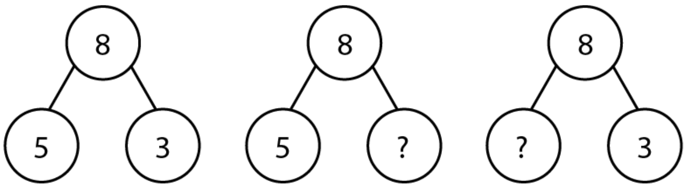
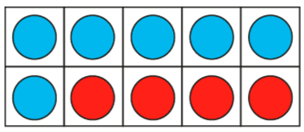
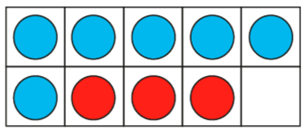
	• how each near-half can be derived from both the previous and next half. *5 + 4 = 9, 4 + 5 = 9, 9 − 5 = 4* and *9 − 4 = 5* may be more easily related to *5 + 5 = 10* since this builds on earlier knowledge of bonds to ten.	*'Fill in the missing numbers.'* Near-doubles: 2 + 2 = 4 3 + 3 = 6 4 + 4 = 8 2 + 3 = □ 4 + 3 = □ 5 + 4 = □ 3 + 3 = 6 4 + 4 = 8 5 + 5 = 10 Near-halves: 4 − 2 = 2 6 − 3 = 3 8 − 4 = 4 5 − 2 = □ 7 − 3 = □ 9 − 5 = □ 6 − 3 = 3 8 − 4 = 4 10 − 5 = 5

9:8	Provide varied practice, including both addition and subtraction for doubles and near-doubles, in the form of:
	• missing number problems (number sentences)
	• part–part–whole diagrams
	• real-world contexts, for example:
	  • *'There are seven children. Four of them have had lunch. How many children have not had lunch?'*
	  • *'I have two five metre lengths of rope. What length of rope do I have altogether?'*
	Note that part–part–whole models enable children to see the relationship between addition and subtraction facts and quickly derive subtraction facts from known addition facts – covering a part with a finger or hand can help to illustrate this.
		'Fill in the missing numbers.'
		5 − □ = 2 □ = 3 + 3 10 − 5 = □
		5 + 4 = □ □ − 4 = 4 3 + □ = 7
		
		
		

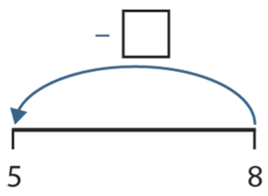
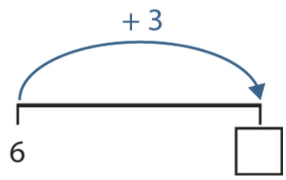
Teaching point 10:

Addition and subtraction facts for the pairs five and three, and six and three, can be related to known facts and strategies.

Steps in learning

	Guidance	Representations
10:1	There are only two pairs of remaining addition facts within ten which have yet to be covered: *5 + 3* and *6 + 3* (and their commutatives). Since these facts don't fit into the other categories, the most pragmatic solution may be to quickly move to memorisation and practice. However, there are useful derived-fact strategies to draw on in the process of moving to memorisation, and if reminders are needed. Children have already come across *5 + 3* in the context of 'five and a bit' numbers (see *1.4 Composition of numbers: 6–10*); children learnt to show eight in this way with their hands. Now, link this implicit knowledge (that if we hold up five fingers and three fingers we are showing eight fingers) to the explicit knowledge that *5 + 3 = 8* and *3 + 5 = 8*. Children should also be able to recognise eight on a tens frame (used 'fives-wise'), so include this representation as well. You can also use both the hands, tens frame and part–part–whole models to demonstrate the related subtraction facts: 8 – 5 = 3 8 – 3 = 5	  
10:2	*6 + 3* (and its commutative) is probably most easily calculated by relating it to *6 + 4 = 10*, which is a known fact (see *Teaching point 2*). Use tens frames and missing number problems laid out to highlight the relationship between	Derived-fact strategy – tens frames:  6 + 4 = 10  6 + 3 = 9

	6 + 3 and *6 + 4*. Ask children to describe the derivation in sentences: '*Six plus four is ten. Three is one less than four, so six plus three is one less than ten.*' Children should simply memorise the corresponding subtraction fact (*9 – 6 = 3*), since derivation is more complex. Note: at this stage children will probably not yet be learning to count in threes but, when they begin, the three, six, nine sequence will reinforce the relationships in this family of addition and subtraction facts.	*'Fill in the missing numbers.'* Derived-fact strategy – missing number problems: 6 + 4 = 10 4 + 6 = 10 6 + 3 = □ 3 + 6 = □ □ + 6 = 10 □ + 6 = 9 Subtraction – missing number problems: 9 – 6 = □ 9 – 3 = □

10:3	Once you have discussed the various derived-fact strategies with children, provide varied practice, including:
	• missing number problems (number sentences and number lines)
	• part–part–whole diagrams
	• real-world contexts, for example:
	  ◦ *'There are nine children. Six of them have scooters. How many children do not have scooters?'*
	  ◦ *'I need five metres of fabric to make some costumes. I need three metres of fabric to make some curtains. How much fabric do I need altogether?'*
		Missing number problems:
		'Fill in the missing numbers.'
		□ = 5 + 3 8 – 5 = □ 9 – □ = 6
		6 + 3 = □ 9 – 3 = □ 8 – □ = 5
		 
		Part–part–whole diagrams: