Primary
& Early Years
Magazine 65

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another edition of our new-look and more compact Primary Magazine. This magazine has been serving primary practitioners for 64 editions with a varied collection of different articles related to maths education and mathematics professional development. We hope you like the new sections - and don't forget that all previous editions of the magazine are accessible through our Primary Magazine Archive.

Contents

In this edition we have a selection of interesting and useful articles. New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study. In this edition we begin with a focuses on *Algebra in KS2*, exploring what this means in Y3-Y5 and not just where it is a specific requirement in Y6.

Where's the Maths in That? shares ideas for ensuring that mathematics is taught and experienced across the curriculum. In the coming months, this series of articles that will explore opportunities for mathematics and mathematical thinking within the new science programme of study. The first theme we explore is *Everyday Materials* for Y3.

Finally, Maths in the Staff Room provides a simple plan for CPD meetings in your school to be led by a member of your staff. These are short meetings that can be used exactly as indicated, or adapted to meet the CPD needs of the school. Editable resources are supplied to enable flexibility of 'delivery'. In this edition we focus on *Progression in Problem Solving*.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

Maths Hubs

Several primary schools are among the list of schools, colleges and institutions selected to take the lead role in 32 new Maths Hubs across England. But this does not mean that only these will concentrate on primary maths. It is the remit of all of the hubs - regardless of the lead school's identity - to work to bring about improvements across all school phases. To that end, all of the hubs will gather together a group of partner schools, colleges and other education institutions in a strategic steering group. Here's a full list of the lead schools.

BERA report on Research in Teaching

In May, the British Educational Research Association (BERA) published a new report, Research and the Teaching Profession – Building Capacity for a Self-Improving Education System, on the importance of schools and colleges becoming research-rich environments in which to work. The report also strongly suggests that teacher researchers and the wider research community work in partnership, rather than in separate and sometimes competing universes. Although this is not a mathematics-specific report, there is much that schools and universities can do to share and contribute to the wealth of mathematics education research already in existence. With a new curriculum to implement, it is important that systemic changes are underpinned with an evidence based foundation.

Don't forget that the NCETM provides access to hundreds of free research papers through its Research Gateway, and there are also some useful self-study materials around some specific pieces of research in the Accessing Research Study Modules.

National Curriculum

Have you explored the NCETM National Curriculum Planning Tool yet? This interactive tool will support you in the following ways: your subject knowledge; making connections within and across the primary curriculum; suggest helpful papers, pupil activities, exemplification of expectations, and links to the suite of NCETM videos.

Mathematics CPD

Don't forget that if you are looking for high quality providers of maths CPD in the next academic year, use our Professional Development Directory to find CPD Standard Holders (gold rosette) or Accredited Professional Development Leads (purple rosette).

New National Curriculum in Focus

New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics

Algebra in KS2

Those who are familiar with the new programme of study will have noticed that there is a specific requirement for algebra in Y6. There is a widely held view that in order to work with higher level algebra (i.e. KS3+) pupils need to have had good experiences of algebra in their primary education (Cai et al 2007). Although there is specific content in the Y6 programme of study there is a significant amount of algebra that is hidden amongst the remaining programmes of study for both KS1 and KS2. In the second part of this two-part series we explore where the hidden algebra from Y1 to Y5 is, and provide you with some thoughts about how your scheme of work/school curriculum might ensure that by the time your pupils reach Y6, algebra is not a daunting new theme to learn in mathematics. You can read more about what we mean by algebra in KS1 in the first article.

For simplicity in analysing the programme of study let us think of algebra as *generalising* and *structuring and restructuring*.

Generalising is finding a way to express a relationship from an observed pattern. From Y3 it is important to try to encourage pupils to repeatedly pattern-spot. Asking them to notice commonalities between mathematical objects and confidently use the language of generalisation such as 'all...' or 'every…'. Pupils should continue to observe and articulate with increasingly accurate mathematical vocabulary about what is the same and different about the mathematical objects they observe.

Structuring and restructuring requires pupils to represent mathematical relationships in different ways. If pupils have had a solid grounding as described in the first article, in KS1, then by the time the pupils reach Y3 they will be confident in representing mathematical ideas in a variety of ways and be flexible in their ability to translate between different representations. A simple example is using Dienes apparatus to represent a two digit number, using place value arrow cards to represent the same number and place value counters to represent the same number. Each of these resources structures the number in a different representation but pupils should understand and articulate the differences between the representations and create them for themselves.

So where can we find examples and opportunities for *generalising* and *structuring and restructuring* in the new KS2 programme of study?

This chart (PDF) is an example for some of the Y4 Number strand.

You will notice that some words are emboldened. This is to emphasise the key vocabulary associated with generalising: always, every, only etc. The chart is a guide to the types of generalisations that Y4 pupils should be able to generate for themselves as a result of their mathematical activity. This can be achieved by the teacher using questions such as "what do you notice?" and "what's the same/ different?". Alternatively you can provide a generalisation to the pupils and ask them whether they think it is **true or false** or **sometimes, always or never true**.

Here are a few further ideas that could also be used in KS2:

- Use pan balances to explore the structure of multiplicative relationships - e.g. using Numicon or linking cubes to explore equivalences for 6 x 8
- Encourage pupils to use multiplicative reasoning to solve missing number problems such as:
  - 4 x 7 = 2 x ☒
  - ☒ x 2 = 3 x 4
  - 96 ÷ 6 = ☒ ÷ 3
  - 54 ÷ ☒ = ☒ ÷ 3 (exploring positive integers that work for this number sentence/ equation)

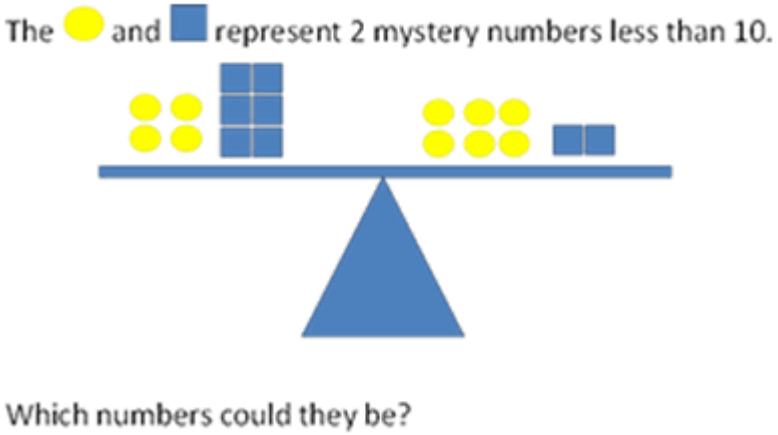
    Observe whether pupils perform the 'complete' calculations or whether pupils notice relationships between the numbers either side of the equals sign. i.e. in the first sentence above "4 is 2 times as big as 2, so the missing number must be 2 times as big as 7" (using multiplication as scaling). Encourage pupils to observe the relationships by asking questions such as "what do you notice?" and "what's the same, what's different?" Return to the pan balances to support the reasoning
- Solve problems of the type below from *World Class Arena*'s sample test questions from their World Class Tests:

*A **ball** and a **cube** together weigh 170 grams*

***Three balls** and **two cubes** weigh 420 grams*

*What is the weight of **one cube**? ..... grams*

- Explore problems represented in similar ways to below:



- Use coloured rods (e.g. Cuisenaire) to explore equivalence and arithmetical relationships leading to the use of symbols. For example:

t = 2r = ?

r = ? p = ?

- Read about how teacher Caroline Ainsworth used coloured rods (e.g. Cuisenaire) with Y2 and Y3 pupils to develop an understanding of proportional relationships and see examples of her pupils working in this way from an NCETM funded project.

Why not work with colleagues to identify other generalisations and opportunities for structuring and restructuring that KS2 children should be able to generate from the other programmes of study for number Y3 to Y5. What about for geometry and measurement?

Further links

NCETM National Curriculum Videos: Algebra

Research Gateway Algebra in KS1 and KS2

Professional Development Directory Algebra

National STEM Centre eLibrary Algebra KS2

NRICH Early Algebra.

References

Cai et al, (2005) The Development of Students' Algebraic Thinking in Earlier Grades: A Cross-Cultural Comparative Perspective ZDM 2005 Vol. 37 (1)

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Where's the Maths in That? – Maths across the curriculum

*In this section of this Primary Magazine we explore how mathematics can be embedded into other subjects in the context of the new curriculum. The subject in this new series is **science** and over the next few months we will explore the different themes for the KS1 and KS2 science programmes of study and how maths can be embedded in and enhance understanding of scientific ideas.*

The In this edition we look at the theme of **Everyday Materials** for Y1 and how a scheme of work for this might incorporate mathematical skills.

The statutory requirements for **Everyday Materials** in the Y1 programme of study are:

Pupils should be taught to:

- distinguish between an object and the material from which it is made
- identify and name a variety of everyday materials, including wood, plastic, glass, metal, water, and rock
- describe the simple physical properties of a variety of everyday materials
- compare and group together a variety of everyday materials on the basis of their simple physical properties.

Below are some ideas for incorporating maths into this science theme

1: Distinguish between an object and the material from which it is made

Identify and name a variety of everyday materials, including wood, plastic, glass, metal, water, and rock.

Investigate what is the most common material that can be found in our school? Go on a school treasure hunt (different groups could go to different parts of the school). Pupils collect a variety of different objects in a fixed time (e.g. 10 mins) and bring them back to class. Pupils then sort the objects according to the material they are made from. Use linking cubes to represent each object to make a 3D block graph or bar chart for each group's treasure. Use a different colour cube for each material. Use this information to collate a whole class bar chart on a simple spreadsheet in view of all pupils so that they can see the bar chart changing with live data. You could use this spreadsheet and amend the group names or materials. As the data builds up ask pupils describe what they see happening to the bar chart. i.e. the numbers changing the heights of the bars growing.

2: Describe the simple physical properties of a variety of everyday materials

Explore with an adult how easily some materials can pour. Fill large clear fizzy drink bottles (all the same size) with the same quantity of different materials. E.g. sand, water and custard. Pupils could measure the quantity using a measuring jug or practise filling the bottle up to a marked line on the bottle. Ask pupils to time with a stop-watch how long it takes for the material to come out of the bottle when tipped upside down.

Read a recipe to bake a cake or cupcakes with an adult. As well as measuring out the ingredients discuss the physical properties of the different ingredients. Compare the amounts needed of each ingredient. How many ingredients can you pour? How many of the ingredients can you slice? Which is the heaviest ingredient? etc.

Investigate what is the best combination of sand and water to make a sandcastle. Demonstrate with dry sand what happens if you try and build a sandcastle with dry fine sand. Ask the children what needs to happen to the sand to make a sandcastle. The children are likely to say the sand needs water. Add a lot of water to the sand and illustrate that the sand is now too sloppy to build a sand castle. This sets the scene for the investigation. To make a good sandcastle we need to have the right amount of sand and water together. Provide a small sandcastle bucket for each group of children, the same amount of sand and a cup to add water to the sand with. Ask the children to experiment by adding one cup at a time and then testing. Ask the children to keep count of the cups of water they use. This can be done using a tally or by dropping a cube into a container each time a cup of water is added. Pupils can measure the cup of water as a non-standard unit or pour an exact amount of water from a measuring jug. Compare the results from the different groups. Did they all need the same number of cups/ quantity of water to build the sandcastle? Use the results to discuss the importance of fair testing and possible errors that might have crept in. You may also introduce the idea of approximate answers using the term 'about'. E.g. "about four cups of water are needed."

3: Compare and group together a variety of everyday materials on the basis of their simple physical properties

Look at a selection of materials and find ways to sort them. Say what is the same and what is different about the materials. Play 'Odd one out' – find reasons why one material is the odd one out when compared to two other different materials. How many different 'odd one outs' can you find?

Sort different materials based on their appearance on a table-top Carroll Diagram e.g shiny/not shiny against natural/not natural.

You can get further general ideas for teaching this theme from the National STEM Centre National Curriculum pages.

Maths in the Staff Room – Short Professional Development Meetings

This section provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school

Progression in problem Solving

Meeting Aims

- Understand how children's problem solving can develop from Y1 to Y6.

Timing

- 1.5 hours

Resources

- Progression Map: Problem Solving
- Progression in Problem Solving Sorting Cards
- Problem Solving Questions.

Introduction

Share the aim of the professional development meeting.

1. Ask teachers to look at the progression map: problem solving taken from the programmes of study for KS1 and KS2. What do they notice about the progression for problem solving across the year groups? [e.g. the number of references to problem solving; lack of distinction between word problems and pure numerical problems; progression in problem solving appears to be about the mathematics getting harder rather than the complexity of problem solving processes etc.]

2. Share the 'problem solving' aim of the new National Curriculum

The national curriculum for mathematics aims to ensure that all pupils:

*can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions*

Explain that there is no other specific strand for problem solving (as there was as part of Using and Applying Mathematics (AT1) in the 2000 National Curriculum). Therefore we need to understand as a school how we ensure that pupils progress through the skills in problem solving (10 mins)

Developing teachers' subject knowledge

3. Provide the set of progression in problem solving sorting cards between small groups of teachers (ideally teachers who teach different ages of children). Ask the teachers to order the statements according to level of sophistication. When most groups have reordered, compare the different lists. Ask the teachers to compare which statements have been

placed in the same position and which statements where there is disagreement? Ask teachers in their small groups to discuss in which year group you might expect to begin to see some children solving problems in each way described on the cards. Organise their cards to indicate this. Compare and discuss similarities and differences, focusing on the differences to ensure that all are happy that some statements might be more difficult to assign to a particular year group. Conclude by reminding them that these year group-points are when children are likely to begin to show signs of this level of sophistication but that pupils will spiral around these statements with more sophisticated mathematics as well in the following years. (15 mins)

4. Share evidence for the importance of mathematical reasoning:

Ofsted (2011) – Good practice in primary mathematics: evidence from 20 successful schools

Para 41. The emphasis almost all of the schools placed on pupils using and applying their arithmetic skills to solving a wide range of problems was striking. Diverse opportunities were provided within mathematics, including measures and data handling, and through thematic and cross-curricular work. Pupils' extensive experience of solving problems deepens their understanding and increases their fluency and sense of number.

Para 42. Problem-solving and cross-curricular use of mathematics were regular and integral to pupils' learning of mathematics, and diverse in nature.
(p20)

Ask teachers to reflect as a whole group on how frequently they plan for pupils to solve problems in their lessons. What form does this take? (Refer to the progression cards). (10 mins)

Developing Practice

5. Use this selection of questions from KS1 and KS2 test papers that the teachers can try out identify the problem solving skills by referring to the progression cards used earlier. (30 minutes)

Embedding in Practice

6. Conclude by asking the teachers to discuss how the elements of problem solving can be built in to their daily teaching by referring them to the NRICH planning frameworks for KS1 and KS2. If time allows, provide some time for teachers to practice searching through the NRICH site for suitable problems to support their planning. Use the discussion to move into a short period where teachers reflect on their own learning from the PD meeting by identifying changes in their practice that they think they can make. Take some time to identify some next steps individually and as a school/ year groups in order to ensure that the school has a consistent approach to developing progression in problem solving. (15 mins)

Further links

- Research Gateway: Problem Solving

- Professional NCETM Professional Development Calendar: Problem Solving
- Using the bar model to solve problems.

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