Secondary Magazine
Issue 75

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to Issue 75 of the Secondary Magazine.

Contents

Contributors to this issue include: Mary Pardoe, Peter Ransom and Aaron Sloman.

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From the editor

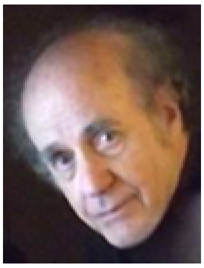
Welcome to Issue 75 of the Secondary Magazine, which is a short issue published just before Christmas, but we have included some activities for the classroom in Christmas Puzzles by Dudeney – just in case you are looking for something different to do in a last lesson of the term! The next issue will include all the usual features.

Both Aaron Sloman, in The Interview, and Professor Sir Christopher Zeeman, in A Christmas lecturer discusses mathematics teaching, raise interesting and important issues about students' experiences of appealing mathematical proofs. We hope their views will provoke fruitful discussion – perhaps not unrelated to concerns raised in the Subject Leadership Diary.

HAPPY CHRISTMAS!

The Interview

Name: Aaron Sloman



About you: Until 2005 I was Professor of Artificial Intelligence and Cognitive Science in the School of Computer Science at the University of Birmingham. I am currently retired, but doing research full time – on a range of topics related to philosophy, cognitive science, artificial intelligence, biology and mathematics – in the School of Computer Science. My first degree was in mathematics and physics in Cape Town (1956), after which I did a DPhil in Philosophy of Mathematics in Oxford (1962), then a few years later discovered that the best way to address many philosophical problems – for example about the nature of minds, and about what mathematics is – is to do Artificial Intelligence, including trying to design and test working fragments of minds.

Do you use mathematics in your work?
I use examples of mathematical ways of thinking, to try to formulate tasks for a designer of intelligent machines, or tasks for someone trying to build a robot that could be a model of mathematical development in young children. For example, how does a child come to understand that it makes no difference whether you count a row of objects from left to right or from right to left? How can a child discover that containment must be transitive? How does a child develop from discovering that some generalisations have no exceptions in our experience to discovering that a subset of those generalisations can be proved to be true in all possible circumstances: for example that three internal angles of a triangle on a plane surface must add up to a straight line?

Why mathematics?
Mathematics was my favourite subject at school and university for various reasons including its power, its depth, and the existence of beautiful short proofs about infinitely varied topics – for instance the ancient proofs that there cannot be a largest prime number, that the square root of 2 cannot be a ratio of two integers, and the purely geometric proofs of Pythagoras' theorem, illustrated in this video and notes about it. When asked my religion on immigration forms etc., I always wrote 'Mathematics'!

Some mathematics that amazed you?
When, many years ago, a school teacher who had been a student at Sussex University 'proved' to me that the internal angles of a triangle add up to a straight line by sliding an arrow, or pencil, round the sides of a triangle, rotating it at each corner through the internal angle, using the five moves labelled "a" to "e", it seemed to me to be a far more memorable proof than the standard one using parallel lines – I was amazed that nobody else had discovered it and used it in the classroom (as far as I knew). Both proofs fail on a curved surface -- but that's also an interesting mathematical fact.

A significant mathematics-related incident in your life?
First learning about transfinite ordinals when I attended lectures given by Hao Wang in Oxford about 50 years ago. These are three examples based on the natural numbers:
'Greater' this way -->

1 2 3 4 .....

.... 4 3 2 1

1 3 5 7 ... 2 4 6 8 ....

It still amazes me that a human mind can think about these infinite structures. I can teach complete non-mathematicians to think about them in about five minutes, so that they can answer some questions about them. Are they discussed in schools? I don't think anyone knows yet how to give machines those human-like abilities to visualise infinite structures and transformations of the structure. We just have not figured out how human brains do it.

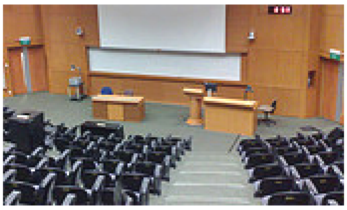
The best book you have ever read?
That's impossible to answer. There probably isn't one. But perhaps Bertrand Russell's *History of Western Philosophy* had the biggest impact on me, leading me to switch from mathematics to philosophy as a graduate student, even though I disagreed with Russell's philosophy on many things, including the nature of mathematics.

Who inspired you?
Immanuel Kant – when I started reading his Critique of Pure Reason many years ago I thought he was closer to the truth about mathematics (as I had experienced it) than Russell, or David Hume, or the philosophers I met in Oxford at the time. My DPhil Thesis was an attempt to show why Kant was right.

If you weren't doing this job you would…
be forced to retire properly! Perhaps I would try to find a primary school that would let me help with teaching of mathematics and artificial intelligence. Or maybe I should try to organise the many half-baked discussion notes on my website into a collection of books, all freely available online.

Any regrets?
In 1978, I thought the tremendous promise of computing to revolutionise many sorts of education (including mathematics, philosophy, and the development of scientific and creative thinking) would be obvious to everyone – as I wrote in The Computer Revolution in Philosophy.

But that promise never materialised. That's partly because it was thought by politicians, employers, parents and teachers, that school kids should be taught to use word-processors, databases, spreadsheets, and other tools that would be useful in their jobs – instead of learning to use computers and programming languages as tools to help them design and test working versions of their own ideas! An example might be inventing a way to express unary arithmetic using lists of symbols. I don't know if it is too late to reverse what happened – unfortunately too many computer experts are now products of that disastrous educational mistake, and have no idea what has been lost.



'Christmas lecturer' discusses mathematics teaching

Professor Sir Christopher Zeeman FRS, who is Honorary Professor at the University of Warwick, was the first mathematician to deliver, in 1978, a Royal Institution series of Christmas lectures - Mathematics into pictures. This was the first series of lectures about mathematics – they are cited by many mathematicians as their childhood inspiration to study mathematics further, and inspired the Royal Institution Mathematics Masterclasses, in which each year 3 000 students take part at venues across the UK. He is also known for founding the Mathematics Department at the University of Warwick. The Christopher Zeeman Medal for Communication of Mathematics was named in his honour, and first awarded in 2008 to Ian Stewart.

In December 2009 Teachers TV published a fascinating 15-minute video, Maths with Professor Christopher Zeeman, in which four mathematics teachers talked with Sir Christopher Zeeman about teaching and learning mathematics.

The discussion, which in the film follows a short introduction to Sir Christopher's RI lectures, focused on the central role of short and appealing formal proofs in mathematical learning in secondary schools. To help you reflect on, and discuss, issues discussed in the film we provide an approximate transcript of the film.

In the transcript, below, each participant in the discussion is indicated in the following way:

CZ - Professor Sir Christopher Zeeman
VK - Vinay Kathotia, Clothworkers Fellow in Mathematics, Royal Institution of Great Britain
T1 - Helene Le Roux, Head of Faculty for Mathematics, St Matthew Academy, London
T2 - Asnat Doza, Mathematics Teacher, Comberton Village College, Cambridge
T3 - Matthew Evans, Head of Mathematics, Herne Bay High School, Kent
T4 - Ruth Tanner, Secondary Mathematics Consultant.

VK 'What do you think is the purpose of mathematics – the nature of mathematics?'

CZ The nature is to understand things better.

VK We'd like to get a sense from you of what mathematics teachers should be engaging with to develop their own mathematical thinking.

CZ Well my main feeling about mathematics is that students should be taught theorems and proofs. Theorems and proofs are the core object of mathematics.

T1 I agree – even for younger people from inner city London, maths is all about getting your thoughts organised.

T2 While I think that this is a fascinating area of mathematics I'm not sure that all my students, some as young as 12 years old, are actually ready to engage in full theorems and proofs – they need to have some background to reasoning before they can actually engage successfully with theorems and proofs – and that needs doing – and it seems we're stuck in that stage throughout secondary school – occasionally we do touch theorems and proofs, but I agree not enough.

T3 The problem is most of us as teachers are in an exam culture where we feel 'I'd love to do more enjoyable maths, but I still feel under pressure to make results'

T4 I think it's an interesting dilemma that teachers are in about doing what they might think is more interesting maths and for fun, but my experience is that if you do that and push students to think more for themselves and get more interested in maths then they ***do*** do well in exams, and I don't think the two things are mutually exclusive.

T3 Do you think a lot of teachers are knowledgeable enough to move to this abstract world?
T2 That might be part of the reason, but I don't think it's the whole reason. The curriculum is divided into very clear categories – where do theorems and proofs go in there?

VK But it's not quite clear how is it is to be done, and whether we have that depth of knowledge and confidence?
CZ It's true that in selecting the theorems and proofs I choose ones that are very appealing – that are easy to state and understand - and easy to explain completely.
VK Do you have an example?
CZ Lots of examples!
VK Let's have a few
CZ A proof that there are an infinite number of primes.

T1 It doesn't matter how many primes there are - that's not the issue – the issue is that reasoning is a way of thinking, and proof is not about showing, for example, a circle theorem - it's about how you organize your mind and thoughts to make a thing that you can't refute.
T4 I've recently done quite a lot of work with students trying to discover some circle theorems for themselves while not actually giving a rigorous proof of them – their proofs were not formal taught proofs - although they were rigorous, they were drawing angles in triangles and definitely dealing with generalities and not just with examples. Do you think that's sufficient for them to come to it that way?
CZ No – they must have complete rigorous proofs.
T4 So would you be happy for them to start in that way – exploring with dynamic geometry, for example, and then taking it on to a rigorous formal proof at the end?
CZ Yes
T1 A proof shows that in any, any case … – that's a concept that in itself is beautiful.
T4 And that's a concept that teachers have to work with all the time in their classrooms at every opportunity – pointing out the difference between example and proof.
T2 And this layer of proof and theorems needs to be there constantly and built over years – and usually we touch it far too late! Coming back to GCSE questions – a question, say, with 6 points on proof - 'here's how you do proof' is probably what's happening quite a lot!
T1 Geometrical proof can be handled in Y7 and be quite formal about it – and you can build it into algebra easily because they've got the idea of how you build a proof. But, because we do very little geometry, we're missing that opportunity - I agree that you need to make it appealing for everyone – but sometimes doing things differently will make it appealing for them – sometimes making them get out of their own world, which is not always a nice place, will make it appealing for them because it's beautiful to be lost in algebra – you're in your own little world of itself – you manage to do things, you learn new things - doing something abstract – something that's not rooted to a world you might not really enjoy or like.
T4 But have a lot of teachers got the confidence to do that yet?
T3 How many times have you heard the question 'Why am I doing this? What's the point? When am I going to use it?' What you're saying is brilliant for one or two percent of the students. Sometimes I'll give them an answer, but they're asking when will THEY use it - not when can ONE use it.

VK So how do we get a whole generation of teachers and students engaging with this sort of approach?
CZ You have to find a subject that is 'playable' – then you exploit the play area, and usually there's some sort of surprise to it.
T2 Some students do have the talent to grasp all this – to generalize properly - perhaps to root out their own misconceptions very early - and they're ready to play with theorems and proofs, and

other things – and some may not – we, as teachers in a classroom, have to cater as well for those who cannot!

CZ **Yes**

T3 We're also trying to break down a barrier of years of a kind of 'Dentist's Theory' - you go to a maths lesson and maths is done on you or to you – and you leave – like going to the dentist – and people don't like going to the dentist – it's so ingrained now – for lots of students at school level their parents say they don't like maths either – at Parents' Evening they say 'I never liked maths – you (to student) don't like maths do you?' You think 'Oh no – they could be such a good mathematician!'

VK It's such a high-stakes game. So you can't introduce the play too much? Because it's there in every testing regime – 'what are they coming out with?'

T4 I firmly believe that if you ***do*** give children opportunities to play with mathematics you won't damage their assessment at the end of it.

T1 I play with them – I say 'who wants to mark mine?' 'Are you going to beat me?' The kids say 'Oh you're doing maths as well' – because they see us telling them or doing things to them, but they don't see us play – doing maths too. If you are part of the game …

T2 Following on from your idea that you're doing maths as well as the students – bringing an idea to the class and taking it wherever it goes without putting a five-minute lesson plan onto it. Bringing a question into class and exploring it together – you're exploring with them. Sometimes I go in with a problem that I don't have the solution for. All I have to have is the competence that I will be able to take it forwards with my students as far as they can go, exploring something together through discussion which leads them to generalisation in mathematics – which will get them ready for theorems and proofs.

VK Some mathematics teachers seem to find it very difficult to engage with the doing, the challenge and the play. You (to T1, T2 and T4) seem to be quite engaged with playing with your children in classrooms, but some teachers find it very difficult.

CZ **Yes**

VK You (to CZ) seem to have engaged, in the master classes, with students and with teachers, with everyone playing with the tasks together.

CZ **Yes**

VK When you started on mathematics was it the play that got you in?

CZ **I've decided on the play – yes – if you stick always to the theory it's no damn good!**

Christmas Puzzles by Dudeney

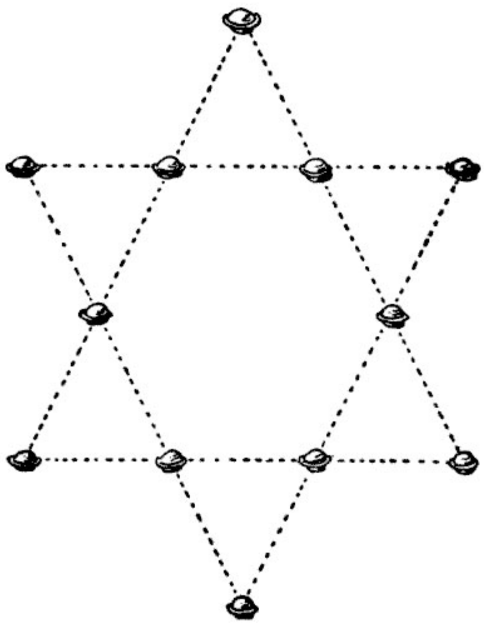
Each of these four seasonal puzzles from Amusements in Mathematics by Henry Ernest Dudeney, published in 1917, might be the starting point for an exploration of some ideas.

Solutions and ideas for extension, and links to classroom presentation and student resource sheets, are further down the article.

Note: Dudeney's original text - both for the puzzles and solutions - is given *in italics*.

The Twelve Mince-Pies

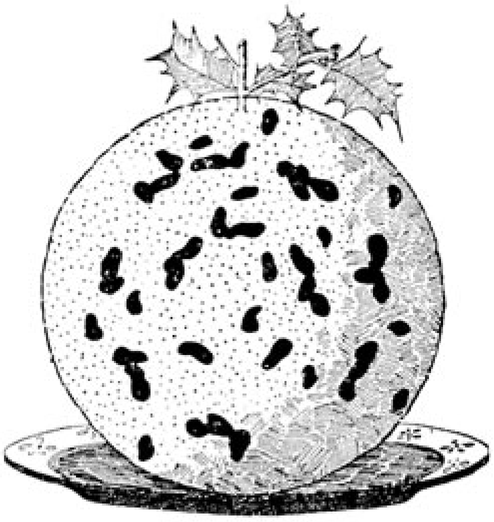
It will be seen in our illustration how twelve mince-pies may be placed on the table so as to form six straight rows with four pies in every row. The puzzle is to remove only four of them to new positions so that there shall be seven straight rows with four in every row. Which four would you remove, and where would you replace them?



Mrs. Smiley's Christmas Present

Mrs. Smiley's expression of pleasure was sincere when her six granddaughters sent to her, as a Christmas present, a very pretty patchwork quilt, which they had made with their own hands. It was constructed of square pieces of silk material, all of one size, and as they made a large quilt with fourteen of these little squares on each side, it is obvious that just 196 pieces had been stitched into it. Now, the six granddaughters each contributed a part of the work in the form of a perfect square (all six portions being different in size), but in order to join them up to form the square quilt it was necessary that the work of one girl should be unpicked into three separate pieces. Can you show how the joins might have been made? Of course, no portion can be turned over.

The Christmas Pudding



"Cut the pudding into two parts, each of exactly the same size and shape, without touching any of the plums. The pudding is to be regarded as a flat disc, not as a sphere."
"Why should you regard a Christmas pudding as a disc? And why should any reasonable person ever wish to make such an accurate division?" asked the cynic.
"It is just a puzzle—a problem in dissection." All in turn had a look at the puzzle, but nobody succeeded in solving it. It is a little difficult unless you are acquainted with the principle involved in the making of such puddings, but easy enough when you know how it is done.

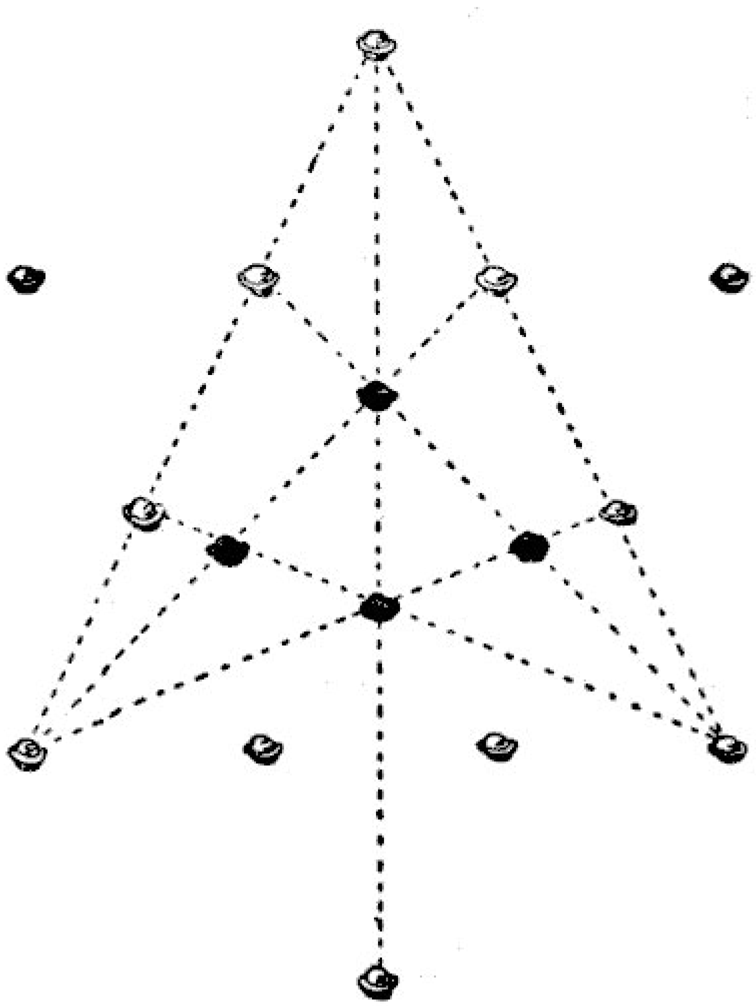
A Calendar Puzzle

> *If the end of the world should come on the first day of a new century, can you say what are the chances that it will happen on a Sunday?*

Dudeney's solutions

The Twelve Mince-Pies

If you ignore the four black pies in our illustration, the remaining twelve are in their original positions. Now remove the four detached pies to the places occupied by the black ones, and you will have your seven straight rows of four, as shown by the dotted lines.



Is this the only solution?

Challenge students to create their own mince-pies puzzles with various different numbers of mince-pies. They should provide at least one solution to each puzzle that they create.

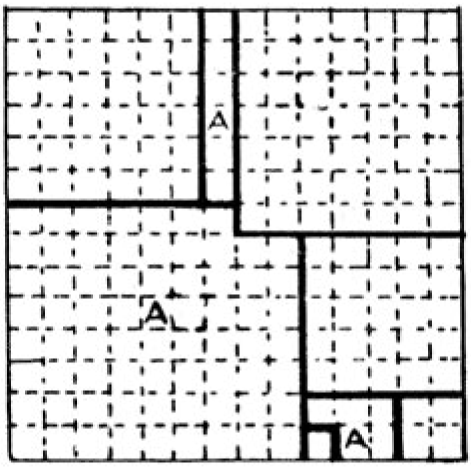
What is the least number of mince-pies with which it is possible to form a puzzle of this kind?

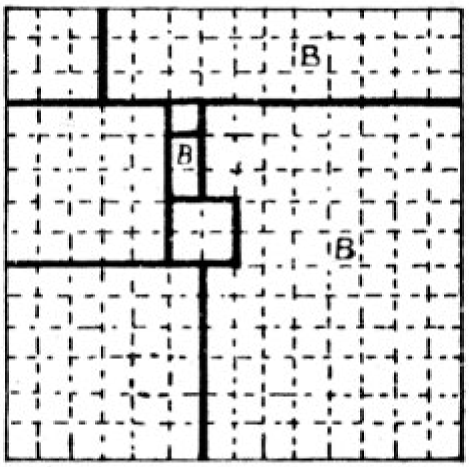
Can they create more than one puzzle with one initial arrangement of mince-pies?

Give mince-pies as prizes?

Mrs. Smiley's Christmas Present

The first step is to find six different square numbers that sum to 196. For example, 1 + 4 + 25 + 36 + 49 + 81 = 196; 1 + 4 + 9 + 25 + 36 + 121 = 196; 1 + 9 + 16 + 25 + 64 + 81 = 196. The rest calls for individual judgment and ingenuity, and no definite rules can be given for procedure. The annexed diagrams will show solutions for the first two cases stated. Of course the three pieces marked A and those marked B will fit together and form a square in each case. The assembling of the parts may be slightly varied, and the reader may be interested in finding a solution for the third set of squares I have given.





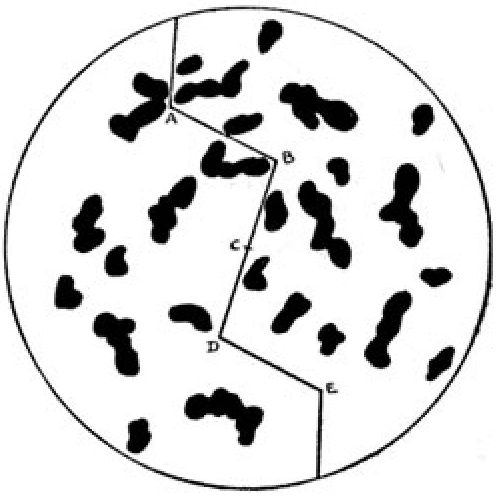
Are there only three solutions? When comparing their solutions students will need to decide what will be their criteria for 'sameness'.

Challenge students to make up their own square patchwork quilt puzzles for quilts with various numbers of small squares on each side.

What is the minimum number of small squares for which it is possible to create a puzzle of this kind? For example, can you make a puzzle for a 4-by-4 quilt?

The Christmas Pudding

The illustration shows how the pudding may be cut into two parts of exactly the same size and shape. The lines must necessarily pass through the points A, B, C, D, and E. But, subject to this condition, they *may be varied in an infinite number of ways. For example, at a point midway between A and the edge, the line may be completed in an unlimited number of ways (straight or crooked), provided it be exactly reflected from E to the opposite edge. And similar variations may be introduced at other places.*



Do you and your students agree with this solution? Why or why not?

Challenge your students to design their own Christmas Pudding puzzles – and provide solutions that they can explain.

(It may be helpful to suggest that they start with puddings with very few plums).

A Calendar Puzzle

The first day of a century can never fall on a Sunday; nor on a Wednesday or a Friday.

Why is this? Can your students explain? Can you?

The puzzles are available to download - as a PDF document, and also in PowerPoint format to display on a whiteboard.

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Mince pie photograph by dichohecho some rights reserved
All black and white drawn illustrations from Amusements in Mathematics by Henry Ernest Dudeney (1917) in the public domain

Subject Leadership Diary

Well, what a week with a White Paper and a white-out! There have been some very difficult decisions we have had to make as a faculty recently. We had many discussions on where we would go, but in the end we decided that this year we would go for a curry for our Christmas do. That way there's generally no problems in getting a suitable venue – and the varied menu allows everyone to have their own choice, rather than the limited 'Christmas menu' that is often on offer at rather inflated prices. Someone volunteers to sort out the date, and someone else searches for any available discount vouchers.

There is much to commend in The Importance of Teaching - The Schools White Paper 2010, as well as parts that need some examination and further thought – the behaviour section contained statements that had me agreeing one minute and disagreeing the next!

It was also interesting to read about teacher education. For example in section 2.21 *We will provide more opportunities for a larger proportion of trainees to learn on the job by improving and expanding the best of the current school-based routes into teaching – school-centred initial teaching training and the graduate teacher programme*. Nobody can argue that we learn 'on the job' and teaching practice is an essential part of learning how to teach. My worry here is the time a mentor spends with a PGCE teacher at the moment. If we are expected to provide more opportunities in school that means more mentor time is needed and our school students get less time with these experienced teachers. We will wait and see what the implications for mathematics teachers are.

Under the curriculum section we see the following:
We will focus central government support on strategic curriculum subjects, particularly mathematics and science
4.25 We will continue to provide additional support for the uptake of mathematics and the sciences. A strong national base of technological and scientific skills is essential to growth and employers continue to report shortages of these skills.
Yes, we know that there is a shortage of students taking up science and mathematics at universities and progressing into teaching, and the STEM initiative has been going for some years now. Hopefully we will see more importance placed on this work in schools and a greater uptake in STEM initiatives. I have a personal interest in this having recently worked with a London school's Y10 students for a day – it is good to see some schools preparing young people for the future by introducing them to the way science, technology, engineering and mathematics are integrated together.

That's the politics out of the way. What will we as mathematics teachers actually do in practice? We will moan, complain about new initiatives and then get on with it, taking on board the new recommendations that we know will work and working out how best to carry out any other initiatives. After all, all our students do as we tell them, don't they?

I've seen some outstanding lessons this past fortnight taught by PGCE trainee teachers. They have been guided by great mentors and their students have been enraptured by the stimuli they have experienced. The lessons have had pace and challenge and I just wish when I was training to be a mathematics teacher I could have produced differentiated materials like those I've seen. One student teacher had three pupils acting as a human fruit machine – with apples, bananas and oranges – so that the class could compare theoretical and experimental probability, AND managed to incorporate '5 a day' thus including the 'be healthy' aspect of Every Child Matters. It reassured me that the future of mathematics teaching is in some safe hands.

Well, nearly December now and GCSE mocks are on the horizon. So it's keeping the Year 11s focussed, and dusting off the Christmaths activities, and organising the annual mathematics Christmas card competition – cards must include something mathematical and something about our school, and we had over 400 entries last year. We use the winning design (with the back of the card featuring the runner-up designs) as the card that goes out from our faculty to a host of people – hits the key concept of creativity on the nose, Rudolph!

Image Credits
Christmas lights in shape of triangles photograph by Rhalah in the public domain