NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Core concept 4.1: Sequences

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

4.1.1 Understand the features of a sequence

How confident are you that you can generate and describe sequences through both term-to-term and position-to-term rules?

1 ☐ 2 ☐ 3 ☐ 4 ☐

A **mathematical sequence** is a list of terms. The sequence has a starting place and a generating rule. The sequence can be generated from numbers, shapes, musical notes, or any other items you can describe and apply a rule to.

When describing the sequence 3, 5, 7, 9, 11, … students may often say, *'It goes up in twos'*. Through discussion, this response should be refined so that students are more explicit regarding the starting number and the amount added each time. For example:

- *'The sequence begins with three, and two is added each time'*
- *'The first term is three, the second term is three plus two, the third term is three, plus two, plus two, etc.'*

Students should also experience sequences where there are multiple ways the sequence could be extended. For example, the terms in the sequence 1, 2, 4, … could be generated by:

- doubling each term to get the next (1, 2, 4, 8, 16, 32, …)
- adding one, then two, then three, etc. (1, 2, 4, 7, 11, …).

Students should explore being able to generate a sequence given a rule expressed algebraically, as a position-to-term rule, for example $4n - 1$. The power of using this approach to find any number in sequence without writing the whole sequence out should be emphasised. Students often find the concept of 'n' challenging and the language of 'term number' and 'position in the sequence' can help this.

Further support links

- NCETM Secondary Professional Development materials: 4.1 Sequences, pages 8–12

4.1.2 Recognise and describe arithmetic sequences

How confident are you that you understand, and can explain, how the structure of an arithmetic sequence can be explored through its position-to-term rule (nth term) including the calculation of any term and determining whether a number is a term of a given sequence?

1 ☐ 2 ☐ 3 ☐ 4 ☐

An **arithmetic sequence** (also known as an **arithmetic progression or linear sequence**) is a sequence of numbers where the term-to-term rule is adding or subtracting a 'fixed number'. This number is the **difference** between consecutive terms.

In general, for an arithmetic sequence with first term a and difference d, the nth term is $T_n = a + (n - 1)d$.

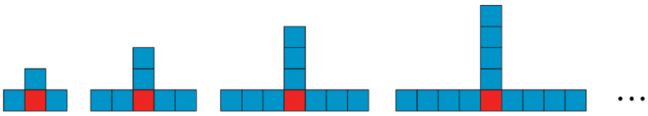
For example, in the sequence 5, 8, 11, 14, 17, … the rule is 'add 3', so $d = 3$. The nth term is $5 + 3(n - 1) = 3n + 2$.

When examining arithmetic sequences, it is natural for students to express the rule in terms of this fixed amount. For example, students may see the sequence 4, 7, 10, 13, … as 'add 3' and think that, therefore, the nth term is $n + 3$ rather than the correct $3n + 1$.

This misconception can be explored by examining the three times table (i.e. 3 (3×1), 6 (3×2), 9 (3×3), 12 (3×4), 15 (3×5), …, $3n$) and a range of other sequences that consist of terms in the three times table with one, two, three, etc. added (or subtracted). Students should be encouraged to notice what's the same and what's different – i.e. it is the '$3n$' that determines the 'increasing by three'. It will then be helpful for students to experience substituting $n = 1, 2, 3$, etc. into the expression '$n + 3$' to realise that this will give a sequence that begins at 4 and increases by one, rather than beginning at 4 and increasing by three.

The fundamental awareness here is that, in the general statement $3n + 1$, the common difference is represented by the '3' (the coefficient of n) because as n increases, the value of the whole expression increases by three. It is important here to explore any potential confusion between multiples of three (for example) and numbers that increase by three, and to recognise these are not necessarily the same thing.

The use of growing shape patterns that generate such sequences of numbers can support students in seeing what is constant in each term and what varies. For example, *'How many squares are needed to make each shape in this sequence?'*



Rewriting each of the numerical terms to reveal the structure can also assist in developing this awareness (e.g., $1 + (1 \times 3)$, $1 + (2 \times 3)$, $1 + (3 \times 3)$, $1 + (4 \times 3)$, …).

An alternative way to identify the expression is that the 3 in $3n + 2$ is the difference between consecutive terms. The $+2$ in $3n + 2$ is the term that would come before the first term, sometimes called the zero term.

Other approaches include linking the sequences to straight line graphs. (The term number is plotted against the x-axis and the solution to the term number is plotted against the y-axis.) This results in the constant difference between the terms corresponding to the constant rate of change (the gradient) of the graph, and the y intercept corresponding to the constant in the nth term expression.

Further support links

- NCETM Secondary Professional Development materials: 4.1 Sequences, pages 13–19
- NCETM Secondary Professional Development materials: 1.4 Simplifying and manipulating expressions, equations and formulae, pages 14–17

4.1.3 Recognise and describe other types of sequences (non-arithmetic)

How confident are you in your knowledge of other sequences such as geometric and Fibonacci sequences, square, triangle and cube numbers?

1 ☐ 2 ☐ 3 ☐ 4 ☐

It is not uncommon for students to notice only additively increasing sequences (i.e., arithmetic sequences where the common difference is positive), so students should experience a varied collection of types of sequence. For example:

- 3, 6, 12, 24, 48, 96, … (geometric sequences, where there is a constant multiple or ratio between successive terms)
- 1, 4, 5, 9, 14, 23, … (Fibonacci-like sequences, where terms are generated by adding the two preceding terms)
- 1, 4, 9, 16, 25, … (square numbers)
- 1, 8, 27, 64, 125, … (cube numbers)

Notes