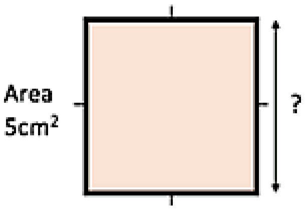
Secondary Magazine 151

National Centre for Excellence in the Teaching of Mathematics

Welcome to the NCETM Secondary Magazine. How do you look for inspiration for the classroom? We hear from a teacher who loves the ideas and support that judicious use of Twitter can provide. And for those teaching mechanics for the first time this year - without turning to Twitter - another teacher shares his inspired ideas for introducing the constant acceleration equations.

Don't forget that all previous issues are available in the Archive.

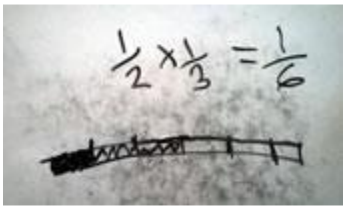
This issue's featured articles



Teaching surds - how Twitter came to the rescue!
Some teachers may know Kathryn Darwin via Twitter. Here, she shares the pleasure of her Twitter 'virtual staffroom' by starting a thought-provoking discussion about where to begin teaching surds.

$$s = ut + \frac{1}{2}at^2$$

Inspiration and ideas for teaching constant acceleration
Teaching mechanics for the first time? A level teacher Mark Dawes offers some insights and imaginative ideas for teaching the constant acceleration equations. How can you derive them from GCSE knowledge of velocity-time graphs? How can you use them in a practical experiment involving a crazy swan-dive?



From our Primary Magazine: Keeping the 'Rapid Graspers' Engaged
More and more primary schools we are working with are moving to teaching maths to whole classes, rather than differentiating work for different 'tables' of children grouped by perceived ability. Often this has gone hand-in-hand with adopting a teaching for mastery approach to the curriculum. As teaching for mastery spreads into secondary schools, you may be interested to read about solutions primary teachers are finding to extending and deepening learning for 'rapid graspers', when they have 'got it'. How many of these strategies could be used in a secondary classroom? You might also like to read the NCETM's position on 'ability' grouping.

And here are some other things for your attention:

NCETM and Maths Hubs news

- A chance to understand what we can learn from Shanghai maths teachers! Early next year, secondary schools are involved in the **England-China Exchange** (14–25 January 2019) with eight schools across the country opening their doors to hundreds of local teachers who want to see teaching for mastery in action. See the list below to find out how to see a Chinese teacher in action.
- We've produced a new **poster** to help teachers understand the range of professional development options, run by **Maths Hubs** and the **Advanced Mathematics Support Programme (AMSP)**, available to anyone teaching maths beyond GCSE.
- What is it like to be in a Maths Hub Work Group? Find out from a new **NCETM podcast**.
- Have you read NCETM Director Charlie Stripp's **latest blog**? In it he suggests that all students should be encouraged to have a go at maths competitions.

- We've added an extra date for the conference for ITE tutors on teaching for mastery approaches in secondary schools: more places in London (28 January), and now also in Birmingham (25 February). **Find out more**.

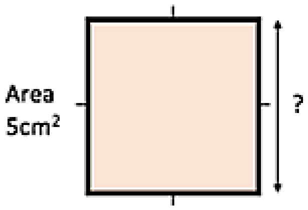
Other news

- The **Association of Teachers of Mathematics (ATM)**/**Mathematical Association (MA)** joint annual conference (15-18 April 2019) is taking bookings. A number of bursaries are being offered through the **ATM**.
- The DfE has published a list of institutions selected to become **FE Centres for Excellence** to improve basic maths for post-16 learners. Where's your nearest?
- The Institute of Mathematics and its Applications (IMA) is again offering **Mathematics Teacher Training Scholarships** (worth £22,000), for those starting their secondary teacher training in 2019/20.
- For those applying for an undergraduate degree considering teaching as a career afterwards, it may be worth considering the **Future Teaching Scholars Programme**, offering £15,000 of support during undergraduate years.

These eight secondary schools will be hosting the 2018/19 Shanghai Showcase lessons. Click the school nearest to you to find out more and register your interest:

- St Cuthbert's Catholic High School, Newcastle
- Loreto Grammar School, Altrincham
- Hungerhill School, Doncaster
- Tudor Grange Academy, Solihull
- Impington Village College, Cambridge
- Harris Academy, Bermondsey (south London)
- Mayfield School, Portsmouth
- The Castle School, Taunton.

Image credit
Page header by Maria Shanina (adapted), in the public domain



Teaching surds - how Twitter came to the rescue!

Kathryn Darwin (@Arithmaticks) regularly uses Twitter as a platform for professional discussion, camaraderie and encouragement. Recently, faced with a dilemma about how to sequence teaching points in a unit of work on surds, she turned to her Twitter colleagues for their thoughts. What prior knowledge should students be equipped with before tackling simplification of surds?

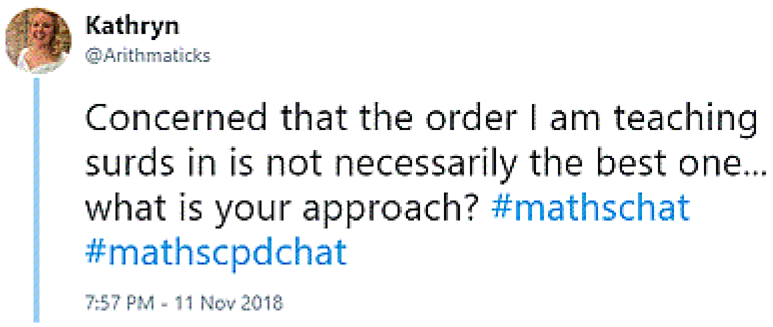
In her own words, Kathryn shares this experience:

It is no secret that I adore "Teacher Twitter". I use it to network with colleagues across the globe and view it as more of a portable staff room than a social media site. I love that I can simply keep track of the highlights of my day via #loveteaching, participate in various other CPD chats or find countless articles about anything from behaviour management to cognitive load.

But most of all I love the comradery of countless teachers. The whole place is fit to burst with teachers sharing resources, discussing pedagogy or just picking each other up when they are down.

As with many other resources, maths teachers are spoiled for choice on Twitter – it makes you wonder if you'll ever have to plan a lesson yourself again. Quite often, I turn to these "tweachers" to find ideas for how to approach a topic and discuss both the mathematics and pedagogy behind these choices.

Most recently, I started to over-think how I teach surds. This was brought on primarily because I was going to be teaching it to Year 8 for the first time – this is the first year it has been included in the Y8 scheme of work, in order to build up procedural fluency and aid teaching problem solving in KS4. I wanted to make sure that the way I introduced it allowed them to succeed in the future. So I tweeted:


Kathryn
@Arithmaticks

Concerned that the order I am teaching surds in is not necessarily the best one... what is your approach? #mathschat #mathscpdchat

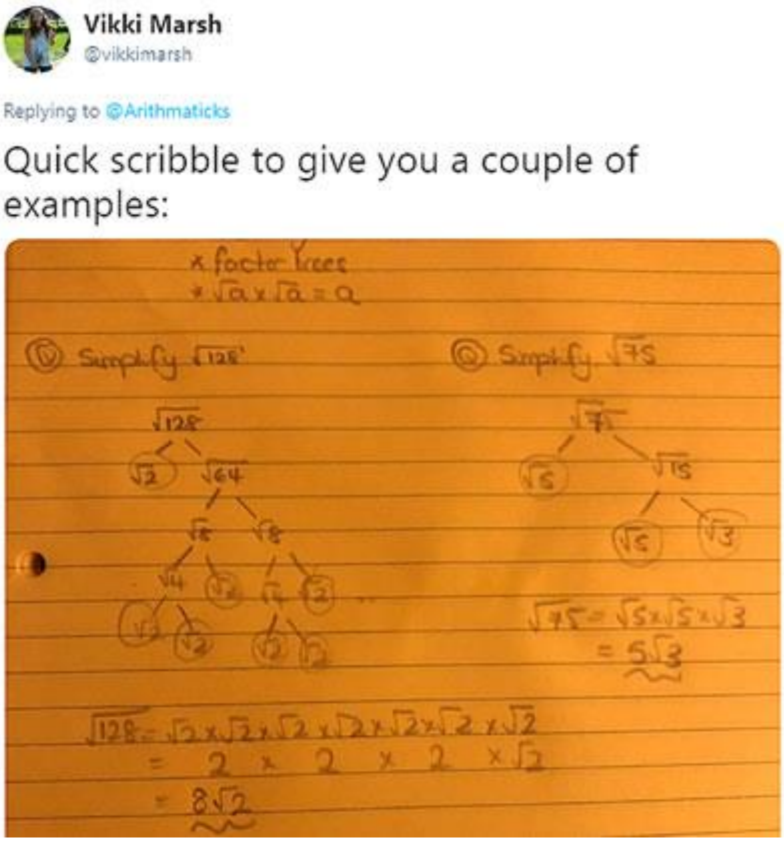
7:57 PM - 11 Nov 2018

I included #mathscpdchat and #mathschat to ensure it would get to the right people, and I quickly got responses. It is probably worth saying that my usual approach, introducing surds to older pupils, would be in this order:

- multiplying with no simplification, e.g. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$
- division by a surd (with a common factor, so no need to rationalise), e.g. $\frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$
- then simplifying, e.g. $\sqrt{8} = 2\sqrt{2}$

I would then move on to more complex calculations using these general principals. However, having spoken to many of my "real life" colleagues and looking at the Scheme of Learning, I was starting to wonder if simplification was the place to start; the replies I was receiving on Twitter echoed this.
One reply in particular caught my eye. Vikki Marsh (@vikkimarsh) suggested that the way she introduces simplification is through factor trees. Both of my Year 8 sets were confident using factor trees for whole

numbers in the last unit and so I asked for more information. She responded with a couple of examples (the discussion thread is here):



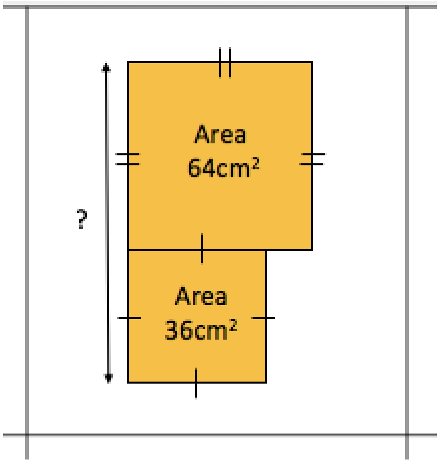
(Ctrl+click to enlarge)

I loved this idea and I quickly decided that I would start with this for the sake of familiarity, and then introduce the formal presentation alongside it once students were confident. I still wasn't sold on this method of 'factorising' before students knew how they could multiply surds. They would need to know that $\sqrt{a} \times \sqrt{a} = a$ at the least, so why not cover general multiplication of surds too? Is $\sqrt{2} \times \sqrt{3}$ less important than $\sqrt{2} \times \sqrt{2}$? I was willing to be proven wrong so carried on chatting to Vikki and others about it.

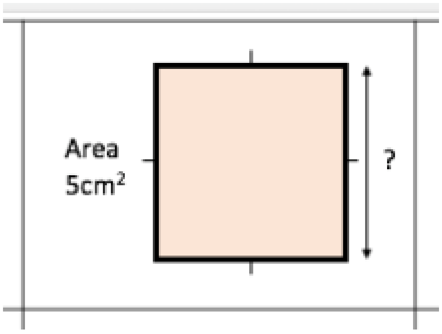
The "simplifying first" team were louder than the "multiplying first" one, and an argument that this was "intuitive" through the factor trees as a result of knowledge of prime factor decomposition won me over. I decided to try it and planned an alternate version of my lesson with the order switched.

While I was doing this Miss Konstantine (@GiftedBA) sent me a tweet with a new resource on her blog (matching area diagrams with numerical expressions) she was working on. It involved looking at the area and side lengths of various squares, in order to get students using roots.

I popped the matching exercise into the start of my lesson to get students thinking about when and where to use surds. This went down well in my lesson – students were confident with the 'root' meaning the inverse of 'squaring'. It facilitated a lovely discussion about why it would be more efficient to use 6 & 8 rather than $\sqrt{36}$ & $\sqrt{64}$.



I then extended this with a square of area 5cm^2 as in the second part of Miss Konstantine's resource:

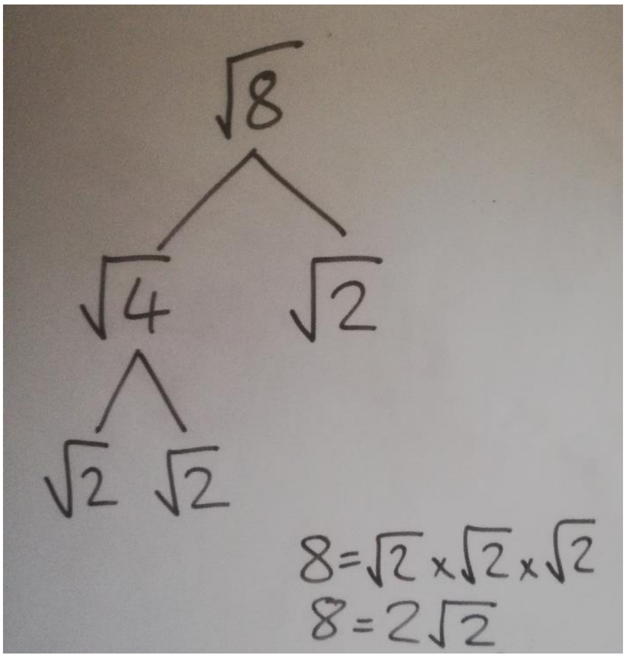


…and we discussed why leaving the side length as $\sqrt{5}cm$ would be more useful than the decimal shown in the calculator. One student asked

> *"Is this like when there is a recurring decimal for 1/3 and we use that instead of rounding it, so we stay accurate?"*

I really felt like we'd cracked it.

So now was the moment of truth… simplifying (before covering multiplying)! Students seemed happy with $\sqrt{a} \times \sqrt{a} = a$ and could show it in a variety of ways. So then I introduced $\sqrt{8}$ and began a worked example using the factor tree:

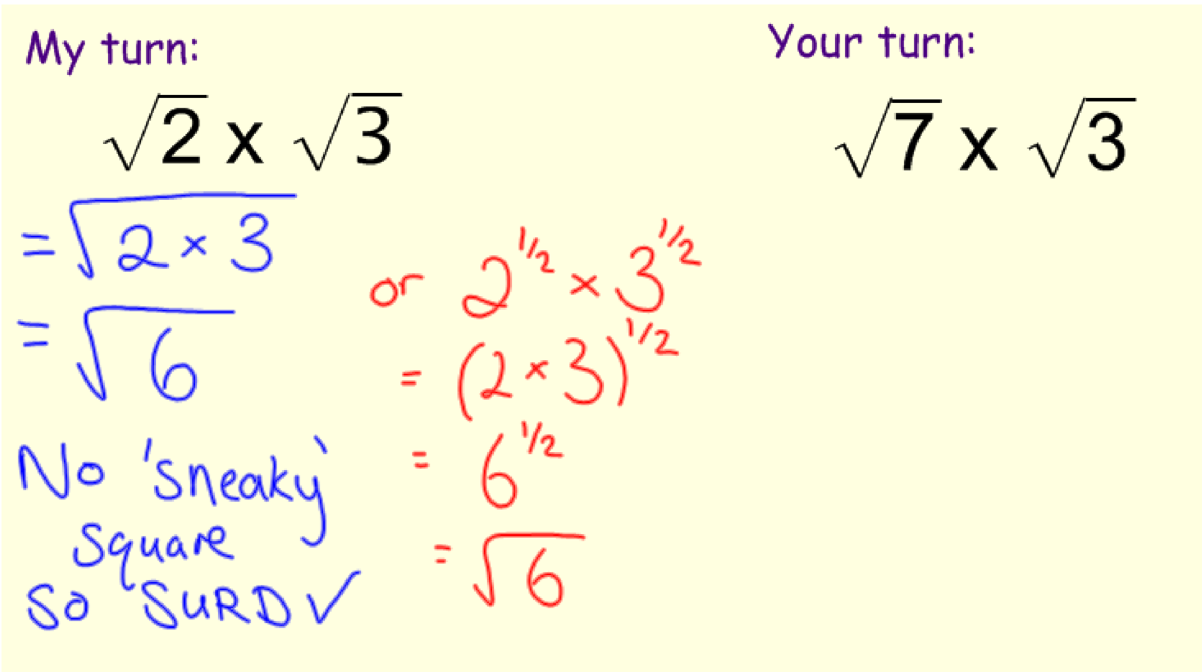


…then the questions started about WHY I can factorise like this:

> *"I know 2 x 2 x 2 is 8 BUT why is there still a root on every one, doesn't it just need one?"…*

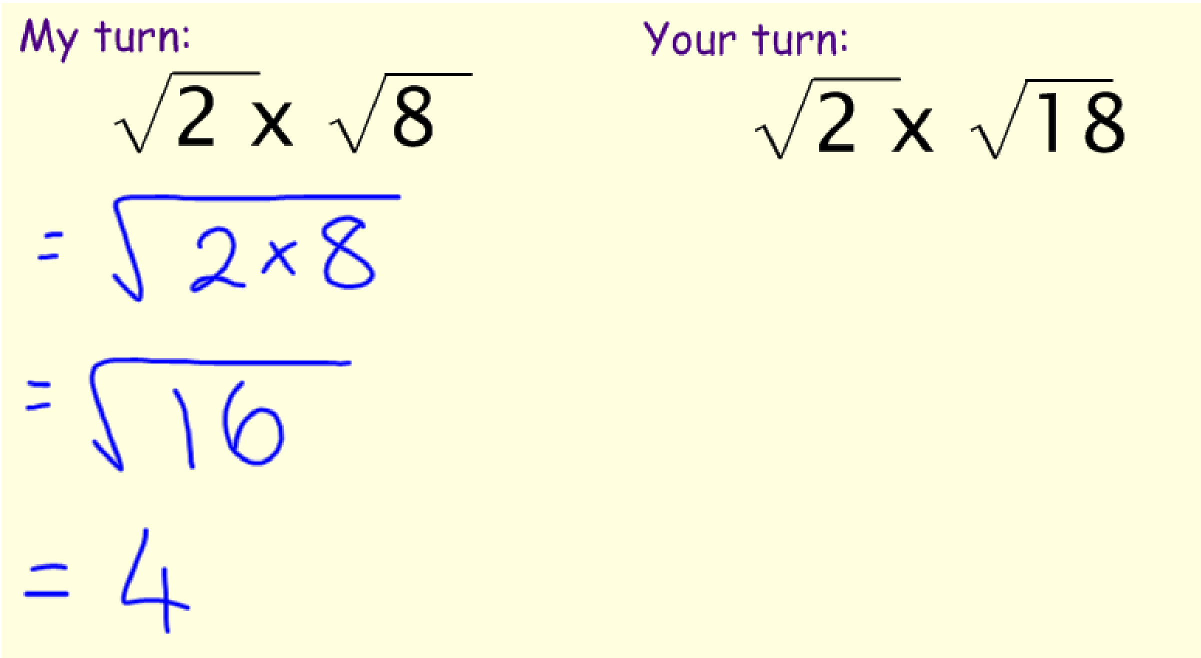
We tried one more ($\sqrt{18}$) but the questions kept coming thick and fast.

I fast-tracked to my original start point, using whiteboards and example problem pairs:

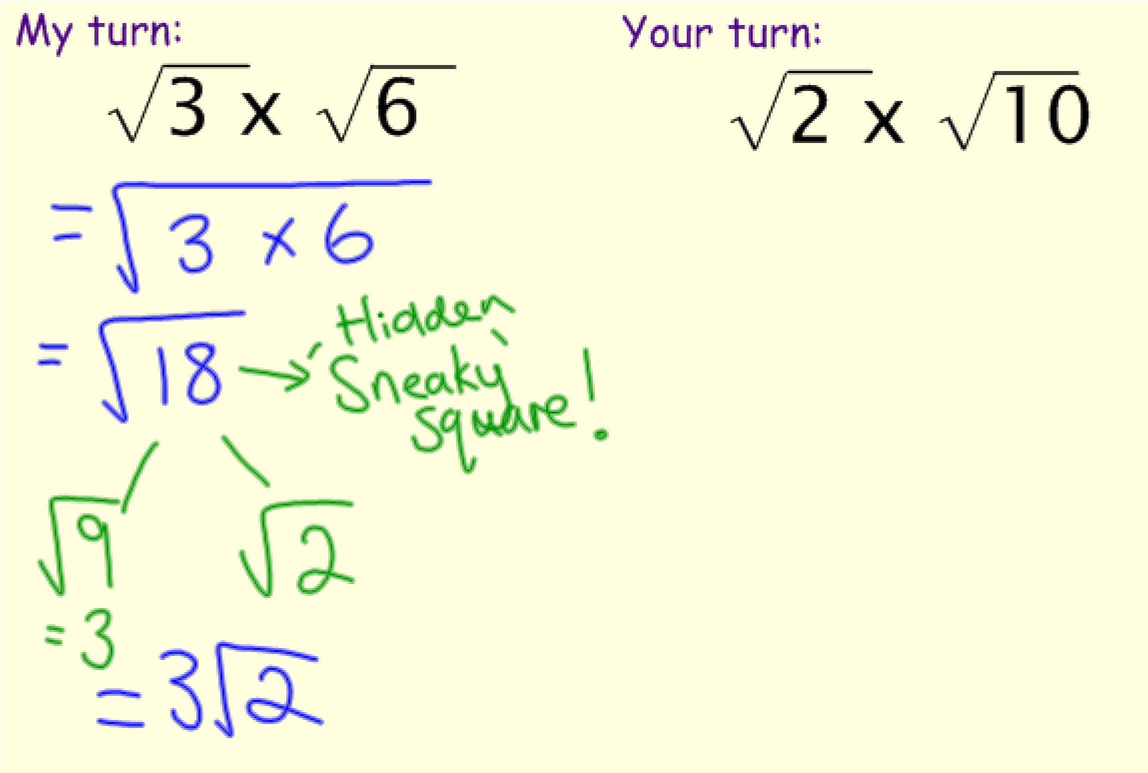


I used their familiarity with our previous topic to my advantage – they were excellent with indices. Quickly students began to nod, and there was a little chorus of "ooooh". Students then tried $\sqrt{3} \times \sqrt{5} = \sqrt{15}$ really successfully.

With them buying in using surds or indices, I pushed them on to $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$, which caused huge excitement as it wasn't a surd anymore! We had an "obvious" square! The class were then comfortable with completing $\sqrt{2} \times \sqrt{18}$ independently:



I moved on to $\sqrt{3} \times \sqrt{6} = \sqrt{18}$. Then one student said "But there's a hidden 9 in there, so can't we use the factor tree now?" So we did!



Suddenly, as a result of understanding multiplication, factorising was making sense. So I carried on from here. We just multiplied for a while – getting students used to the root "sticking around", and a few students started to simplify surds with the factor tree. I'm hoping next lesson will bring ALL students using this method and then formalising their notation.

Despite the fact that the lesson didn't go perfectly, I'm glad I used the ideas I gathered from Twitter. It was the best form of CPD, pushing me out of my comfort zone and encouraging me to reflect. My surds lessons in the future will definitely use the factor tree… but I will be using my original sequence next time. As Cat (@CMaths3) said, we need to discuss multiplying first "otherwise I don't think simplifying makes sense".

Kathryn helped her students to understand the rule for multiplying surds by linking it back to previous work they had done on indices. But the discussion made us, at NCETM, made us wonder about other ways of helping students to understand deeply why: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. *We'd love to hear from anyone that might have visual (or other) representations to help with this understanding. Email or tweet us.*

Kathryn's Seven Top Tips for getting started on Teacher Twitter…

- Create an account for your 'teaching' personality only – You will get less out of the account if you post pictures of your lunch than if you engage in conversation about education and sharing resources/ideas regularly.
- If you think you may want to whinge, make it your account anonymous!
- If you want to be a #loveteaching ambassador, share every success you have and spread the positivity.

- Share successes of your classes and tag your school in them if they have an account! They will share these more widely with students and parents too.
- Follow the likes of @mathsjem (Jo Morgan/Resourceaholic), @JustMaths, @AccessMaths, @PixiMaths and other resource creators/collators as a start. From them you will find many more maths teachers to follow as they interact with them.
- Get involved with weekly themed 'chats'. To join in, reply to tweets with the hashtag during the designated time slot. Click it to follow all the replies.
    - #mathscpdchat – Tuesday 7-8pm – run by the NCETM, hosted by various maths teachers across the UK (including me!) with a different theme each week
    - #MathsChat – Wednesday 8-9pm – affiliated with AQA, but discussions of all kinds of maths related topics, hosted by @BetterMaths
    - Many others including #SLTChat, #ITTChat, #NQTChat for specific points in your career.

- If you're not sure about joining in yet, just lurk! Read what people say and absorb ideas, one day you will just reply and never look back!

Inspiration and ideas for teaching constant acceleration

With applied maths units now a compulsory component of the new A level, there will be many teachers picking up applied maths teaching for the first time. We asked Mark Dawes (teacher at Comberton Village College and Deputy Lead for the Cambridge Maths Hub) what suggestions he might have for teachers teaching mechanics for the first time.

If you use any of his ideas, we'd love to hear how it went. And if you'd like more articles for those new to teaching stats and mechanics, please let us know.

One of the brilliant things about teaching A level maths is being able to introduce students to new and exciting areas of mathematics. This is as true in mechanics as it is in other part of the new A level. It might therefore seem to be a shame that GCSE mathematics now expects a certain level of familiarity with the constant acceleration formulae, because these won't be new when studied at A level. On the other hand, there is an opportunity when revisiting these formulae not just to practise how to use them, but also to explore where they come from.

I like to begin with a review of what my Year 12 students can recall from GCSE. They sometimes refer to these as the 'suvat equations', because this is a reminder of the letters that are involved.

They may know some or all of them:

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$s = vt - \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \frac{1}{2}(u + v)t$$

It is worth looking at these and discussing some of the key features. Some people refer to these as 'Newton's equations of motion', and others as 'the constant acceleration formulae'. There are five different quantities involved:

s – the displacement
u – the initial velocity
v – the current velocity
a – the acceleration
t – the time.

Each equation uses four of the variables and each equation is missing a different one of them. That is very helpful because, for example, if we don't know and don't care about the displacement, we can use the first of the equations. If we know any three of the quantities we can calculate a fourth.

We need to have the values in consistent units, so usually s is measured in metres, u and v in ms^{-1}, a in ms^{-2} and t in seconds. The name 'constant acceleration formulae' reminds us that they only work if the acceleration is constant.

Many students are surprised that *s* does not stand for speed. These equations are related to vector equations, so the direction of the velocities and the acceleration is important. We therefore use velocity and *not* speed and displacement and *not* distance.

After collecting together the formulae, I use them to calculate the height of the school lecture theatre. One student practises throwing a tennis ball vertically upwards until it almost touches the ceiling. Other students time how long it takes for the ball to fall from its highest point to the ground. We can then calculate the height using a formula.

From the highest point (and taking the vertical down direction to be positive) we have:

$$u = 0$$
$$a = 9.8$$
$$v = ?$$
$$t = time\ recorded$$
$$s = ?$$

We want to calculate the height, which is the displacement and don't care about the velocity, so we need the equation that doesn't involve *v*. That is:

$$s = ut + \frac{1}{2}at^2$$

Substituting into this gives us the displacement of the tennis ball, so the height of the hall.

If you don't have a suitable high building then a video of the extraordinary 'Professor Splash' can be used. He holds the world record for diving from a tower into a paddling pool of water.

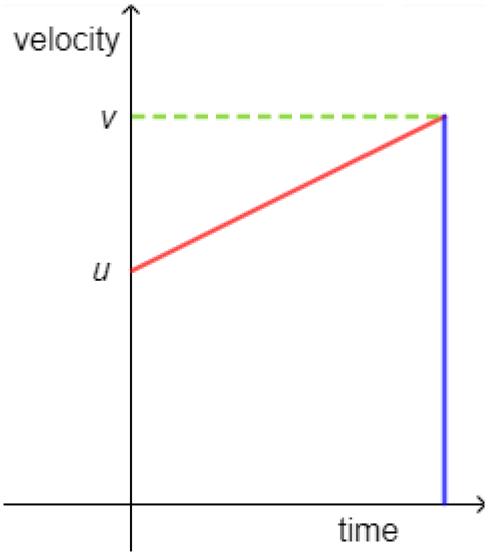
When I use this, I hide the text and ask the students to time the dive. This gives us an opportunity to use a range of times (or the average) to calculate the height dived, before checking our answer.

So far then we have reminded ourselves of the formulae, have made explicit things that the students may not have realised were important at GCSE, such as that the acceleration must be constant, the units compatible, etc, and used the formula in a practical experiment.

Where do the equations come from?

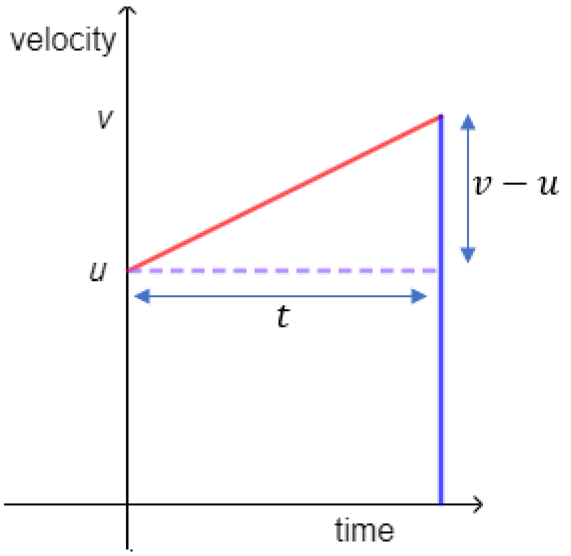
It is not always necessary (or possible) to derive every equation we use in A level mathematics, but here it seems useful to do so. Partly this is because it helps students to understand them, partly because they may then be able to recall them better and partly because they need to know about the graph that produces them in any case.

Most of them come from a velocity-time graph:



The initial velocity, *u*, is when the time is zero, so that is where the graph crosses the y-axis. The red line shows the velocity of the object at each time, so here the velocity is increasing.

What do we know about this graph? The gradient of the graph is the change in velocity compared to time, so this is the acceleration. We can work out what this is:



The gradient is therefore

$$a = \frac{v - u}{t}$$

Rearranging this gives

$$v = u + at$$

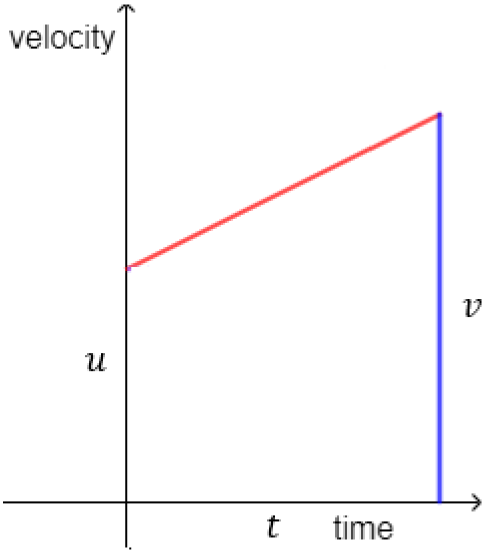
That's a good start. But there's more to just this first equation. We know we can represent a straight line using $y = mx + c$. Here the x-axis is time, the y-axis is velocity, the gradient is *a* and the y-intercept is *u*. Substituting these into $y = mx + c$ gives us $v = at + u$

That's rather good: the equation $v = u + at$ is actually the same as $y = mx + c$!

This also shows us why we have to have constant acceleration for these formulae to work: we can only write the gradient in this way if the line is straight.

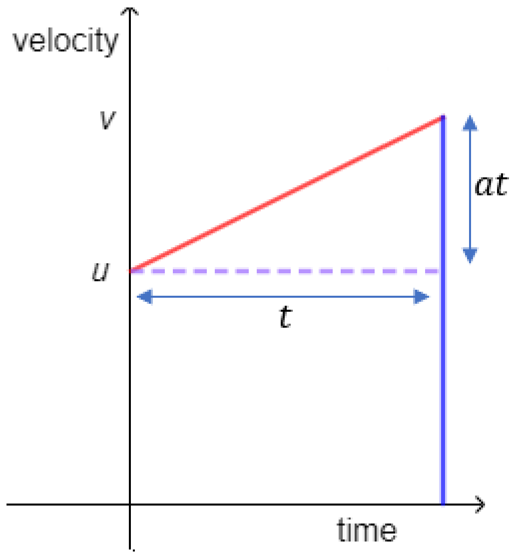
The area under the graph is the displacement. (The area of a rectangle can be calculated by multiplying the base by the height. On the velocity-time graph the units can help us to see that something measured in m/s multiplied by something measured in seconds gives us an answer measured in metres.)
There are several ways to work out this area and three of them are useful for our purposes.

We start by treating it as a trapezium (with the two parallel sides running vertically):



The area of the trapezium is $s = \frac{1}{2}(u + v)t$, which is another of our equations.

Next, we consider the area as a rectangle added to a triangle:

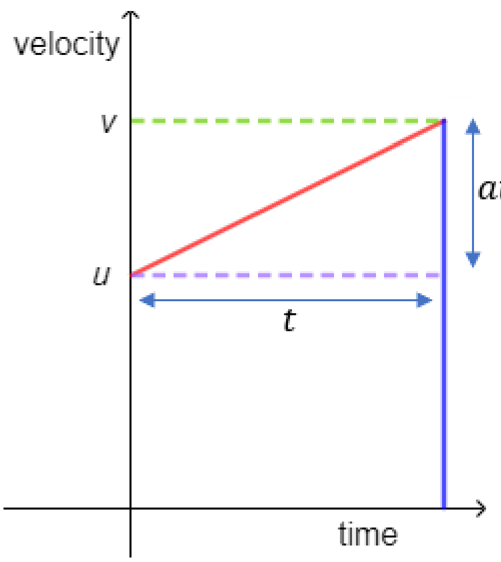


There are several ways to write the height of the triangle. Here, to involve the acceleration (the gradient of the hypotenuse of the triangle), we will write it as at. This is the height because when we divide it by the base of the triangle we get the gradient. The area of the rectangle is ut and the area of the triangle is $\frac{1}{2}t \times at$

The area under the graph is therefore

$$s = ut + \frac{1}{2}at^2$$

A related diagram shows the area under the graph as a larger rectangle (with area vt) subtract a triangle that is congruent to the one we used in the previous version:



This gives us

$$s = vt - \frac{1}{2}at^2$$

The graph has given us four of the five equations.

The final equation is the one that does not involve time. The velocity-time graph has time at its core, so we cannot easily use the graph to derive it.

Taking any two of the equations and eliminating *t* will work.

I will use $s = \frac{1}{2}(u + v)t$ and $v = u + at$

Making *t* the subject of each one gives us $t = \frac{2s}{u+v}$ and $t = \frac{v-u}{a}$

Equating these gives $\frac{2s}{u+v} = \frac{v-u}{a}$

Multiplying through by *(u+v)* and by *a* results in $2as = v^2 - u^2$

This therefore gives us the final equation of $v^2 = u^2 + 2as$

This article has looked at an area of mechanics that the students will have met briefly at GCSE. Later in their A level course they will extend these ideas, firstly to be able to work with scenarios where there is variable acceleration and subsequently to work on problems with constant acceleration in two or three dimensions. This involves combining their understanding of constant acceleration with vectors (another area of A level mathematics that students will have come across at GCSE).