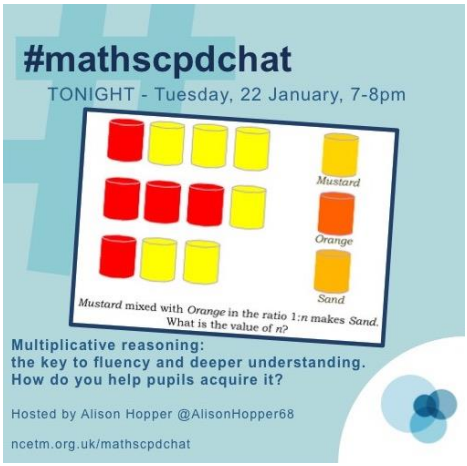
#mathscpdchat 22 January 2019

Multiplicative reasoning: the key to fluency and deeper understanding. How do you help pupils acquire it?

Hosted by @AlisonHopper68

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focused were:

- how pupils **reveal that they are 'stuck in additive reasoning'** and that it is impeding their learning … how they reveal their inability to use multiplicative reasoning in situations involving multiplicative relations;
- **pupils at the end of KS2** are expected to be able to differentiate between, and use, both additive and multiplicative relationships;
- pupils' responses to example-tasks based on Kath Hart's **Curly Ks item** taken from the CSMS ratio test (1981);
- pupils who lack deep understanding (and therefore ability to make crucial connections between mathematical ideas) **becoming overwhelmed by trying to**

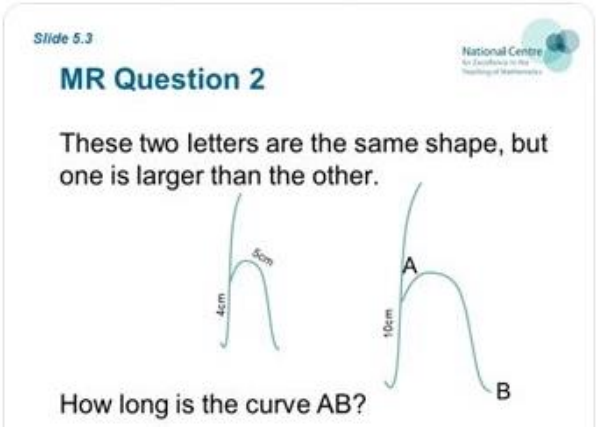
remember separate 'sets of rules'/procedures to apply in situations that are actually structurally similar;

- how pupils' inabilities to use multiplicative reasoning appropriately (or at all) **puts obstacles in the way of their learning to understand and use simple algebra**;
- effective teaching approaches and resources to help pupils who **habitually respond to multiplicative situations using additive reasoning** … eg approaches to help pupils who use repeated addition/subtraction to solve multiplication/division problems;
- research evidence, support and advice available on the **Multiplicative Reasoning pages of the ICCAMS Maths website**;
- examples that **help pupils distinguish between** seeing a numerical difference as indicating an **additive relation** and seeing the 'same' difference as indicating a **multiplicative relation** … e.g. compare operating on 103 to make 109 with operating on 3 to make 9;
- **implications for later learning** of introducing doubling as 'adding again' … and times tables as 'adding on' …;
- **coping with 'Just tell me how to do it!' responses** from pupils to 'explanations' that use pictorial/concrete representations;
- the challenge for teachers of **pupils regarding the use of pictorial/concrete representations as a 'backward step'**;
- **a Venn-diagram task** designed to generate thought about fractions, factors, multiples and simplification;
- the innate tendency of humans to **reason/think logarithmically** before they are 'taught' additive reasoning … evidence of remnants of this inclination in adult thought**!**

An interesting 'conversation' of tweets, about pupils using additive reasoning to get wrong answers to ratio questions, teaching strategies to try to remedy this, and sources of teaching-guidance, followed from this tweet by Alison Hopper:

Alison Hopper @AlisonHopper68 · 18h

A little something to get us started. This is being used in some Multiplicative Reasoning focused Y5-8 Continuity work at the moment. What are the most common wrong answers and why? #mathscpdchat

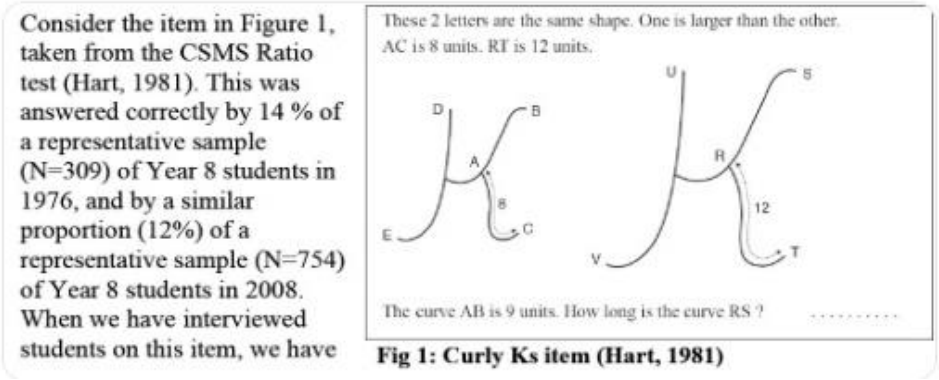


including these from Professor Smudge, Alison Hopper and Salop Herefordshire:

Professor Smudge @ProfSmudge · 17h

Replying to @AlisonHopper68 @SandH_MathsHub

based on Kath Hart's Curly Ks item



Alison Hopper @AlisonHopper68 · 18h

@SandH_MathsHub what have been the main observations when this has been used with students in your experience? #mathscpdchat

Salop Herefordshire @SandH_MathsHub · 18h

Replying to @AlisonHopper68

Responses based on additive reasoning, giving the incorrect answer of 11 are very common...at Primary and Secondary... #mathscpdchat

these from Alison Hopper and Andy Parkinson:

Alison Hopper @AlisonHopper68 · 18h

How can this reliance on additive reasoning be addressed? #mathscpdchat

Andy Parkinson @andyparkinson31 · 18h

Replying to @AlisonHopper68

We need to look at situations where we see a multiplicative difference. Ie A=101m B=102m, look the same, C=1m, D=2m same difference but look very different as D is double C.

Alison Hopper @AlisonHopper68 · 18h

Would you represent this diagrammatically as well as symbolically? #mathscpdchat

these from Andy Parkinson, Alison Hopper and Mary Pardoe:

Andy Parkinson @andyparkinson31 · 18h

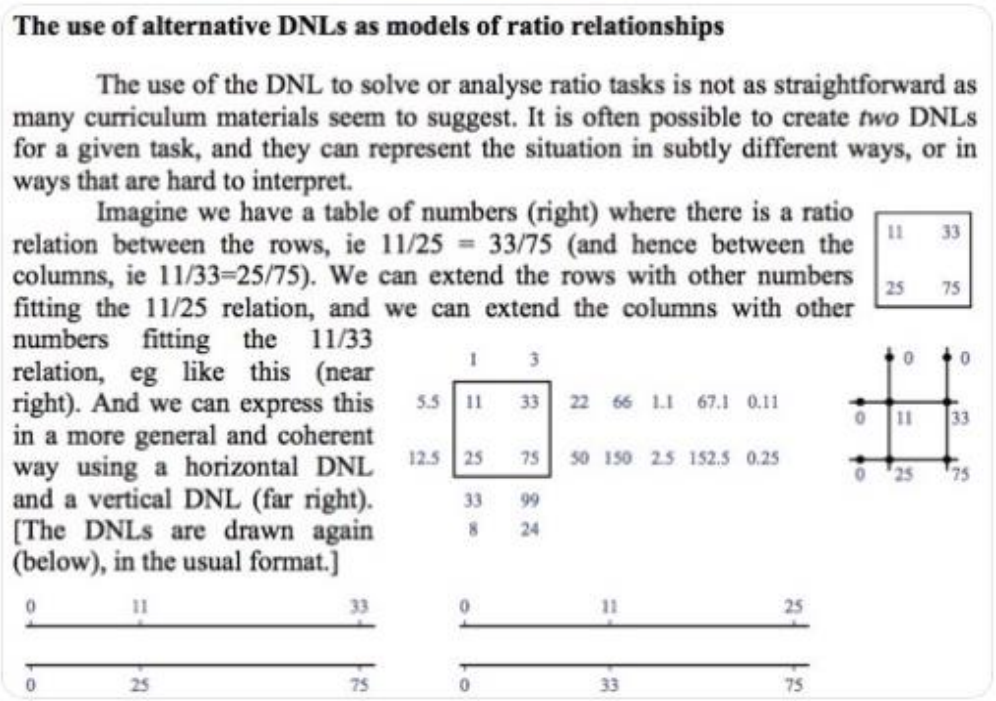
It must be represented pictorially and concretely for pupils to be able to effectively build conceptual understanding #mathscpdchat

Alison Hopper @AlisonHopper68 · 18h

Completely agree! Battled with a Y8 today (my own) when I insisted on using a bar to explain the method for working out the original price after a 10% rise ... I was not popular "Just tell me how to do it!" Perils of being son of a maths person! **#mathscpdchat**

Mary Pardoe @PardoeMary · 17h

There is some useful research/advice re using double-number-lines in one of the research papers that you can download from ICCAMS ..... #mathscpdchat 'The use of alternative double number lines as models of ratio tasks and as models for ratio and scaling' iccams-maths.org/wp-content/upl...



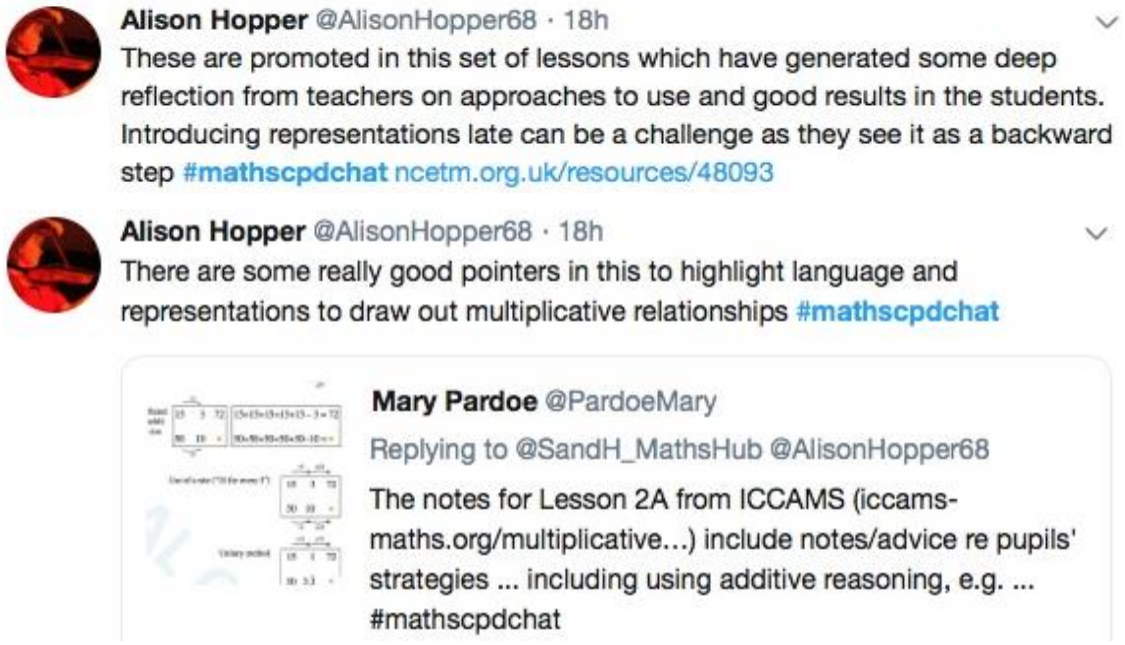
The use of alternative DNLs as models of ratio relationships

The use of the DNL to solve or analyse ratio tasks is not as straightforward as many curriculum materials seem to suggest. It is often possible to create *two* DNLs for a given task, and they can represent the situation in subtly different ways, or in ways that are hard to interpret.

Imagine we have a table of numbers (right) where there is a ratio relation between the rows, ie 11/25 = 33/75 (and hence between the columns, ie 11/33=25/75). We can extend the rows with other numbers fitting the 11/25 relation, and we can extend the columns with other numbers fitting the 11/33 relation, eg like this (near right). And we can express this in a more general and coherent way using a horizontal DNL and a vertical DNL (far right). [The DNLs are drawn again (below), in the usual format.]

	1	3					
5.5	11	33	22	66	1.1	67.1	0.11
12.5	25	75	50	150	2.5	152.5	0.25
	33	99					
	8	24					

and these from Alison Hopper:



(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Multiplicative Reasoning, which is an NCETM microsite to support teachers in developing their learners' multiplicative reasoning. It provides materials, advice about how the materials might be used, and suggested teaching approaches, and it was shared by Alison Hopper

From the Library: ICCAMS Maths, which is an article in the NCETM Secondary Magazine 134. Professor Jeremy Hodgen, who at that time (June 2016) was Director of the ICCAMS Maths (Increasing Competence and Confidence in Algebra and Multiplicative Structures) project at the University of Nottingham, explains what ICCAMS Maths is. It was shared by Alison Hopper

Revisiting Mr Tall and Mr Short which is a PDF version of an illustrated article in Mathematics Teaching in the Middle School, Vol.20, No 4, November2014. Suzanne M. Riehl and Olof Bjorg Steinthorsdottir discuss pupils' strategies when dealing with multiplicative relationships in the context of the classic Mr Tall and Mr Short problem. It was shared by Sharon Malley

ICCAMS Maths: Multiplicative Reasoning, which is a page on the ICCAMS Maths website from which you can download materials to support the teaching and learning of multiplicative reasoning. It was shared by Mary Pardoe

The struggle to achieve multiplicative reasoning 11-14, by Margaret Brown, Dietmar Küchemann and Jeremy Hodgen, is a BCME 2014 research paper. It presents, and discusses, evidence that the poor understanding of multiplicative reasoning achieved by many students in the 1970s had not improved by 2014. It was shared by Mary Pardoe

The use of alternative double numberlines as models of ratio tasks and as models for ratio relations and scaling, by Margaret Brown, Dietmar Küchemann and Jeremy Hodgen, is another BCME 2014 research paper. The authors draw on ICCAMS project materials that used the double number line (DNL) to develop secondary school students' understanding of multiplicative reasoning. It was shared by Mary Pardoe

Standards Unit N6: Developing proportional reasoning which provides guidance about effective approaches and materials, shared by Jenny Hill-Parker

Vision in Elementary Mathematics by W.W. Sawyer in which the author describes techniques of visualising, dramatising and analysing numbers that attract and retain the attention and understanding of students, shared by Andy Parkinson

Slide Rule Sense: Amazonian Indigenous Culture Demonstrates Universal Mapping of Number onto Space which is an article in Science Daily, May 2008, derived from Harvard University. 'The ability to map numbers onto a line is universal. But for an Amazonian tribe, this mapping is not linear but logarithmic.' It was shared by Sharon Malley