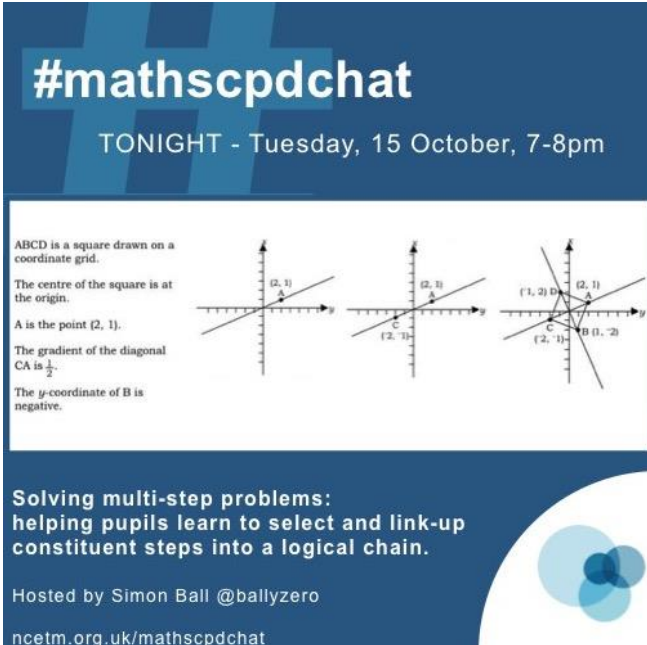


## #mathscpdchat 15 October 2019

**Solving multi-step problems: helping pupils learn to select and link-up constituent steps into a logical chain.**

Hosted by [Simon Ball](#)

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag #mathscpdchat in Twitter*



**#mathscpdchat**  
TONIGHT - Tuesday, 15 October, 7-8pm

ABCD is a square drawn on a coordinate grid.  
The centre of the square is at the origin.  
A is the point (2, 1).  
The gradient of the diagonal CA is  $\frac{1}{2}$ .  
The  $y$ -coordinate of B is negative.

**Solving multi-step problems:  
helping pupils learn to select and link-up  
constituent steps into a logical chain.**

Hosted by Simon Ball @ballyzero  
nctm.org.uk/mathscpdchat

Some of the areas where discussion focussed were:


- that when pupils are trying to solve a multi-step problem it requires of the teacher **so much patience not to 'give anything away' (not to 'lead' them)** ... **prompts** that are very effective when used repeatedly are 'What do you know?', 'What can you write down?', 'What do you see now?' 'What can you do now?';
- encouraging pupils to draw, annotate, and **use diagrams**;

- **giving pupils a diagram without an associated question to answer** ... invite them to say what they know that is relevant to the diagram, and what knowledge they can derive from it;
- **giving pupils some information about a mathematical 'situation'** ... challenging them to represent (or show) that information in a sketch ... they may realise that by using their existing mathematical knowledge they can **reason to more information** that they can then also show on their sketch;
- **changing a multi-step problem** (possibly taken from a past GCSE exam paper) **into a 'goal-free' version of it** by removing the challenge to work-out/find whatever specific 'answer' was required in the original 'question' ... seeing what mathematical ideas are generated in the pupils' minds by the given information/situation/diagram alone;
- **observing what pupils can work out when they tackle 'goal-free' problems;**
- challenging pupils to solve multi-step problems that each require pupils to use knowledge of **mathematical facts from different (although most likely related) 'topics'** (eg knowledge of both trigonometrical ratios and of Pythagoras' Theorem), and to use **more than one distinct procedure** (rather than the same procedure repeatedly);
- **pointing-out to pupils (and praising) the progress that they have made** in trying to solve a problem even when they have not yet reached a solution ... that this **encourages them to 'keep going'**;
- habitually saying **'See how far you can go with this problem'** rather than 'Solve this problem' ... that using this strategy consistently, over some time gradually **switches pupils' mindsets positively** ... that consequently pupils often surprise themselves by 'getting to the end' when initially they 'didn't know where to start';
- showing pupils a 4-mark multi-step GCSE question, and **inviting them to consider (and discuss) what they could do for 1 mark, 2 marks, and so on** ... that this develops a **'give-it-a-go' attitude** instead of a 'look-at-it-and-move-on-because-you-don't-like-the-look-of-it' attitude ... that being able to 'get started' is a 'big leap of faith' for some pupils with some problems;
- that when pupils are trying to solve a multi-step problem, **effective prompts** are 'Have you thought of all the possibilities?', and 'Is there something else you should be considering here?' ... that by the teacher using such prompts repeatedly, **pupils gradually learn to prompt themselves in the same way;**
- **giving pupils both the 'question' and the 'answer'** and challenging them to think-of/show two or more steps to 'get from the question to the answer';

- **pupils' initial perceptions of problems** ... pupils look through all the questions (of an exam paper), tick the ones that they like the look of, and attempt those first, then they move on to problems that look harder to them ... pupils rating each question as red, amber or green according to how they 'feel' about it ... whether pupils get their ratings right (that is, do they get full marks for questions that they rate as green?);
- **regularly setting-aside a whole lesson** (for example once in every two weeks) for pupils to work on multi-step problems;
- **using a spotlight-tool on the board** to show only a specific part of the given information of a problem;
- that even 'Higher' students struggle to see where **forming an equation (to solve in order to reveal 'new' information) is a useful/essential step** in the solution of a multi-step problem;
- that many pupils struggle to understand **what the fraction  $a/(a + b)$  represents** when they are **given the ratio  $a : b$** ;
- **that the making of strategic decisions about the selection of concepts and procedures, and the order of their application, is a manifestation of mathematical reasoning.**

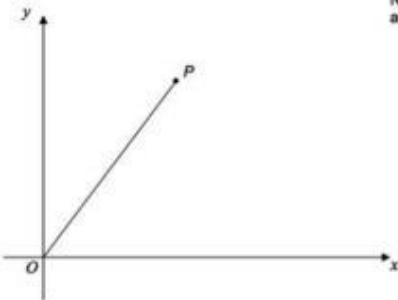
In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a 'conversation' of tweets that was stimulated by the host showing an example of a multi-step problem. Participants made suggestions of the kinds of prompting and questioning that might enable pupils to learn from thinking about this example. The conversation was generated by this tweet from [Simon Ball](#):

 **Simon Ball** @ballyzero · Oct 15  
 How would you help students through this problem? #mathscpdchat [All tonight's problems are AQA GCSE Maths questions]

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28 The diagram shows a line joining  $O$  to  $P$ . Not drawn accurately

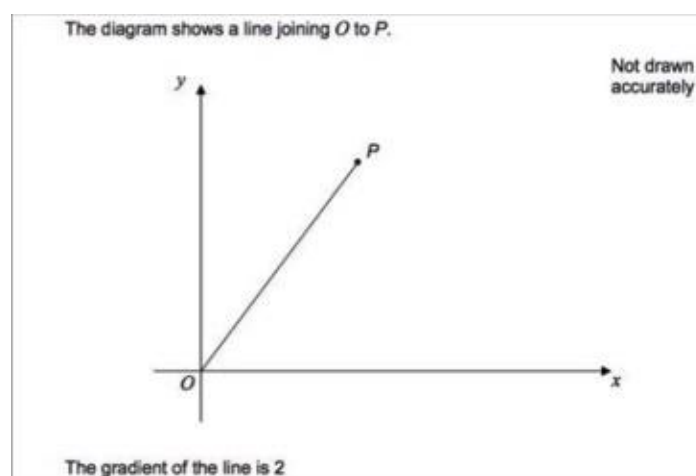


The gradient of the line is 2  
 The length of the line is  $\sqrt{2645}$

Work out the coordinates of  $P$ . [4 marks]


and included these from [Gerry McNally](#), [Heather Scott](#), [Simon Ball](#) and [Heather Massey](#):

 **Gerry McNally** @mcnally\_gerry · Oct 15  
Replying to @ballyzero  
How about giving this goal-free version first and seeing what mathematical ideas arise? #mathscpdchat




 **Heather Scott** @MathsladyScott · Oct 15  
Replying to @ballyzero  
#mathscpdchat - My approach would be to ask them what they know and to write something of what they know on the diagram. 🤔🤔


 **Simon Ball** @ballyzero · Oct 15  
What would you be looking for them to add to the diagram? #mathscpdchat

 **Heather Scott** @MathsladyScott · Oct 15  
#mathscpdchat That would be giving too much away! That's another strategy- don't give students any answers as the only thing they learn then is to wait for the answers & the teachers will give the answers- so draw a diagram & write something on the diagram from info in question 😊

 **Heather Massey** @HeatherMassey2 · Oct 15  
Replying to @MathsladyScott  
#mathscpdchat Is there any mileage in giving them the question AND the answer, and ask them what 2 or more steps gets you from Q to A?

these from [Tess Maths](#), [Simon Ball](#), and [Heather Scott](#):

 **Tess Maths** @tessmaths · Oct 15  
Replying to @ballyzero  
#mathscpdchat  
I'd perhaps ask what the 4 marks are for? Analysis shows that grade 3/4 students will not even attempt any part of these type of q's - so help them consider what they would do for 1 mark...2marks and so on 😊

 **Simon Ball** @ballyzero · Oct 15  
That's a great thought! Enough 1-2 mark responses on these sorts of questions can make a difference. Does that build students' confidence with other kinds of questions? #mathscpdchat

-  **Tess Maths** @tessmaths · Oct 15  
Exactly...it develops that 'give it a go' attitude instead of look at it and move on because you don't like the look of it...
-  **Heather Scott** @MathsladyScott · Oct 15  
Being able to 'get started' is also the biggest leap of faith some of them need to make. #mathscpdchat 😊😊
-  **Simon Ball** @ballyzero · Oct 15  
Replying to @MathsladyScott  
So very, very true! That initial hurdle can be so daunting for some. I see it often in GCSE resit students. #mathscpdchat

and these from [Heather Scott](#), [Angelos Sphyris](#), and [Simon Ball](#):

-  **Heather Scott** @MathsladyScott · Oct 15  
#mathscpdchat - it takes so much patience not to give anything away. I'd keep repeating something like "What do you know what can your write down?" Then "What do you see now?" "What can you do now?" It also never ceases to amaze me we usually get there with just those prompts 😊
-  **Angelos Sphyris** @knightofmaths · Oct 15  
I would be looking for them to add a dotted line from P perpendicular to the x-axis, so as to then write the definition of the slope as the ratio of the coordinates of P and so on... #mathscpdchat
-  **Simon Ball** @ballyzero · Oct 15  
Can only agree with this! Do you think the length being a surd is enough of a clue, or would you have to give more hints? #mathscpdchat
-  **Angelos Sphyris** @knightofmaths · Oct 15  
It should be enough. I think, however, that the very presence of a right-angled triangle together with a known hypotenuse will contribute to driving their minds towards Pythagoras. #mathscpdchat
-  **Heather Scott** @MathsladyScott · Oct 15  
#mathscpdchat ... which may lead to another prompt - have you thought of all the possibilities? 😊 Is there something else you should be considering here?

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

[Goal Free Problems](#) which is a place to find and share goal free problems for use with pupils. The site was set up by Peter Mattock. It was shared by [robyn afford](#)

[Growth Mindset Maths](#) which presents a strategy that removes the emphasis in mathematical problem-solving, for both students and teachers, from arriving at the correct

answer to understanding the thinking needed to arrive at the solution. It was shared by [McBec](#)

[EMMAths Learning and Teaching](#) which is a colourful display designed by [Em Bell](#) to support students who are preparing for mathematics exams. It was shared by [Em Bell](#)

[PISA 2021 Mathematics Framework](#) which is the PISA website where you can click on interactive framework components, or download the full PISA 2021 Mathematics Framework Draft in PDF format. It was shared by [Mary Pardoe](#)

[Preparing for GCSE Problem Solving](#) which is a free-to-download PDF copy of *Teaching Mathematics at Secondary Level* by Tony Gardiner. It was shared by [Mary Pardoe](#)