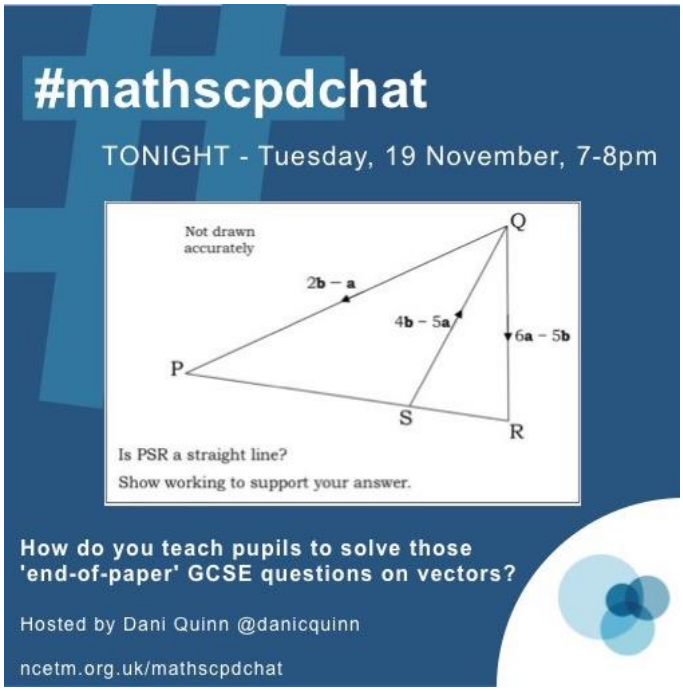
#mathscpdchat 19 November 2019

How do you teach pupils to solve those 'end-of-paper' GCSE questions on vectors?
Hosted by Dani Quinn

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

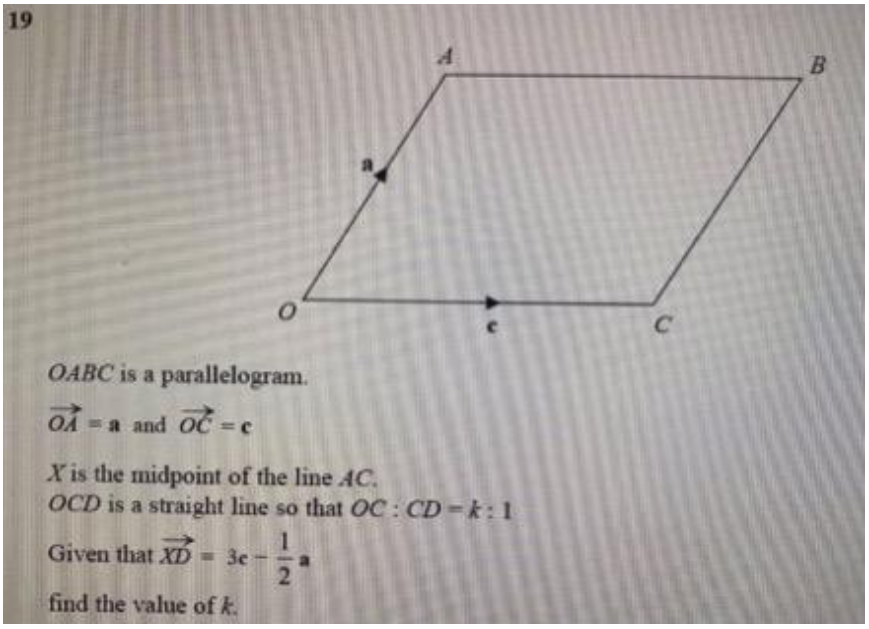
- the **point/stage/time in pupils' mathematical learning journeys when they first encounter the idea of a vector** … 'during Year 11' … 'alongside translations' … 'for plotting routes' … 'in Year 8 in the context of journeys (on maps drawn on square grids)' … 'as column vectors, followed by translations and enlargements described using column vectors' … 'when learning about translations or gradients in Key Stage 3' … 'it's the first topic of Year 11' … 'we find out where science introduce the idea (with forces or with velocity) and do it just before';

- that vectors might be more easily understood if they were introduced in **the context of transformations** (other than enlargement) **of single points** rather than of shapes;
- that **one of the most important ideas** that students need to grasp is that **vectors represented by parallel line-segments of equal length (magnitude) are identical** … understanding that (partly) concurrent line-segments are parallel … that the ability to visualise parallel vectors is important when reasoning from given information and diagrams;
- the crucial understanding that on coordinate grids **column vectors name movements, whereas coordinates name points**;
- 'laying the groundwork for those end-of-paper GCSE vector questions' by factorising algebraic expressions, thus focussing on multiples, for example seeing 20x + 30y as 10(2x + 3y) … using fractional and negative factors, as in (x/2) – (y/4) = -(1/4)(y – 2x);
- **moving from column vectors to 'generalised' vectors** … for example 'if $\mathbb{OA}$ takes (1, 0) to (6, 9), and $\mathbb{OA}$ is **a**, to where does **–a** take (0, 0)?' … that it helps to introduce ('general') vector notation sooner rather than later … that it is a 'large leap' from using column vectors to working with 'abstract' vectors using letter notation;
- whether it might be helpful to introduce vectors **when the focus is on fractions and ratio** … for example, '$\mathbb{OA}$ is vector **a**, P is a point on OA such that OP : OA is 2 : 3, express $\mathbb{OP}$ in terms of **a**';
- finding or creating **material likely to build students' confidence and 'root-out confusion'**;
- **designing simple vector tasks/items involving ratios**, before moving on to using vectors to solve problems … for example using this kind of variation: 'express $\mathbb{OP}$ in terms of **a** if $\mathbb{OA}$ is **a**, and P is a point on OA such that OP : PA is: 1 : 1, 1 : 2, 1 : 3, 2 : 3, …';
- that **introducing vectors when describing enlargements with fractional and negative scale-factors** may 'prepare the way' for seeing the sense in writing fractional and negative multiples of 'general' vectors;
- that **students who are 'put off' by the topic of 'Vectors'** need 'lots of exposure to vectors over time' … 'underestimating the power of slowly weaving-in vector ideas, building confidence gradually' … using 'goal-free' vector problems to help students overcome their fear of them;
- the value of both teachers and students **expressing their reasoning in detailed step-by-step written explanations**;

- that students **annotating vector diagrams** helps them reason to conclusions/solutions … that it can sometimes be helpful to reproduce a vector diagram several times, each time with different annotations corresponding to different steps of reasoning;
- teachers who were contributing to the discussion being able to describe what they did in order to solve a 'vector problem', but **being unable to explain how they knew what to do**;
- **various different methods used by teachers contributing to the discussion** to solve relatively complex vector problems … that an obstacle to students being able to equate coefficients and solve the resulting equation as successive steps in a solution is their inability to form and solve equations in any situation … the strategy of equating ratios of coefficients, rather than equating coefficients;
- not under-valuing **the role of geometrical reasoning in solving 'those end-of-paper GCSE vector problems**' … that using a geometrical method of solution may avoid the need to equate coefficients and solve the resulting equation;
- challenging students to **create their own 'vector patterns'** as a way to get them thinking and talking about what happens when vectors are combined.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a 'conversation' of tweets about various different ways, adopted by contributors to the discussion, of approaching the task of solving a particular vector problem that the host presented. The conversation was generated by this tweet from Dani Quinn:

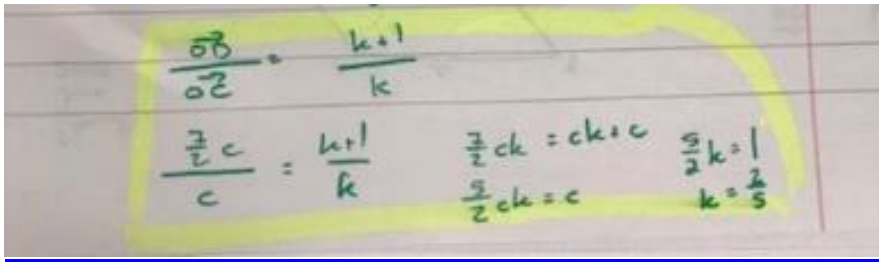


and included these from Dani Quinn, David Chart and Catherine van Saarloos:

Dani Quinn @danicquinn · Nov 19

Replying to @danicquinn

Here is my quite confused working...turned out the bits in yellow were all I needed 😳 And no idea why I initially got -3/5 😳😳😳 Different approach to equating coefficients: setting up a fraction #mathscpdchat @tallerteacher @mathsjem



David Chart @tallerteacher · Nov 19

I'm not sure how comfortable I am with the concept of "dividing" vectors here. #mathscpdchat

Catherine van Saarloos @CoreMathsCat · Nov 19

Replying to @danicquinn

I like to just show diagram and ask them to tell me how to get from one point to another using a and c. Or just say "from this diagram what else do you know?" #mathscpdchat

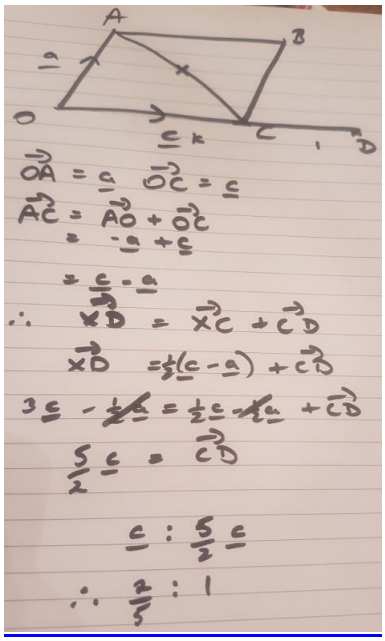
Dani Quinn @danicquinn · Nov 19

So treating it as a goal-free problem? (@mrbartonmaths talks about them here - about halfway down: mrbartonmaths.com/teachers/resea...) #mathscpdchat How do you eventually move them to the tricky final leap?

Catherine van Saarloos @CoreMathsCat · Nov 19

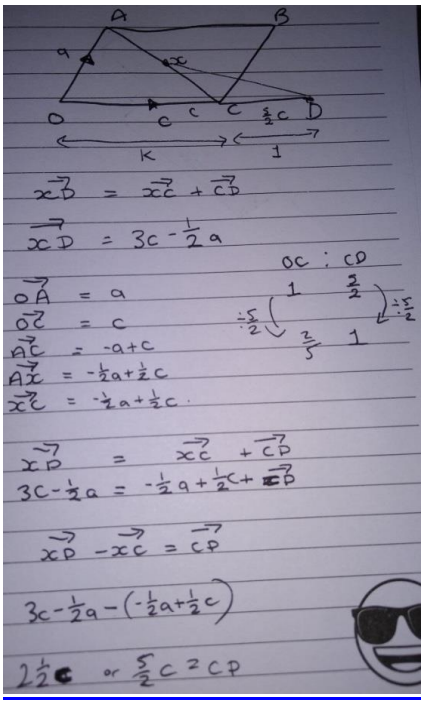
I would certainly be building up to this. In this one they should know how to get a form of XD (mixture of c, a and CD) and they are given XD so can compare. Very hard to explain in tweet sorry. I love goal free though because it enables students to access any qn (even just st)

Catherine van Saarloos @CoreMathsCat · Nov 19
Replying to @danicquinn
Sorry this is messy (big fat felt tip).

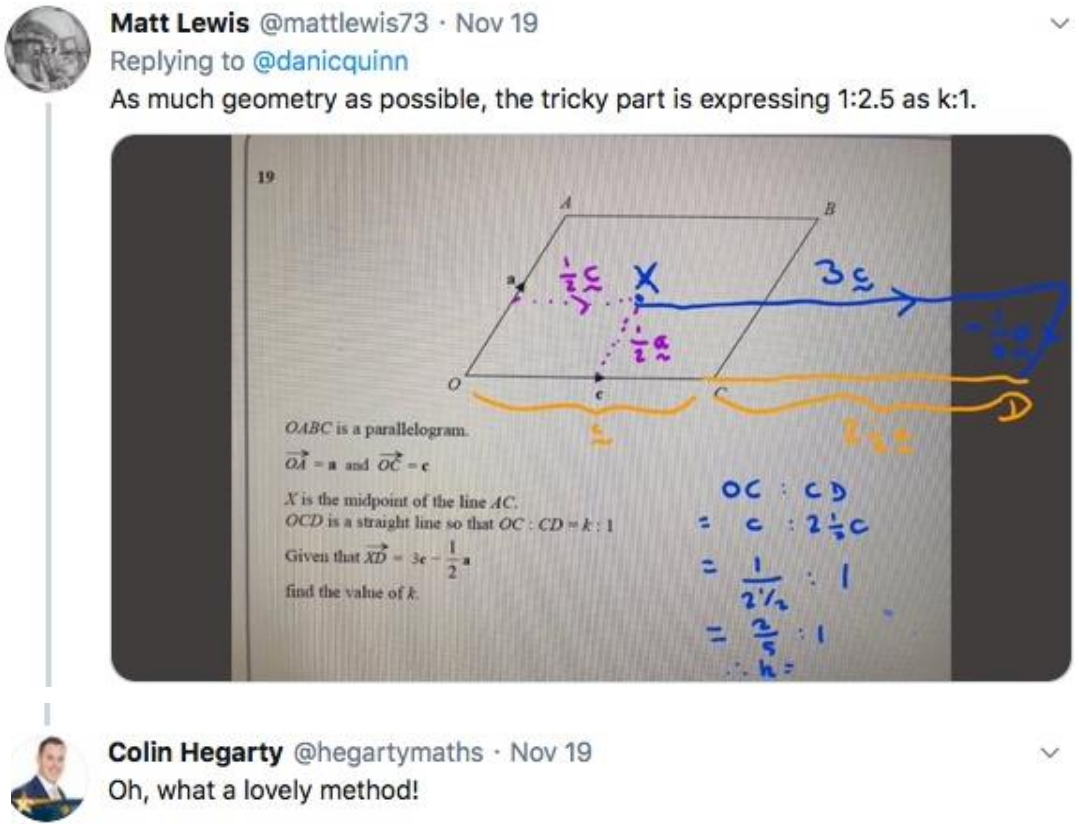


and these from Kerry, Dani Quinn, Matt Lewis and Colin Hegarty:

kerry @kerryleicester · Nov 19
Replying to @danicquinn
I couldn't explain how I did it which is why I don't like teaching it!



Dani Quinn @danicquinn · Nov 19
I FEEL THE SAME WAY! I can explain what I did, not how I knew to do it.



(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

An Introduction to Vectors which is an NRICH article (2011) by Toni Beardon that describes what vectors are and how to add, subtract and multiply them by scalars, and it gives some indications of why they are useful. It provides a summary of the elementary ideas about vectors that are usually met in school mathematics. It was shared by Dani Quinn

Grid Moves which are tasks by Don Steward designed to involve students in simplifying expressions although the letters are not used as variables, and they constitute a simple introduction to vectors used to find 'as the crow flies' direct journey lengths. It was shared by Dani Quinn

Harder GCSE Vector Questions which is material that includes tasks by Don Steward that require students to use vectors to prove results. It was shared by Dani Quinn

Vector Geometry: Review question: Solution which is material from *Underground Mathematics* that shows clearly a way of solving a vector problem. It was shared by Dani Quinn

Equating Coefficients in Identities which is 'practice material' designed for GCSE (AQA Higher). It was shared by Dani Quinn

Equating Coefficients which is more 'practice material' for students. It was shared by Dani Quinn

Goal Free Problems which is a place to find and share goal free problems. It was shared by Dani Quinn

Shinglee Examination Questions which are textbooks mentioned during the discussion. It was shared by Lee Overy

Vectors: tasks which is a collection of NRICH tasks involving vectors. It was shared by Mary Pardoe

Secondary School Students' Difficulties With Vector Concepts which is an article by Eleni Demetriadou and Constantinos Tzanakis. It was shared by Mary Pardoe

Vectors and Geometry which is practice material by Mr Hodgson CMA. It was shared by Mr Hodgson CMA