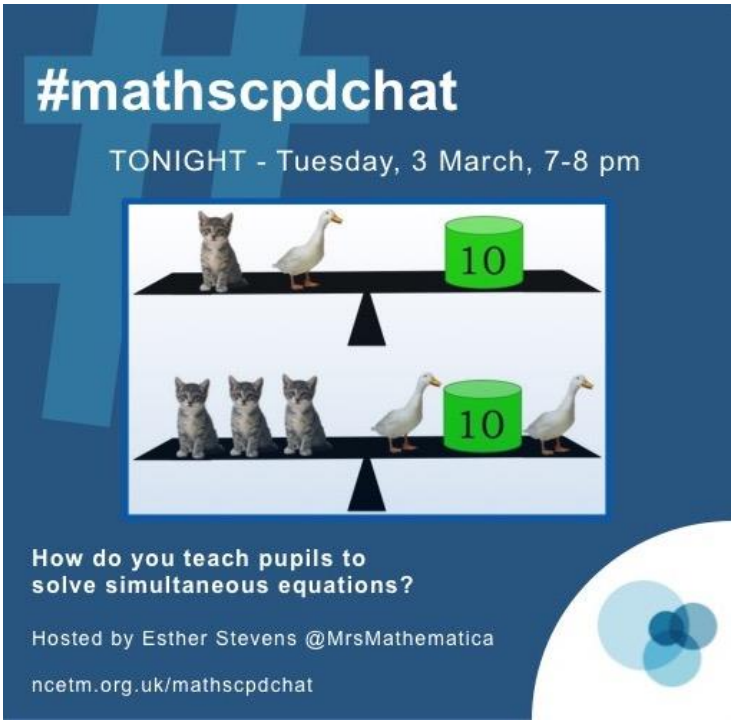
#mathscpdchat 3 March 2020

How do you teach pupils to solve simultaneous equations?
Hosted by Esther Stevens

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

ways of introducing students to pairs of simultaneous linear equations, and to solving them:

- required prior learning includes the achievement of **understanding of what can and cannot be done to a linear equation and it still be valid** … tasks that reveal whether or not students have acquired such understanding;

- **starting from arithmetical statements of equality** (such as 3 + 5 = 8) and seeing what can be done to them while retaining truth (for example, 2 × (3 + 5) = 2 × 8, 3 + 5 + 9 = 8 + 9);
- focussing students' attention so that they see that either **one of a pair of simultaneous linear equations** (such as y = x + 3), taken on its own, has **infinitely many 'solutions'**;
- establishing a firm foundation by making as sure as possible that students can comfortably (fluently) **substitute values and simple algebraic expressions for variables in linear equations**, and can **transform linear equations**;
- that many teachers introduce students to **simultaneous equations via contextual problems** such as '2 coffees and 1 tea cost £2. 2 coffees and 2 teas cost £2.40. What does one coffee cost?' … asking pupils how they might **represent given information so that it is easier to think about** … when presenting information (possibly using 'pictures') looking out for opportunities for pupils to suggest ways of **representing the given information more 'succinctly'** (for example a pupil said 'oh miss just write 'f' instead of that picture of a tub of fries')
- that in Scotland many teachers **introduce the solution of two simultaneous linear equations as the coordinates of the point of intersection of two straight-line graphs**;
- that the graphical solution of simultaneous linear equations can be experienced by students as a **'next step on from' drawing graphs of linear equations**;
- starting teaching about solving pairs of simultaneous linear equations by presenting students with two intersecting straight lines drawn on a Cartesian grid, and challenging the students to **think about, and explain the significance of, the coordinates of the point of intersection**;
- **students using Geogebra** in order to explore on a Cartesian grid the effects on the graphs of linear equations of 'playing with' (varying) the coefficients and constants of the equations, and to compare the graphs of several different linear equations all drawn on the same grid;
- starting by **'giving' students graphs of pairs of simultaneous linear equations that have integer solutions** … then moving on to examples in which the graph of only one of a pair of simultaneous linear equations is already drawn (so that the challenge for students is to **draw the graph of the other equation and 'read-off' the (integer) values of x and y that are the solution**) … eventually moving on to 'awkward' examples (for which the values of x and y that satisfy both equations are not 'easy-to-see' integers) thus **providing a reason to look for an alternative (algebraic) method of solution;**

moving on:

- **using graphs to identify (find) values of x and y that satisfy simultaneous linear inequalities** … that is, 'seeing' the boundaries of areas on a graph in which lie all the points the coordinates of which satisfy given inequalities (for example 'where are the points for which it is true to say about the coordinates (x, y) that x < y AND y + x < 4?');
- with GCSE maths resit students **using elimination methods**, rather than 'substitution methods', to solve pairs of simultaneous linear equations (eg solving the pair of equations x + y = 10 and 2x + y = 13 by subtracting 2x + y = 13 from 2x + 2y = 20) … with students who intend to progress to A level maths also **using substitution methods** (e.g. from the two equations above forming and then solving the equation 2x + 10 – x = 13);
- **using bar models** to help students 'see' (reason to) ways of solving pairs of simultaneous linear equations;
- in Key Stage 5 encouraging students to look out for **opportunities to substitute expressions into equations in order to create 'elegant' ways of solving** pairs of simultaneous equations … for example solving the pair of equations $y^2 = 3x + 7$ and $y^2 = (x + 3)^2$ by writing, and then solving, $3x + 7 = (x + 3)^2$;
- that challenging students to solve problems that require them to (form and then) solve simultaneous sometimes reveals **surprisingly creative reasoning abilities** in students who 'usually find maths difficult';
- that **students in Y12** are 'reassured' when they see that 'the-scary-looking' A level 'linear programming' **builds on their previously acquired abilities fluently to solve simultaneous equations** … that, therefore, being able to solve simultaneous equations is an important aspect of preparation for success at A level;
- that even KS5 students 'who have done GCSE Additional Maths' do not always **recognise where they can solve puzzles and problems by forming (themselves), and then solving, a pair of simultaneous equations**.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is a part of a conversation about the value of using a variety of images and tasks to spark students' interest and provide opportunities for reasoning. The conversation was generated by this tweet from Esther Stevens:

Esther @MrsMathematica · Mar 3

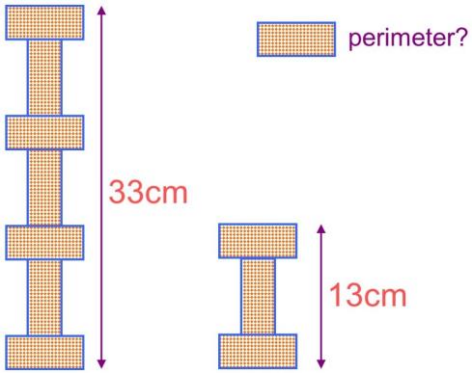
We've already had a few mentions of Don Steward and @Kris_Boulton - following on from them what specific resources and websites are your go to for purposeful practice and developing student understanding and confidence? #mathscpdchat

and included these from Ashling Dolan and Richard Perring:

Ashling Dolan @ASHMD16 · Mar 3

Replying to @MrsMathematica

From Don Steward's website. I have used this for elimination.



Richard Perring @LearningMaths · Mar 3

Replying to @MrsMathematica

Has anyone mentioned this RISP (from @therispguy's site risps.co.uk) which is totally bloomin' magic! Awe and wonder anyone?!

Risp 8: Arithmetic Simultaneous Equations

...1, 3, 5, 7, 9, 11....

...-16, -5, 6, 17, 28, 39...

...78, 76, 74, 72, 70, 68...

Each of the above sequences is called ARITHMETIC; the terms go up or down by a constant amount each time.

Pick six consecutive terms from an arithmetic sequence, and place them in order into the squares below. (Keep the numbers as simple as you can to start with!)

□x + □y = □

□x + □y = □

Now solve the pair of simultaneous equations you have created.

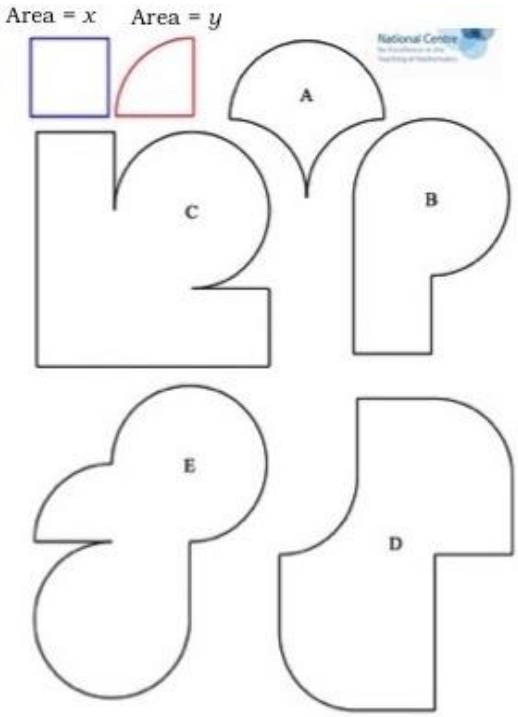
What do you discover?
Can you make a conjecture? Can you prove it?

www.risps.co.uk

these from Mary Pardoe, Heather Scott and Richard Perring:

Mary Pardoe @PardoeMary · Mar 3

the NCETM 'Departmental Workshop' on simultaneous equations (here: ncetm.org.uk/resources/10340) uses areas of shapes ... asking question such as 'if shape C has area P and shape D has area Q, what are the values of x and y?' **#mathscpdchat**



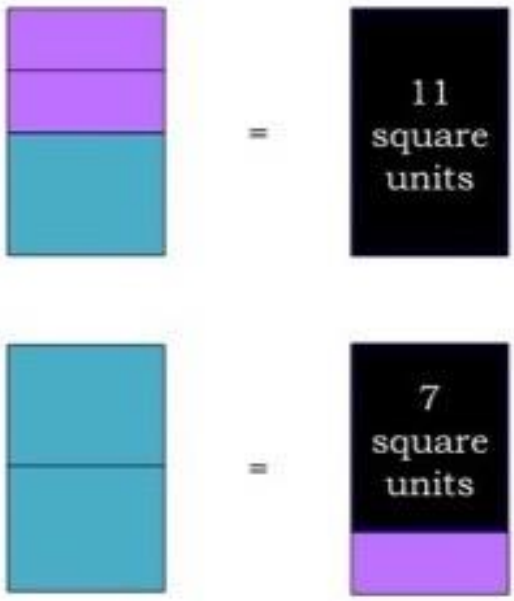
Heather Scott @MathsladyScott · Mar 3

#mathscpdchat I like the connection between algebra and geometry 😍

Mary Pardoe @PardoeMary · Mar 3

Replying to @MathsladyScott

I've sometimes used just rectangles! #mathscpdchat



Richard Perring @LearningMaths · Mar 3

Rectangles/bar models would work just as well (perhaps better?)! This resource was written when Taktiles were sometimes being used for collecting like terms so used that familiar idea and extended it.

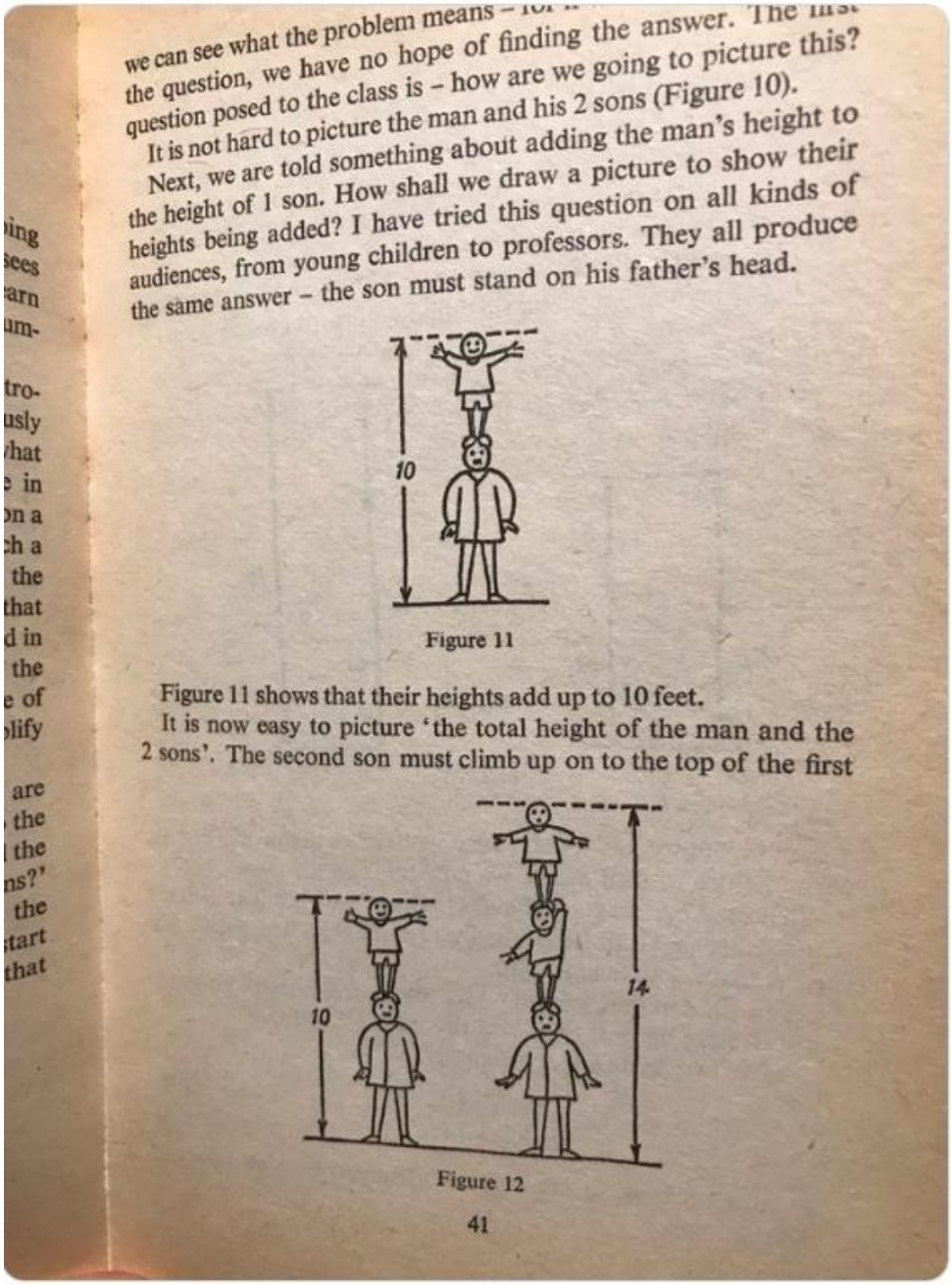
Richard Perring @LearningMaths · Mar 3

I'm thinking that the bars could be to scale so that the equality means something both algebraically AND geometrically. I'm not sure that it's so meaningful in the Taktiles task?

and these from Tim Stirrup:

Tim Stirrup @timstirrup · Mar 3

Just got my W.W Sawyer books out - he used what he called "an unorthodox method of entry"...



we can see what the problem means – [illegible]. The first question, we have no hope of finding the answer. The question posed to the class is – how are we going to picture this?

It is not hard to picture the man and his 2 sons (Figure 10).

Next, we are told something about adding the man's height to the height of 1 son. How shall we draw a picture to show their heights being added? I have tried this question on all kinds of audiences, from young children to professors. They all produce the same answer – the son must stand on his father's head.

Figure 11

Figure 11 shows that their heights add up to 10 feet.

It is now easy to picture 'the total height of the man and the 2 sons'. The second son must climb up on to the top of the first

Figure 12

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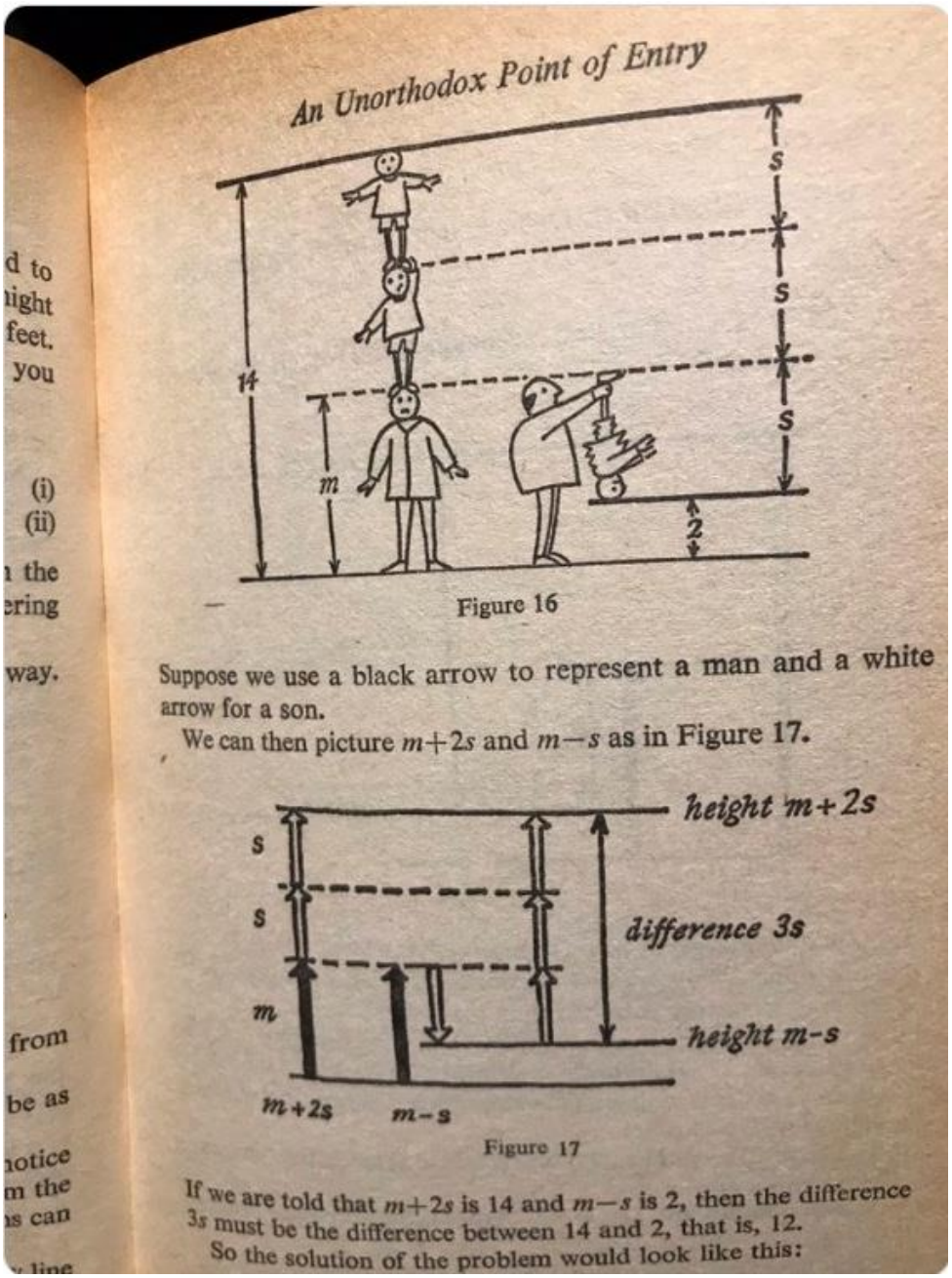
Tim Stirrup @timstirrup · Mar 3

Leading to something like this #mathscpdchat



An Unorthodox Point of Entry

Figure 16

Suppose we use a black arrow to represent a man and a white arrow for a son.

We can then picture $m+2s$ and $m-s$ as in Figure 17.

Figure 17

If we are told that $m+2s$ is 14 and $m-s$ is 2, then the difference $3s$ must be the difference between 14 and 2, that is, 12.

So the solution of the problem would look like this:

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Risp 8: Arithmetic Simultaneous Equations which is a very interesting exploratory task that, while providing opportunities for students to reason, conjecture and prove, requires them to practise solving simultaneous linear equations. It was shared by Richard Perring

Simultaneous equation deductions which are lovely illustrated starting points from Don Steward. They are excellent resources to use when students are starting to solve simultaneous linear equations by considering what can be deduced from two initial statements. It was shared by Heather Scott

Simultaneous equation tasks which is a collection of interesting and varied tasks from Don Steward. In order to complete each task students have to set-up, and then solve, simultaneous equations. It was shared by Sam Blatherwick

Teaching to Mastery Mathematics Bar Modeling which is a book by Yeap Ban Har. It provides guidance for teachers about many different ways that bar models can be used effectively to support the learning of mathematics. It was shared by Lee Overy

Mathematics Departmental Workshops: Simultaneous Equations which is NCETM material that was created by teachers several years ago to support other teachers working together to develop their practice in secondary schools. It was shared by Mary Pardoe