MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.2 Structures: multiplication representing equal groups

Teacher guide | Year 2

Teaching point 1:

Objects can be grouped into equal or unequal groups.

Teaching point 2:

When describing equally grouped objects, the number of groups and the size of the groups must both be defined.

Teaching point 3:

Equal groups can be represented with a repeated addition expression.

Teaching point 4:

Equal groups can be represented with a multiplication expression.

Teaching point 5:

Multiplication expressions can be written for cases where the groups each contain zero items, and for cases where the groups each contain one item.

Overview of learning

In this segment children will:

- learn to identify when objects are grouped equally
- practise identifying the number of equal groups and the size of those groups in a variety of contexts, using precise language to describe the groups
- represent equally grouped objects as both repeated addition expressions and multiplication expressions
- practise interpreting repeated addition expressions and multiplication expressions.

By the end of this segment, children should be able to represent equally grouped items (for example, three groups of four apples) using repeated addition expressions (e.g. $4 + 4 + 4$) and multiplication expressions (e.g. 3×4). Children should be able to recognise the link between repeated addition and multiplication.

Throughout, in order to keep the focus on the structures (*group size* + *group size* + *group size*… and *number of groups* × *group size*), the sum/product (i.e. the total number of objects) is not enumerated, described or included in abstract representations; *expressions*, rather than *equations*, are used to represent the contexts, for example:

$4 + 4 + 4$	3×4
not	not
$4 + 4 + 4 = 12$	$3 \times 4 = 12$

The segment begins by exploring the idea that groups are equal when they each contain the same number of objects and, conversely, that groups are unequal when they contain a different number of objects. Unequal groups are considered at this stage only to the extent that they help to define what is meant by the term '*equal groups*'. Attention is drawn to the concept (equal groups) and non-concept (unequal groups) with children given the opportunity to redistribute objects from unequal into equal groups. When children are comparing groups, they should be encouraged to subitise (see *Spine 1: Number, Addition and Subtraction*, segments *1.1* and *1.3*), and apply their understanding of the composition of number to efficiently enumerate the number and size of the groups.

In *Teaching point 2*, children will progress from identifying equal groups, to describing how many groups there are and how many objects are in each group (number of groups and size of groups). Precise language will be modelled and practised, forming the foundation for learning in later teaching points and segments.

In *Teaching points 3* and *4*, children progress from describing equal groups in words, to representing them with abstract expressions, as both repeated addition and as multiplication. First, children will write repeated addition expressions, building on their learning from *Spine 1*, segment *1.11*, in which they started to write expressions with more than two addends; then they will write multiplication expressions.

Teaching point 5 explicitly deals with 'group sizes' of zero and one (leading to expressions such as 4×0 and 4×1), which can be challenging to visualise. Situations where the *number of groups* is equal to zero or one are avoided at this stage, as they are even more challenging to visualise; expressions such as 0×4 and 1×4 are explored in segments *2.3 Times tables: groups of 2 and commutativity (part 1)* and *2.5 Commutativity (part 2), doubling and halving*.

Throughout segments *2.2–2.12*, the language of '*multiplied by*' is not used, as this implies that the multiplicand (the group size) is written first followed by the multiplier (the number of groups). In this

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segment, children will write expressions in the order *number of groups* × *group size*, as they link multiplication to the structure of repeated addition. In the upcoming segments, children will begin to explore commutativity as well as use the generalisation '***factor times factor equals product***'; they will learn that the factors can be written in either order. However, in this segment, where children are first developing their understanding of multiplication, it helps to keep the factors in the same order (*number of groups* × *group size*). Therefore, the terms *'groups of'* and *'times'* (*not 'times* by') are used instead of *'multiplied by'*, with children describing multiplication expressions such as 4×2 with the phrase *'four times two'*, in preparation for the times-table language used in the upcoming segments.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast*. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.*

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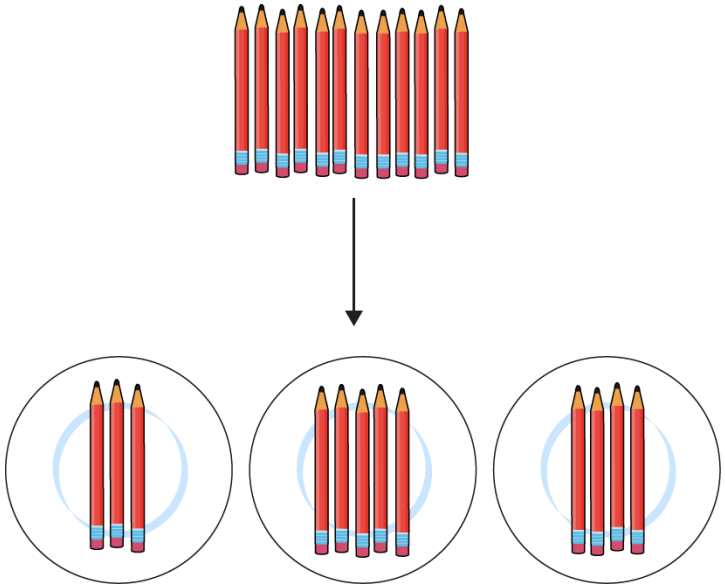
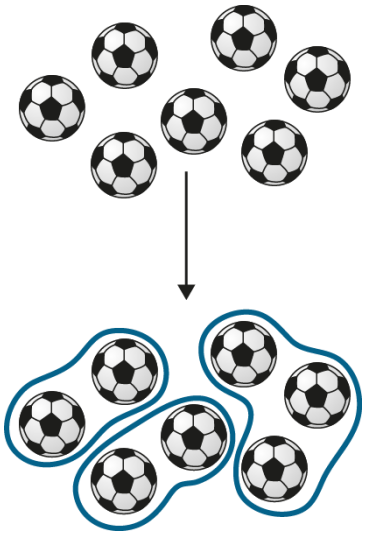
Teaching point 1:

Objects can be grouped into equal or unequal groups.

Steps in learning

	Guidance	Representations
1:1	The purpose of this teaching point is for children to be able to identify equal and unequal groups. Begin by looking at example collections of identical objects (concrete or pictorial) within which groups can be identified. For now, include a mixture of equal and unequal groups, focusing only on the idea that the objects have been put into groups. For each context, show the entire collection of objects together, and then show them being grouped. For concrete representations, move each of the groups onto printed circles, or paper plates, to clearly demarcate the groups. For pictorial representations, animate movement of the items and draw rings around the resulting groups to clearly demarcate them. It is important, in these early stages of exploring multiplication, to use a ring/circle to enclose each group, marking the previously separate objects as one 'thing'. Encourage children to describe what is happening, using the following stem sentences: • Before grouping: ***'There are some ___.'*** • After grouping: ***'The ___ have been grouped.'***	Grouping objects – example 1: *'There are some pencils.'*  *'The pencils have been grouped. '* Grouping objects – example 2: *'There are some footballs. '*  *'The footballs have been grouped. '*

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1:2

Now move on to the concept of equal and unequal groups, continuing to use collections of identical objects.

Present a context, such as the example with the apples shown opposite. Note that the apples have been arranged in columns to make it easier to see whether the groups are equal or unequal; this is the first use of arrays to represent grouping of items, but at this stage the groups are still clearly demarcated. Describe the scenario and show the grouping, then ask children:

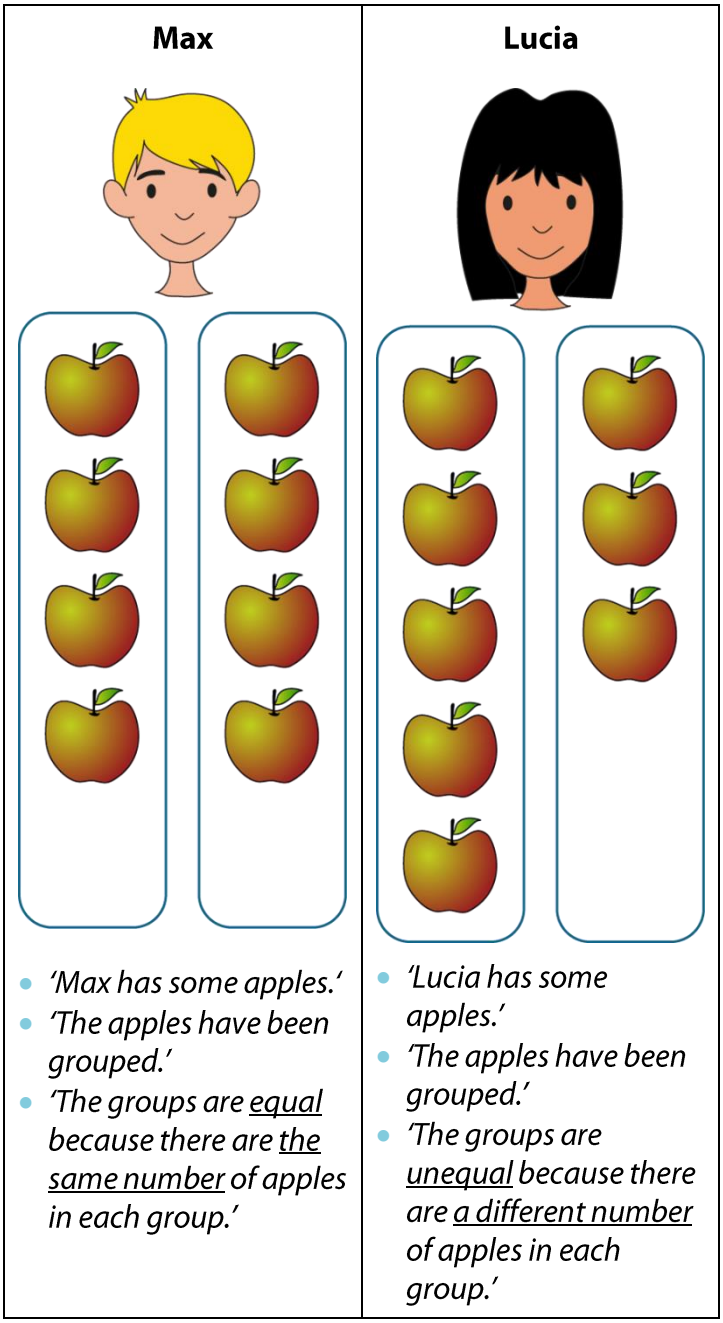
- *'What do you notice about Max's and Lucia's apples?'*
- *'What's the same?'*
- *'What's different?'*

Having elicited that Max has two groups each containing the same number of apples, while Lucia has two groups each containing a different number of apples, use the following stem sentences to establish the equal/unequal groups terminology:

- ***'The groups are <u>equal</u> because there are <u>the same number</u> of ___ in each group.'***
- ***'The groups are <u>unequal</u> because there are <u>a different number</u> of ___ in each group.'***

Repeat for other examples, until children can understand and use the terms *'equal groups'* and *'unequal groups'* as described in the stem sentences.

'Max and Lucia each have some apples. Each of them has put their apples into two groups.'



- *'Max has some apples.'*
- *'The apples have been grouped.'*
- *'The groups are <u>equal</u> because there are <u>the same number</u> of apples in each group.'*

- *'Lucia has some apples.'*
- *'The apples have been grouped.'*
- *'The groups are <u>unequal</u> because there are <u>a different number</u> of apples in each group.'*

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1:3

Provide children with practice working with equal and unequal groups, including:

- identifying whether identical objects (concrete or pictorial) have been arranged into equal or unequal groups (vary the number of groups and the size of the groups)
- creating equal or unequal groups themselves (either from classroom manipulatives or through drawing; in both cases ensure the separate groups are clearly demarcated).

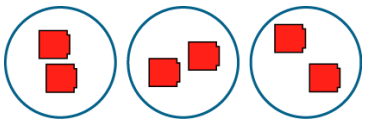
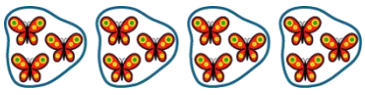
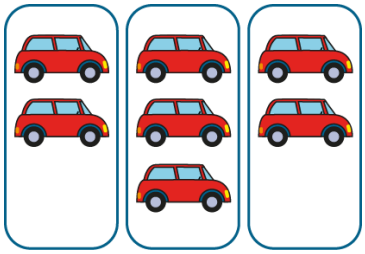
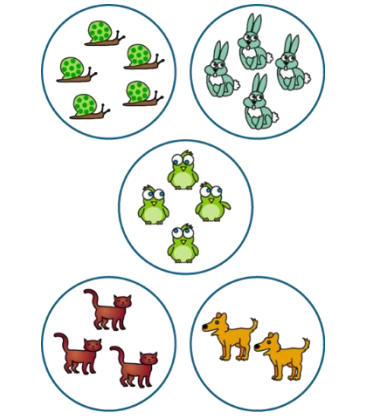
Throughout, continue to encourage children to use the stem sentences in step *1:2*.

Once children are confident working with identical objects, you could introduce some situations in which the objects in each group are different (such as the final pictorial example in the table opposite). The key is for children to be able to recognise the group size irrespective of secondary features.

Use true or false dòng nǎo jīn problems to promote and assess depth of understanding; for example, present an image with a completed stem sentence, and ask children to reason whether the stem sentence is true or false.

Identifying equal and unequal groups:

'For each example, tick the correct column to say whether the objects are in equal or unequal groups.'

	Equal groups	Unequal groups
		
		
		
		
		
		

Creating equal or unequal groups:

- *'Jess is arranging twelve cubes into equal groups. Can you complete her drawing?'*

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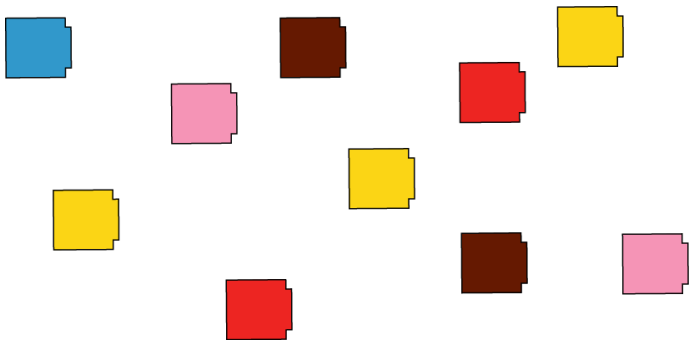
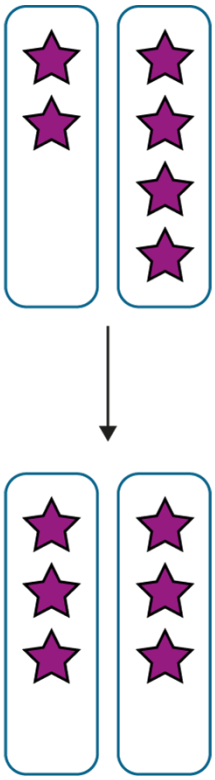
		• *'Now, Jess is arranging twelve counters into unequal groups. Can you complete her drawing?'*  • *Count out ten cubes. How many different ways can you find to arrange them into equal groups?* 
1:4	Now that children can confidently recognise and create equal and unequal groups, look at redistributing objects from unequal groups into equal groups. Use manipulatives so that children can physically move the objects around until the groups are equal. As described in step *1:1*, clearly demarcate the groups using, for example, paper plates or sheets of printed circles/boxes. Begin with *two* unequal groups, asking children to describe the groups using the language introduced in step *1:2*. Then ask children to make the groups equal. Encourage children to subitise when rearranging, and to then check their newly arranged groups by counting, reasoning that the groups are now equal because there are the same number of objects in each group. Then extend to examples with *more than two* unequal groups. Notice that in the examples opposite, the objects are arranged in columns to make it easier to see the relative sizes of the groups; as you progress you can remove this scaffolding, with children	Redistributing from unequal to equal groups – example 1: *'The stickers are grouped. The groups are <u>unequal</u> because there are <u>a different number</u> of stickers in each group.'*  *'The groups are now <u>equal</u> because there are <u>the same number</u> of stickers in each group.'*

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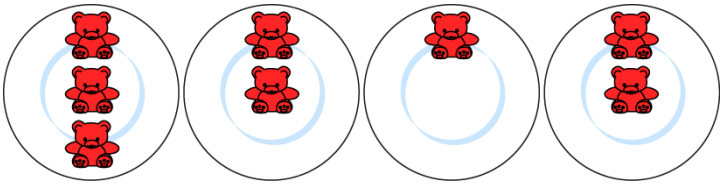
	either working without the column-wise arrangement, or rearranging the objects into columns themselves for easier comparison.	Redistributing from unequal to equal groups – example 2: *'Make these groups equal.'* 

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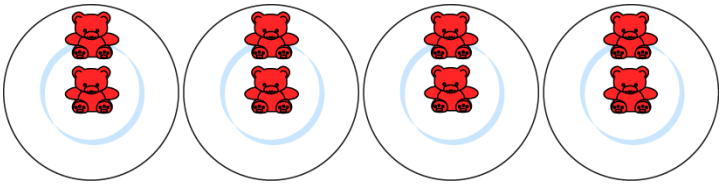
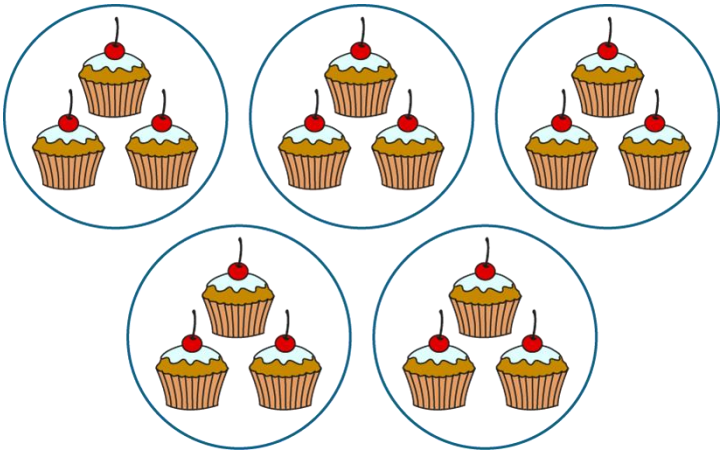
Teaching point 2:

When describing equally grouped objects, the number of groups and the size of the groups must both be defined.

Steps in learning

	Guidance	Representations
2:1	Now that children can identify when objects have been grouped equally, they can look at equal groups in more detail. This teaching point focuses on the correct use of language to describe the size and number of equal groups. Begin with a familiar context from step *1:4*, such as the counting bears (now grouped equally). Ask children • *'How many groups are there?'* • *'How many bears are there in each group?'* Model counting how many groups there are, using the language of groups; for example *'one group of two, two groups of two, three groups of two…'*. Then encourage children to describe the groups using the following stem sentences: • ***'There are ___ equal groups of ___.'*** • ***'There are ___ ___ in each group.'*** • ***'There are ___ groups of ___.'*** Work through a range of contexts, modelling counting the groups and encouraging children to describe them using the stem sentences.	Describing equal groups – example 1: • *'How many equal groups are there?'* • *'How many bears are there in each group?'*  • *'There are four equal groups of bears.'* • *'There are two bears in each group.'* • *'There are four groups of two.'* Describing equal groups – example 2: • *'How many equal groups are there?'* • *'How many cakes are there in each group?'*  • *'There are five equal groups of cakes.'* • *'There are three cakes in each group.'* • *'There are five groups of three.'*
2:2	Now turn the problem around, describing a context and asking children to make or draw the groups, for example, *'Take twelve cubes. Arrange them into three groups of four.'* Ensure that children use plates/circles to demarcate each group. Once children	

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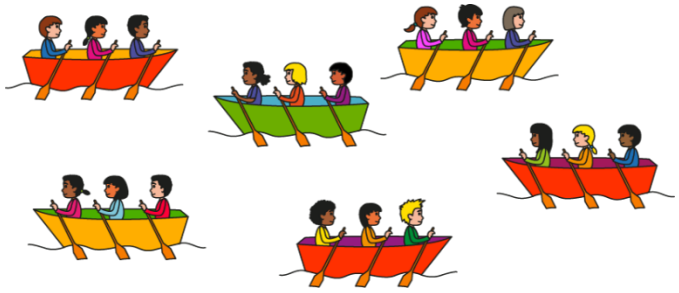
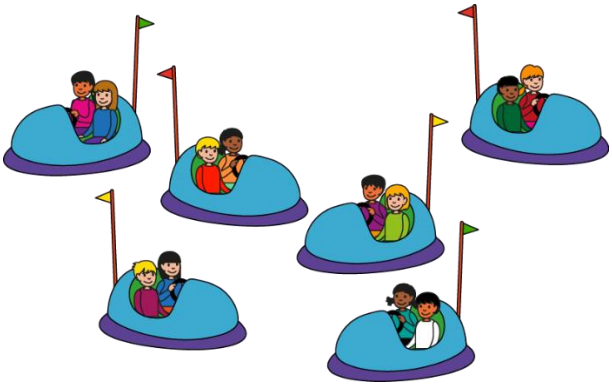
	have made/drawn the groups, encourage them to summarise by repeating the description of the context, using the language described in step *2:1*.	
2:3	Provide children with varied practice, including: • making/drawing a representation to match an existing description (as in step *2:2*) • choosing the correct representation to go with a description and vice versa • completing written sentences to describe a representation. Note that in some of the examples, the noun is not included; for example *'there are six equal groups'* rather than *'there are six equal groups of children'* for the first context opposite. This means that children have to think about the sentence to interpret which value represents group size and which value represents the number of groups. As children progress, you can use visual devices to group the items (as with the boats, cars, ice-cream cones and nests opposite). However, with 'loose' items, continue to clearly demarcate the groups by circling, so that each group is seen as one group. Provide some open questions, such that children need to come up with their own contexts, for example: *'I have five equal groups. There are two in each group. Show me with a drawing.'* You could then compare different children's answers, asking them to identify the similarities and differences to draw out the key features (number of groups and size of each group).	Choosing a description to match a representation: • *'Underline the sentence that correctly describes the picture.'*  • There are 3 equal groups. • There are 6 equal groups. • *'Underline the sentence that correctly describes the picture.'*  • There are 2 groups of 6. • There are 6 groups of 2. Drawing equal groups to match an existing description: *'James started to draw some equal groups. Complete his drawing.'* I have 4 groups of 3. 

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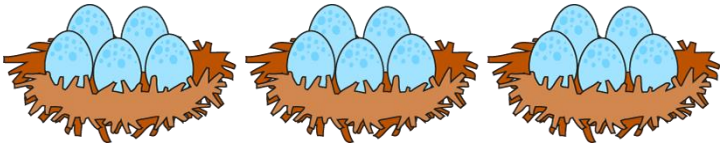
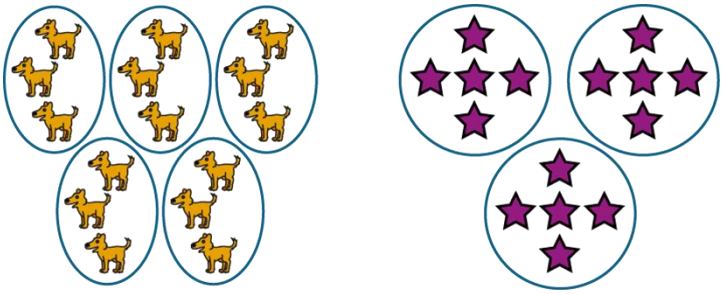
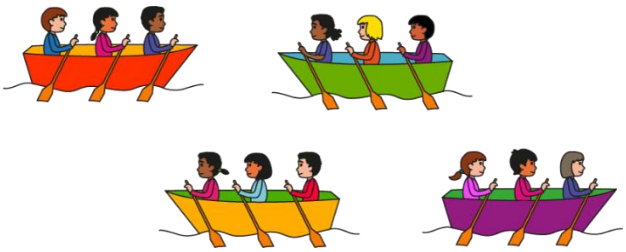
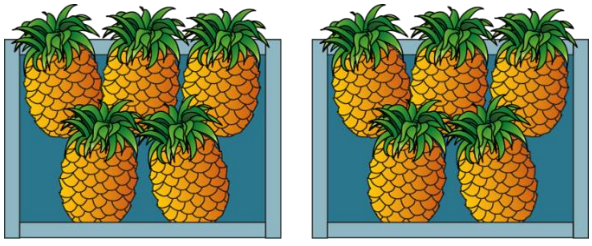
		Completing written sentences to describe a representation: *'Complete the sentences.'*  • There are ___ equal groups of eggs. • There are ___ eggs in each group. • There are ___ groups of ___. Choosing a representation to match a description: *'Tick the representation that matches the description.'* There are 3 groups of 5.  A B
2:4	To complete this teaching point, present children with problems such as those shown opposite, to ensure there are no misconceptions when describing the number of equal groups and the size of the groups. Use a dòng nǎo jīn problem to promote and assess depth of understanding: *'Ralphie says the number of groups and the number of objects in each group <u>cannot</u> be the same. Lily says the number of groups and the number of objects in each group <u>can</u> be the same.* *Who is correct? Convince me. '*	• *'True or false?'* • There are 4 groups of 3.  • There are 5 groups of 2. 

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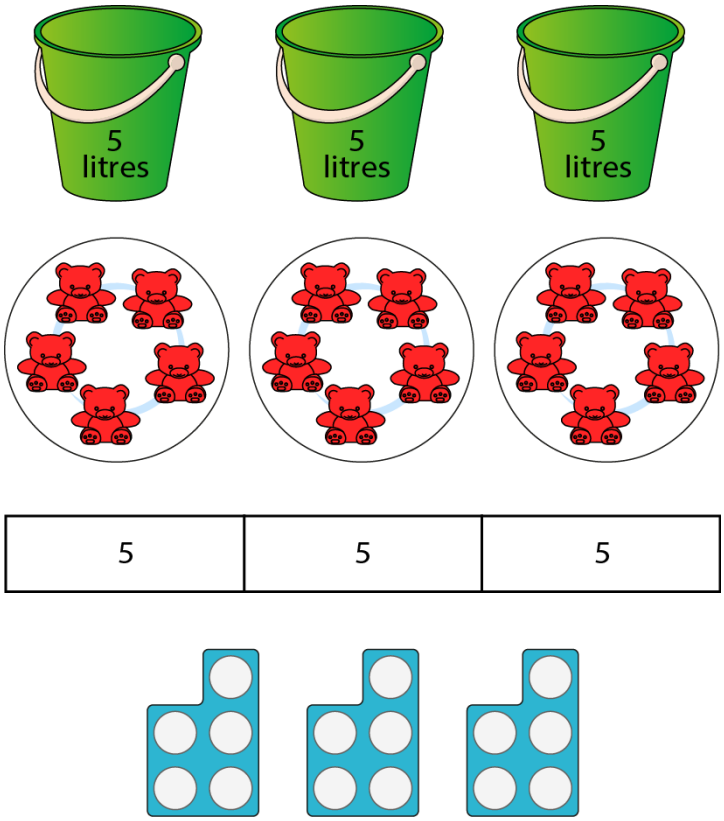
		• *'Look at the representations below.'* • *'What's the same?'* • *'What's different?'* 

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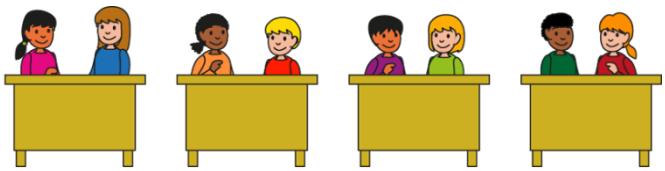
Teaching point 3:

Equal groups can be represented with a repeated addition expression.

Steps in learning

	Guidance	Representations
3:1	This teaching point will build on children's understanding of how equal groups can be described; they will now use repeated-addition expressions to represent various grouping contexts. Since we are now looking exclusively at *equal* groups, the final form of stem sentence from *Teaching point 2* will be used, with 'equal' inferred: ***'There are ___ groups of ___.'***	Example 1 – practical context:   2 + 2 + 2 + 2 • *'There are four groups of two.'* • *'There are two and two and two and two.'* • *'We can write this as two plus two plus two plus two.'* • *'What does each "2" represent?'* • *'Each "2" represents the number of children at each desk. There are two children at each desk.'* • *'Why are there four "2"s?'* • *'There are four "2"s because there are four groups of two.'*
	Begin with a practical context; for example have eight children sit in pairs in the classroom. Describe the grouping, beginning with the familiar stem sentence above, but then extending the description to move towards repeated addition: • ***'There are ___ and ___ and ___ and…'*** • ***'We can write this as ___ plus ___ plus ___ plus…'***	
	Write the associated repeated addition expression on the board, and represent it using a bar model to link to children's previous work on addition. The equal-sized parts in the bar model act as a visual reminder that we are dealing with equal groups.	
	Notice that, at this stage, the focus is entirely on representing the equal groups using repeated addition. Do not include the sum at this stage (for example, in the context opposite, we write $2+2+2+2$, not $2+2+2+2=8$).	
	Work through a range of contexts, both concrete and pictorial, as a class. When working pictorially, you could initially write the group size next to/on each	Example 2 – pictorial measures context:   5 + 5 + 5 • *'There are three groups of five.'* • *'There are five and five and five.'* • *'We can write this as five plus five plus five.'*

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	equal group, as exemplified opposite (the buckets example), helping children to link the picture to the addition expression. Include measures contexts, such as the second example opposite. At this stage, ensure that the measures contexts consider each group/addend as a separate quantity, and avoid scaling scenarios; for example, here we have three separate five-litre buckets of water, and although the cardinality within each 'group' can't be seen, there are still three separate 'groups' each with a 'value' of five. Similarly you could have three 5 p coins, or three five-metre lengths of ribbon. As you work through a variety of examples, deepen children's understanding by asking what each number in the expression represents and why that number is repeated the given number of times (as shown in the first example opposite).	
3:2	Now turn the problem around, presenting a repeated addition expression and asking children to make or draw the groups and describe them using the stem sentences; for example: • *'Use cubes to show me four plus four plus four.'* 4 + 4 + 4 • *'I have some groups of apples…'* 3 + 3 + 3 + 3 + 3 + 3 *'Draw a picture to show the apples.'* Ensure that children use plates/circles to demarcate each group.	

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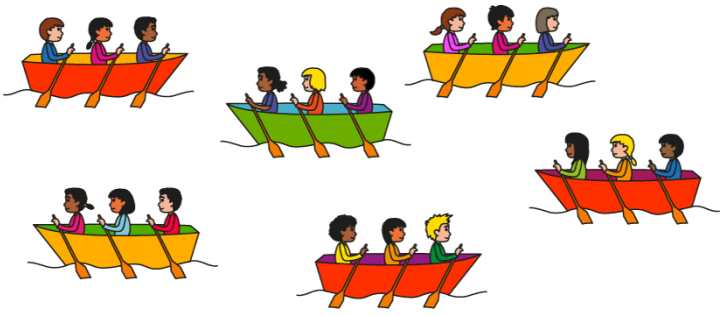
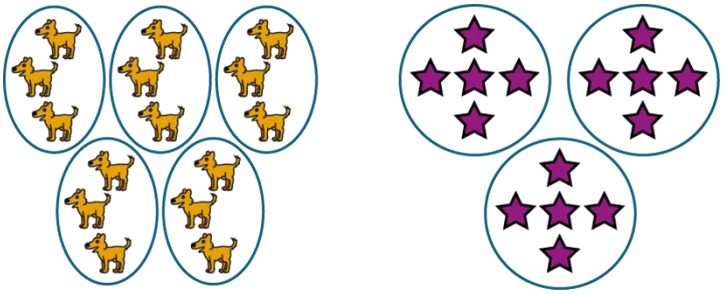
3:3	Provide children with varied practice, including: • making/drawing a representation to match an existing expression (as in step *3:2*) • choosing a repeated addition expression to match a contextual representation and vice versa • completing or writing a repeated addition expression to describe a contextual representation • choosing or writing a repeated addition expression to match a written/spoken description. As in step *2:3*, provide some open questions, such that children need to come up with their own contexts.	Choosing an expression to match a pictorial representation: *'Circle the expression that correctly describes the picture.'*  6 + 6 + 6 3 + 3 + 3 + 3 + 3 + 3 Choosing a pictorial representation to match an expression: *'Tick the picture that matches the expression.'* 5 + 5 + 5  A B Drawing equal groups to match an existing expression: *'James started to draw some equal groups. Complete his drawing.'* I have 3 + 3 + 3 + 3 

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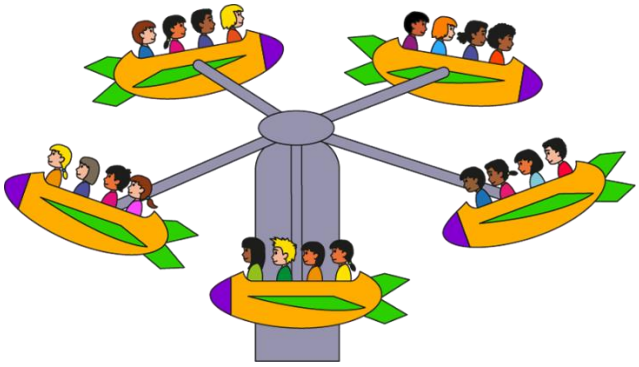
		Completing/writing a repeated addition expression to describe a pictorial representation: • *'Complete the expression to match the picture.'*  □ + □ + □ • *'Write a repeated addition expression to match the picture.'*  • *'Write a repeated addition expression to match the bar model.'* 7 | 7 | 7 | 7 | 7 Choosing an expression to match a written/spoken description: There are 2 groups of 5. *'Circle the expression that matches the sentence.'* 2 + 5 5 + 2 5 + 5 2 + 2 + 2 + 2 + 2
3:4	To complete this teaching point, encourage children to reason about representations and expressions, as shown opposite.	• *'What's the same? What's different?'*  two 2 p coins 2 + 2 4

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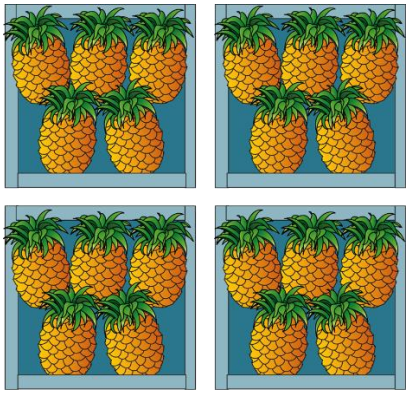
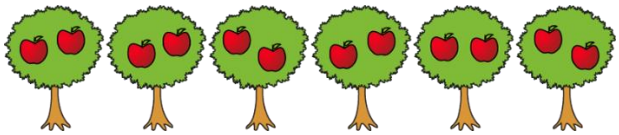
		• *'True or false?'* • This drawing shows 5 + 5 + 5 + 5  • There are 6 groups of 2. We can write this as 6 + 6 

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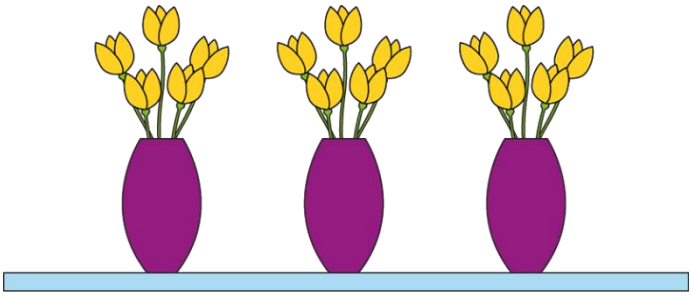
Teaching point 4:

Equal groups can be represented with a multiplication expression.

Steps in learning

	Guidance	Representations
4:1	Now that children can confidently identify the size and number of groups, it is a relatively simple matter to introduce the multiplication symbol to represent, for example, *'three groups of four'* as *'3 × 4'*. Since children can already represent equal groups using a repeated addition expression, they can also relate multiplication to repeated addition. Note that, at this stage, all multiplication expressions will be written with the number of groups as the first factor and the group size as the second factor. Children will be introduced to the idea of writing the factors in the opposite order in segment *2.3 Times tables: groups of 2 and commutativity (part 1)*. Begin by using concrete or pictorial representations, linking the stem sentence: • ***'There are ___ groups of ___.'*** to the multiplication expression: • ___ × ___ Explain to children that this is called a *multiplication* expression. Continue to also write the repeated addition expression, so that children can link the appearance of, for example, *three* 5s in the addition expression with the '3' in the multiplication expression. When verbalising the multiplication expressions, use the language of *'times'* or *'groups of'*, for example: • *'We can write this as three times five.'*	Example 1:  • *'There are three groups of five.'* • *'We can write this as five plus five plus five.'* 5 + 5 + 5 • *'We can also write this as three times five.'* 3 × 5 • *'What does the "3" represent?'* *'The "3" represents the number of groups.'* • *What does the "5" represent?* *'The "5" represents the number of flowers in each group.'* Example 2 – measures context: *'A snail moved 10 cm in the morning, another 10 cm in the afternoon, a further 10 cm in the evening and 10 cm overnight.'*  • *'There are four groups of ten.'* • *'We can write this as ten plus ten plus ten plus ten.'* 10 + 10 + 10 + 10 • *'We can also write this as four times ten.'* 4 × 10

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<table>
<tr>
<td></td>
<td>

‘There are three groups of five.’
(This is the already familiar stem sentence, and so shows how the multiplication expression directly represents the context.)
‘There are three fives.’

This language is used to help children make connections to the times-table facts in the upcoming segments. The phrase ‘multiplied by’ will be introduced in the context of scaling (segment 2.12 Multiplying by 10 and 100).

As you work through a variety of examples, deepen children’s understanding by asking what each number in the multiplication expression represents (as shown in the first example opposite). As discussed in step 3:1, when using measures contexts, make sure that the discrete ‘groups’ can be seen, even though the cardinality of those ‘groups’ is no longer evident. Again, avoid scaling scenarios.
</td>
<td></td>
</tr>
<tr>
<td>4:2</td>
<td>
Now spend some time linking addition and multiplication expressions in the abstract form.

Use variation, as exemplified opposite, to help children to notice how changing the number of groups affects the multiplication expression.

To assess and promote depth of understanding, present a dòng nǎo jīn problem such as the one opposite; ask children to explain how they know the value of the missing numbers.
</td>
<td>

‘Fill in the missing expressions.’

<table>
<tr><td>3</td><td>1 × 3</td></tr>
<tr><td>3 + 3</td><td>2 × 3</td></tr>
<tr><td>3 + 3 + 3</td><td></td></tr>
<tr><td></td><td>4 × 3</td></tr>
<tr><td>3 + 3 + 3 + 3 + 3</td><td></td></tr>
</table>
<table>
<tr><td>4</td><td>1 × 4</td></tr>
<tr><td>4 + 4</td><td>2 × 4</td></tr>
<tr><td></td><td>3 × 4</td></tr>
<tr><td>4 + 4 + 4 + 4</td><td></td></tr>
<tr><td></td><td>5 × 4</td></tr>
</table>
</td>
</tr>
</table>

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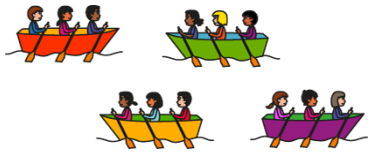
		• *'Match up the multiplication and addition expressions.'* 3 + 3 2 + 2 + 2 + 2 5 + 5 4 × 2 2 × 5 2 × 3 • *'Convince me that:'* 6 + 6 + 6 = 3 × 6 Dòng nǎo jīn: *'Fill in the missing numbers.'* 4 + ☐ + 4 = 3 × ☐ 7 + ☐ + 7 + 7 = ☐ × 7
4:3	Provide children with practice directly linking multiplication expressions with concrete/pictorial representations (without going via the repeated addition expressions). Similarly to step *3:3*, practice should include: • making/drawing a representation to match a multiplication expression • choosing a multiplication expression to match a contextual representation and vice versa • completing or writing a multiplication expression to describe a contextual representation • choosing or writing a multiplication expression to match a written/spoken description. As in steps *2:2* and *3:3*, provide some open questions, such that children need to come up with their own contexts, for example: *'Draw a picture to show six times four.'*	Matching expressions with pictorial representations: *'Draw lines connecting each picture with the correct multiplication expression.'*  4 × 3  3 × 5  2 × 5

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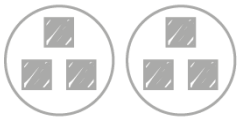
Making/drawing equal groups to match existing expressions:

- *'Antonella started to draw some equal groups. Complete her drawing.'*

I have 4 × 3



- *'Use counters to represent the following expressions.'*

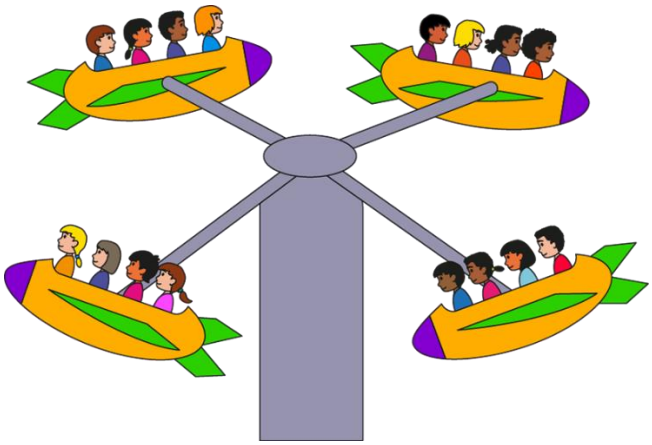
2×2 3×4 5×3

Completing/writing a multiplication expression to describe a pictorial representation:

- *'Complete the expression to match the picture.'*

$\square \times 3$

- *'Write a multiplication expression to match the picture.'*



- *'Write a multiplication expression to match the bar model.'*

8	8	8

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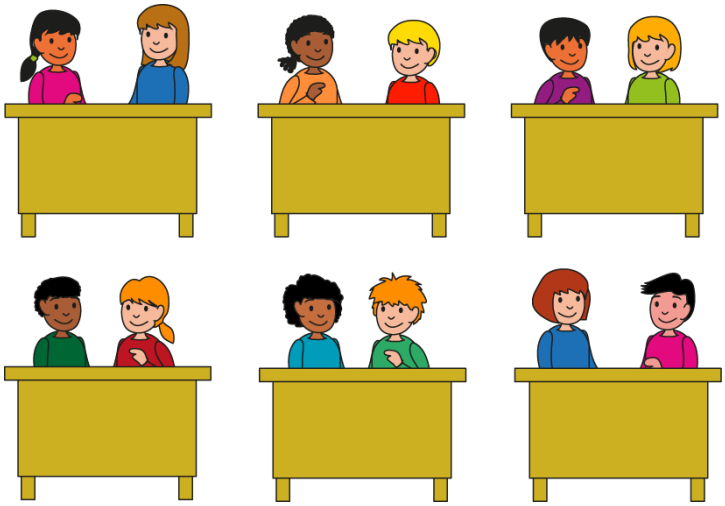
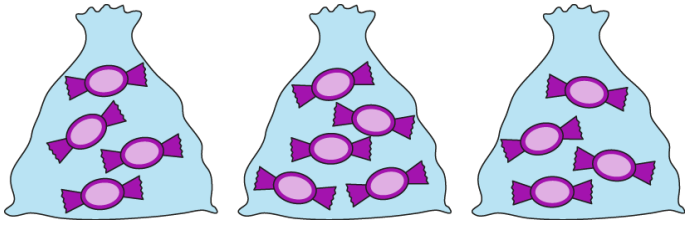
		Choosing an expression to match a written/spoken description: There are 4 groups of 7. *'Circle the expression that matches the sentence.'* $4 + 7$ $7 + 4$ 4×7
4:4	To complete this teaching point, encourage children to reason about representations and expressions, as shown opposite. You can begin to touch on the idea of commutativity and arrays, using the dòng nǎo jīn problem opposite.	• *'Does this show 3×5 or 5×5?'*  • *'Agree or disagree?'* This drawing shows 2×6  • *'True or false?'* There are three groups of four. We can write this as 3×4. 

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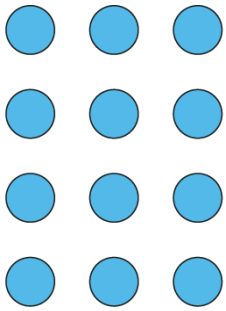
		Dòng nǎo jīn: Does this show 3×4 or 4×3? 

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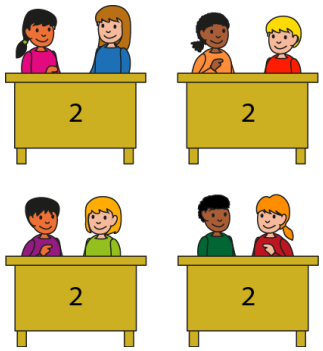
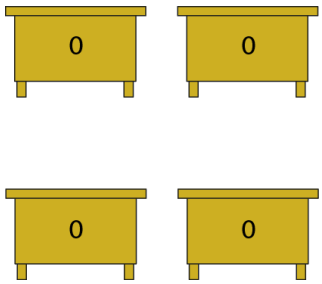
Teaching point 5:

Multiplication expressions can be written for cases where the groups each contain zero items, and for cases where the groups each contain one item.

Steps in learning

	Guidance	Representations
5:1	In this teaching point, we focus on examples with a group size of zero (steps *5:1* and *5:2*) or one (steps *5:3* and *5:5*), applying learning from the rest of the segment. Note that for now, in both cases, we are just considering a *group size* of zero or one, and not zero or one groups; the former can be more easily visualised and a repeated addition expression can still be written; the latter will be considered when we explore commutativity in later segments. To consider groups of zero, begin with a familiar context with a non-zero group size, and then change the group size to zero, as shown opposite. It is useful to use a context where the number of groups is still visible when the group size is zero. However, ensure that the sentences you use to describe the context make it clear which objects are being enumerated (opposite, the *children,* not the desks, are being enumerated; it is important to make this distinction since the four empty desks could be seen as four groups of one). Use the familiar stem sentences to describe both situations, and write both the repeated addition expressions and the multiplication expressions for each situation. By linking the non-zero and zero group size examples, children can more easily apply what they already know about writing expressions. The focus here is still on describing the contexts and writing expressions;	Example 1 – practical context: **Group size > 0**  • *'There are four groups of two children.'* • *'We can write this as <u>two</u> plus <u>two</u> plus <u>two</u> plus <u>two</u>.'* $2 + 2 + 2 + 2$ • *'We can write this as four times two.'* 4×2 **Group size = 0**  • *'There are four groups of zero children.'* • *'We can write this as <u>zero</u> plus <u>zero</u> plus <u>zero</u> plus <u>zero</u>.'* $0 + 0 + 0 + 0$ • *'We can write this as four times zero.'* 4×0

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<table>
<tr>
<td></td>
<td>children will begin to see that if the group size is zero, the sum/product is zero, but this generalisation will be explored explicitly in segment 2.5 Times tables: Groups of 10 and of 5, and factors of 0 and 1.

As in step 4:1, work through a variety of examples, deepening children's understanding by asking what each number in the expressions represent.</td>
<td>Example 2 – pictorial context:
<table>
<tr><th>Group size > 0</th><th>Group size = 0</th></tr>
<tr><td> 
• 'There are three groups of five eggs.'
• 'We can write this as <u>five</u> plus <u>five</u> plus <u>five</u>.'
5 + 5 + 5
• 'We can write this as three times five.'
3 × 5</td><td> 
• 'There are three groups of zero eggs.'
• 'We can write this as <u>zero</u> plus <u>zero</u> plus <u>zero</u>.'
0 + 0 + 0
• 'We can write this as three times zero.'
3 × 0</td></tr>
</table>
</td>
</tr>
<tr>
<td>5:2</td>
<td>Now, following the same progression as step 5:1, explore examples with a group size of one.

Children will begin to see that if the group size is one, the sum/product will be equal to the number of groups (multiplicative identity property), but this generalisation will be explored explicitly in segment 2.5 Times tables: Groups of 10 and of 5, and factors of 0 and 1</td>
<td>
<table>
<tr><th>Group size > 1</th><th>Group size = 1</th></tr>
<tr><td> 
'There are four groups of two footballs.'
<table>
<tr><td>2</td><td>2</td><td>2</td><td>2</td></tr>
</table>
• 'We can write this as <u>two</u> plus <u>two</u> plus <u>two</u> plus <u>two</u>.'
2 + 2 + 2 + 2
• 'We can write this as four times two.'
4 × 2</td><td> 
'There are four groups of one football.'
<table>
<tr><td>1</td><td>1</td><td>1</td><td>1</td></tr>
</table>
• 'We can write this as <u>one</u> plus <u>one</u> plus <u>one</u> plus <u>one</u>.'
1 + 1 + 1 + 1
• 'We can write this as four times one.'
4 × 1</td></tr>
</table>
</td>
</tr>
</table>

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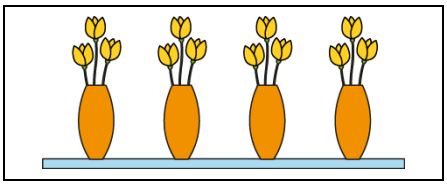
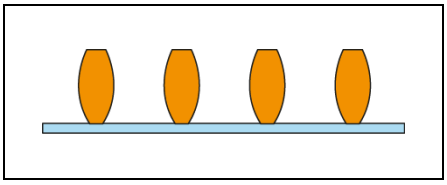
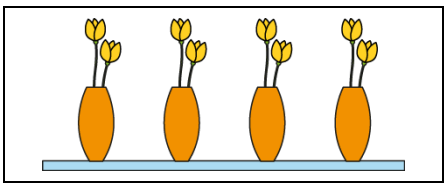
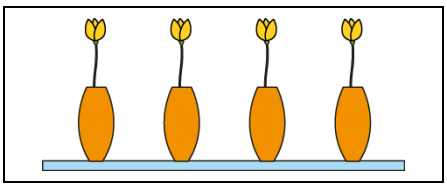
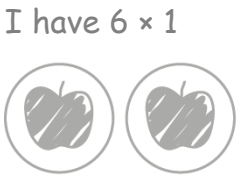
5:3	Provide children with practice directly linking multiplication expressions with concrete/pictorial representations for group sizes of zero and one. Similarly to previous teaching points, practice should include: • making/drawing a representation to match an existing multiplication expression • choosing a multiplication expression to match a contextual representation and vice versa • completing or writing a multiplication expression to describe a contextual representation • choosing or writing a multiplication expression to match a written/spoken description.	Match multiplication expressions with pictorial representations: *'Draw lines connecting each picture of flowers with the correct multiplication expression.'*  4 × 1  4 × 3  4 × 0  4 × 2 Drawing equal groups to match an existing multiplication expression: *'Aisha started to draw some equal groups. Complete her drawing.'* 

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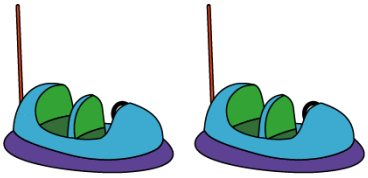
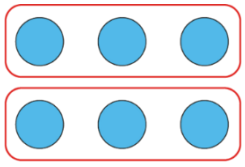
		Completing/writing multiplication expressions to describe a pictorial representation: • *'Complete the expressions to describe the children in the dodgem cars.'*  □ × 3  2 × □ • *'Write a multiplication expression to match the bar model.'* 1 | 1 | 1 Choosing an expression to match a written/spoken description: There are 7 groups of 0. *'Circle the expression that matches the sentence.'* 0 + 7 7 + 0 7 × 0
5:4	To round off the entire segment, provide problems that encourage children to summarise their understanding.	*'Tanek has written this.'*  This represents 3 + 3 *'How could he improve on what he has written?'*

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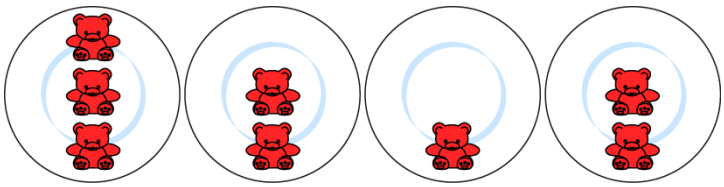
		Dòng nǎo jīn: *'Can you write a multiplication expression for this image?'*  *'What would we need to do to make it match a multiplication expression?'*

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