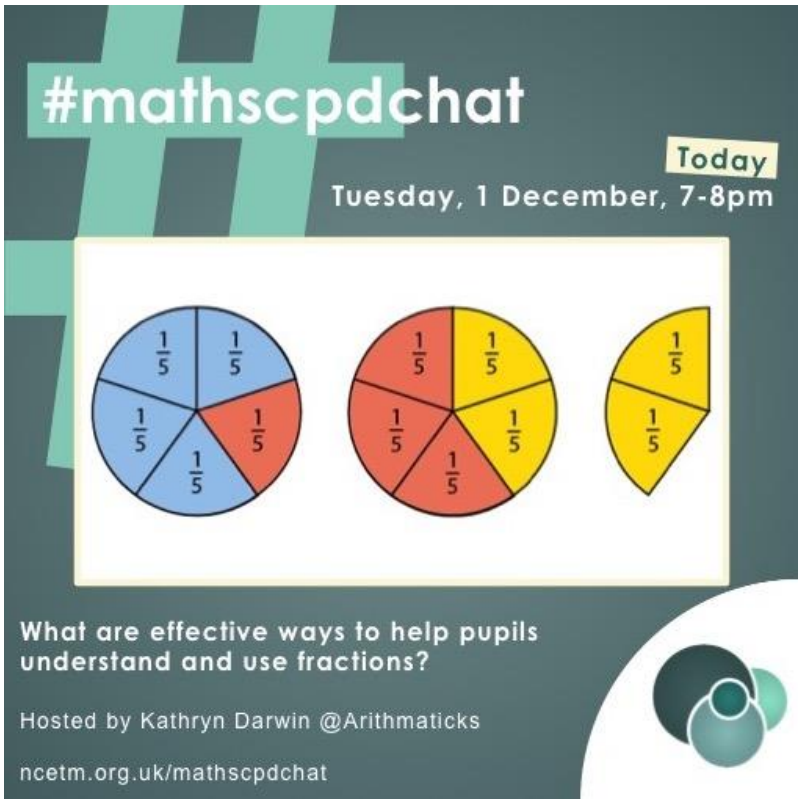
#mathscpdchat 1 December 2020

What are effective ways to help pupils understand and use fractions?
Hosted by Kathryn Darwin

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

how teachers introduce pupils to fractions:

- at least one **Year 1** teacher starts by '**talking about parts of something,** e.g. parts of a monkey' … then moves on to **equal parts of a whole**, always considering a **'fraction of' something**, e.g. seeing a half as one of two equal parts of a whole … then looking 'at it as **counting in halves**' … using 'the small steps suggested by the NCETM's *Guidance on the teaching of fractions in Key Stage 1*' (link provided below) … one teacher wondered if 'this goes too fast for most' … and 'focuses too much on the written form for KS1';

- a secondary teacher starts in **Year 7** by **challenging pupils to say in various ways what a fraction is** … e.g. 'tell me what a quarter is', 'show me a quarter represented differently' … **some pupils respond with 'pictorial' representations** (such as a circle split into four equal parts), others **show the fraction on a number line**, and some show a calculation involving the particular fraction (such as ½ + ¼ = ¾) … another secondary teacher **starts in Year 7 by putting a chocolate bar under the visualiser** and breaking it into (eventually equal) pieces … showing that a whole something can be broken into any number of equal parts, and saying that 'a fraction describes how many parts of a certain size there are';
- at least one teacher **generates 'some interesting discussions' in Key Stage 3** by presenting images of rectangles split into parts that are not all equal (for example four twentieths, one tenth and one thirtieth might be shaded red), and asking 'What fraction of the rectangle is shaded red?';
- **in Key Stage 3 at least one teacher starts work on fractions by spending time prompting students to think about what happens when the denominator of a fraction is increased/decreased** … prompting pupils to generalise … 'comparing 1/n to 1/(n + 1) or a/6 to (a – 1)/6' … encouraging students 'to think like this' by getting students to generate examples, 'such as sharing a particular number of cakes with 7 or 8 or 9 or … people – which situations gives them more cakes each?' … **'looking at intuitive approaches'** (for example comparing 3/11 with 5/11, and comparing 11/3 with 11/5);
- quite early in the discussion one teacher asked other teachers to **'please everyone, stop using shapes chopped into pieces'** … other teachers asked 'what's wrong with shapes?' … that 'the issue is the loss of relationship to 1. As soon as kids internalise a fraction as two numbers it is so hard to work with' … that, instead, it is effective to show **a complete set of Cuisenaire® rods**, and challenge pupils to 'write down the fraction that each rod represents if, for example, … the black rod is 1 … the yellow rod is 1 … the purple rod is 1 … and so on' … preparing pupils to work in this way with Cuisenaire® rods by 'first looking at equivalences between the rods' … that, in this way of exploring fractions with Cuisenaire® rods, pupils are comparing different lengths to a particular unit length, rather than shading a part … a teacher asked 'Where are the snags in understanding for the students with this approach?' … the reply was 'None that I have come across, best way to introduce fractions by a country mile for me';
- **some teachers reported that their 'practice in teaching fractions' was changed by a significant 'event'** … for example, when one teacher read *Teaching Fractions and Ratio for Understanding* by Susan Lamon, and when another took part in an NCETM workshop led by Mike Ollerton (which included inspiring young children to enjoy, and gain insight into, fractions by folding paper);

representations that teachers use when introducing fractions, and that are useful as students' learning progresses:

- Cuisenaire® rods … that when 'using pies and pizzas' it is 'so hard to ensure the splits are equal in size' … at least one teacher 'is putting (her) foot down … we can't abolish pies and pizzas … I think students should understand that any shape can be cut into equal pieces' … a response to that was that 'pies and pizzas should only come when pupils understand how fractions relate to the value of 1' … that 'it shouldn't be how fractions are introduced … if pupils lose sight of the fact that a whole shape represents 1 they can think that, for example, 5 out of 12 parts plus 1 out of 12 parts is 6 out of 24 parts, i.e. that 5/12 + 1/12 = 6/24, or even that ¼ + 1/5 = 2/9' … 'we need constant reference to what represents one whole to think correctly about fractions' that it is easier to lose sight of that when chopping up shapes, rather than when comparing Cuisenaire® rods … some teachers liked this thinking, but commented that 'unfortunately many paid resources use just a circle' … and when children try to use circles they 'can't make thirds and fifths easily let alone other fractions' … another teacher agreed that, although 'people should stop chopping up cakes and pizzas', 'there's lots of Don Steward tasks that involve fractions of shapes and a *Smile* task or two that I particularly like';
- that **bar models** 'mimic the number line more effectively than other shapes/manipulatives' … that 'the leap to understanding a fraction as a number is less difficult';
- **representing multiplication of fractions (as determining a fraction of a number) by showing (aligning) bars of different lengths that are all split into equal same-sized parts** … for example representing 1 as a brown bar split into 8 parts, with a shorter pink bar aligned below it which is split into four of the same-sized equal parts, and with a further even shorter 3-part blue bar aligned below that, and then asking 'What is three quarters of one half? (3/4 × ½ = …?)' … a contributor to the chat responded that such a 'multiplication model only suits certain values. Very difficult to extend to 2/3 × 7/11';
- when one teacher's young (KS1) pupils were **using Cuisenaire® rods to make/show fractions by aligning pairs of rods (seeing the upper rod as 1 and the lower rod as the fraction)** 'some (of those) students were puzzled by the fractions that were bigger than one, but **puzzlement that comes from something you yourself have produced is no bad thing**';
- students acting out a **task in which 6 chocolate bars are distributed on 3 tables (3 bars being placed on one table, 2 bars on another table, and only 1 bar on the third table**) … students, one at a time, go to a table of their own choosing … when they are all 'contentedly' seated the chocolate at each table will be shared equally between the students at the table… students can choose to move from one table to another before

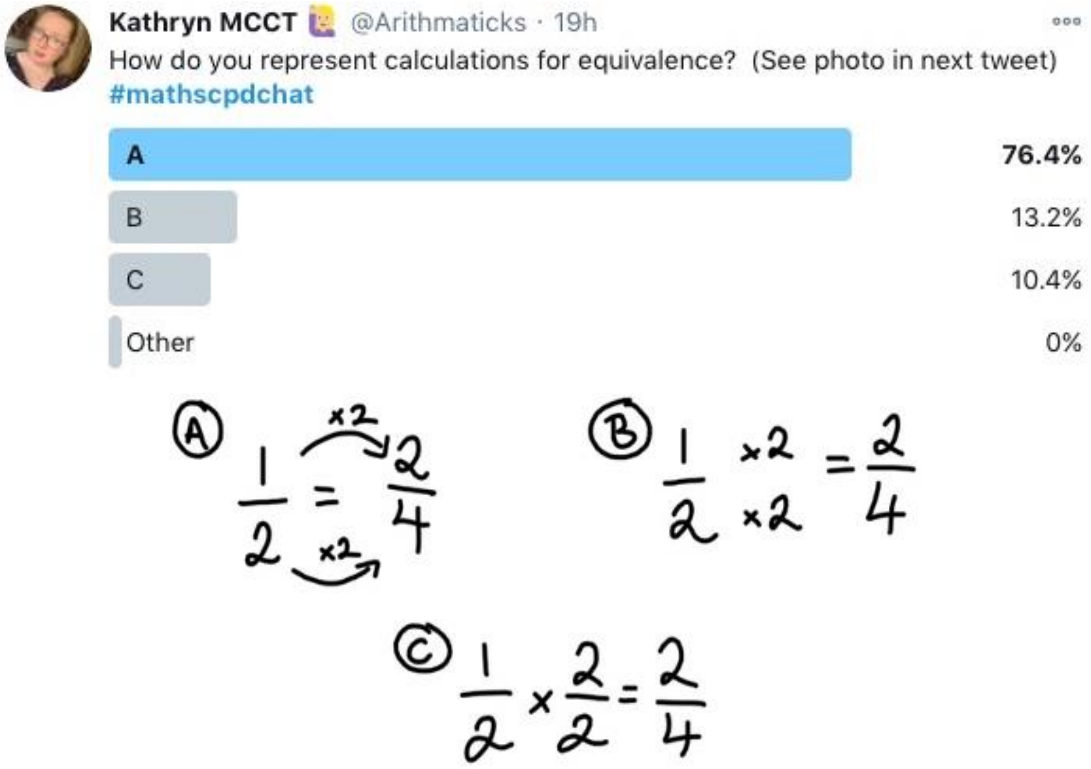
the chocolate is shared … the task usually prompts lots of discussion and some reasoning … 'it certainly makes pupils think' (link provided below);

- some teachers use **'multilink cubes in a row** – similar idea to Cuisenaire® rods';
- some teachers use **'fraction walls'** (e.g. eight 'bars' of all the same length aligned one below the other, and split into 24, 12, 8, 6, 4, 3, 2 and 1 equal parts respectively in order from top to bottom) and **blank hundred-squares**;
- some teachers use **double-sided counters** … 'Cuisenaire and double-sided counters are my go-to with fractions' … for example, showing 3 yellow counters and 6 red counters, and trying to get pupils to see the yellow counters as '1 out of every 3' … there was some discussion about the danger that if the teacher tries to demonstrate 1/3 + 1/3 by showing 2 red counters and one yellow counter, and then showing another 2 red counters and one yellow counter, pupils may interpret what they see as 1/3 + 1/3 = 2/6 … the teacher replied that he would say 'show me one third of six' … a further reply was that the 'change of whole can be tricky, which is why I prefer Cuisenaire for early introduction';
- at least one teacher likes to '**teach demonstrations first until that's really solid** … linking to division … making sure pupils understand that the link to dividing something is clear … only introduce numerators later focussing on the **difference between, for example, two thirds of a pancake and two thirds of ALL the pancakes'**;

manipulatives that teachers use to aid learning about fractions when pupils first focus on them:

- that **Cuisenaire® rods** are really useful … 'all our primary-teacher students at @Roehampton learn how to use them to teach fractions' … that the books by Gattegno 'are great', such as *Now Johnny Can do Arithmetic* (link provided below);
- some teachers have used **Lego** in the past … others mentioned **chocolate bars**, **Numicon**, **paper-folding** and a **100-bead string** … someone remarked that pupils should understand the difference between a fragment and a fraction;

the host tweeted a poll about ways in which teachers show relationships between equivalent fractions:



much discussion followed:

- '**trying to be realistic about what kids actually write and actually think about'** … that although representation C (above) 'feels so pure', students prefer representation B because 'it causes less hassle with 'crowded' working later (eg when working with algebraic fractions) … **multiplying by a fraction equal to 1, a/b × c/c, and knowing that it is the same as multiplying the numerator and the denominator by the same number, (a × c)/(b × c**) … that all three ways of expressing equivalences 'come up at different times with different students depending on the situation';
- **whether teachers encourage students to show the generation of equivalent fractions when simplifying fractions like this a/b ÷ c/c or like this a/b × (1/c)/(1/c)** … teachers discussed whether they ever 'feel comfortable' writing the denominator and/or the numerator of a fraction as itself a fraction, or as a decimal;
- examples (originally from @ProfSmudge) were provided of interesting facts, in the expression of which, the numerator of a fraction is a fraction … **2/5 = (2 & 2/5)/6, and 7/9 = (7 & 7/9)/10**;
- that writing p/q = (c × a)/c × b) = a/b '**links so much better to things like dividing with factorials/standard-form/prime factorisation/algebra**' … some teachers do not expect 'full writing' (as (a × c)/(b × c) = a/b × c/c = a/b × 1) 'once they're confident';

- that **writing a/b × c/c is a 'better long-term habit'** … e.g. it is helpful when 'rationalising denominators with surds', when 'cross cancelling for multiplication' … 'and then algebraic fractions';
- that at least some secondary teachers 'have **never had a departmental discussion' about ways of expressing justifications of fraction equivalence**;

what teachers usually 'move-on-to' once pupils have a good grasp of what a fraction is:

- some teachers move on to working with **improper and mixed numbers**;
- some teachers believe that **multiplication of fractions 'has to come before simplifying'** … using an area model … e.g. 'splitting a rectangle in half vertically and then splitting it in thirds horizontally in order to represent ½ × 1/3 or ½ × 2/3 or ½ × 3/3';
- that **primary level children need a strong grasp of unit fractions** … knowing that ¾ is the same as 3 lots of ¼ … also placing and comparing fractions on a number line … using unit fractions to reason from a whole;
- that **by the end of primary school pupils should have a good grasp of fractions in whole-part comparison, in measurement, as operators, as quotients, and as ratio**s … that is 'a really good foundation for secondary school' … some secondary teachers commented that not all pupils can 'work across' these five aspects of fractions when they arrive in Year 7;
- some teachers like to move on to the idea of a **fraction of a fraction**;
- distinguishing between, and relating fractions to, **part:whole and part:part ratios**;

what teachers believe is the 'correct' order in which to teach fraction arithmetic:

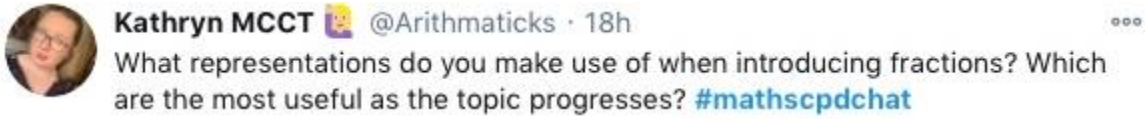
- one teacher this year 'did **addition, subtraction and division with common denominators, and then finished on multiplication**' … that it 'went down well';
- some secondary teachers do **'add and subtract, then divide, and then multiply'** … others asked **'Why would there be a correct order? They are all just ideas pupils have met before?'**;
- at least one teacher believes that when working with fractions there is not usually enough focus on **division as the inverse of multiplication** … that students can learn by looking at related operations together (e.g. by, whenever they see one of 2/3 × 4/5 = 8/15, 2/3 = 8/15 ÷ 4/5, or 4/5 = 8/15 ÷ 2/3, bringing to mind the other two) … supplementing seeing division by a fraction (of a whole number or of a fraction) both as 'how many there are in' and as the inverse of multiplication;
- some teachers prefer to 'do' **multiplication first as 'a fraction of a fraction'** … then **division (quotitively) … as, for example, 'how many fifths make 2 wholes and 3 fifths?'** … and then, lastly adding and subtracting (with fractions and measures);
- **creating helpful (possibly dynamic) images to represent multiplication of fractions** (for example to represent 5-&-1/2 × 5-&-1/2;

when teachers look at fraction-decimal-percentage equivalents:

- the comment was made that 'it is interesting that the primary DfE guidance for mathematics from the NCETM has **moved decimal numbers into *place value* instead of *fractions*'**;
- some teachers use the **equal divisions of a 'hundredths' square** where the whole square represents 1;
- that for many teachers, bringing in decimals and percentages (when fractions are in focus) **happens when relevant** … that whole lessons specifically on fractions/decimals/percentages 'never seem to go well for me' … it is not seen as 'a topic'.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

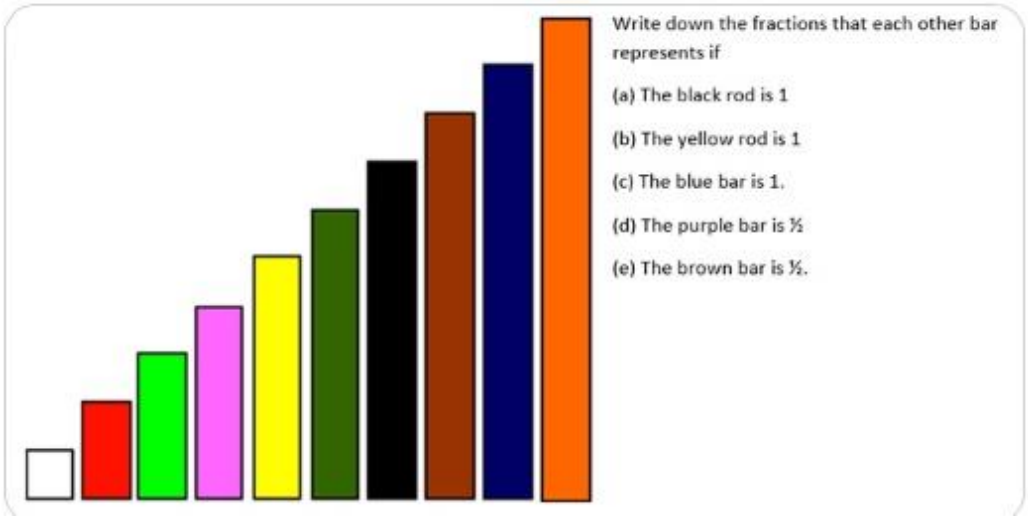
This is a part of a conversation about representations and manipulatives that are used to support the understanding of fractions. The talk is mostly about Cuisenaire® rods, and one teacher, Laura, asks a question that many teachers were asking in this thread of the chat. The conversation was generated by this tweet from Kathryn Darwin:



and included these from Pete Mattock, Charlotte Hawthorne and Kathryn Darwin:

Mr Mattock FCCT NPQSL @MrMattock · 17h

Don't need lots of activities - something as simple as this will do:
The issue is the loss of relationship to 1. As soon as kids internalise a fraction as two numbers it is so hard to work with #mathscpdchat



Kathryn MCCT 🙋‍♀️ @Arithmaticks · 17h

See I think this without a lot of lead in is really hard... how do you get students ready to go it alone beforehand? #mathscpdchat

Mr Mattock FCCT NPQSL @MrMattock · 16h

Lots of whole class stuff first, looking at equivalencies between the rods, how the relationship between a rod and a white rod informs the fraction family it can represent etc.

these from Kathryn Darwin and Pete Mattock:

Kathryn MCCT 🙋‍♀️ @Arithmaticks · 16h

So this is my issue. How is this different to 'one box' shaded? Is it not just replacing the 'one box' with the 'white bar'? #mathscpdchat

Mr Mattock FCCT NPQSL @MrMattock · 16h

A small change that leads to a big difference in how we conceptualise fractions - when breaking a shape into parts it is so easy to lose the fact that the whole shape represents 1. If a fraction is just how much shaded out of how many parts, then logically 1/4 + 1/5 = 2/9

Mr Mattock FCCT NPQSL @MrMattock · 16h

Because it is not shaded/total (i.e. two separate values) it is different lengths compared to unit length.

Kathryn MCCT 🙋‍♀️ @Arithmaticks · 16h

That's really true re wholeness... As I say, curse of knowledge. Definitley something to think about, I'll be doing some more reading and playing about with this, for sure. #mathscpdchat

and these from Laura and Pete Mattock:

(to read the discussion sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Guidance on the Teaching of Fractions in Key Stage 1 which is part of the NCETM Mastery Professional Development materials. From this page you can download a 20-page illustrated PDF document containing guidance that covers the Key Stage 1 statutory requirements for fractions. It was shared by Martyn Yeo

Teaching Fractions and Ratios for Understanding which is a book by Susan Lamon published in 2012. All the material offered in the book has been used with students, and the text is designed to help teachers build the comfort and confidence they need to begin talking to children about fractions and ratios. It was shared by Pablo R Mayorga

Now Johnny can do arithmetic which is a book by Caleb Gattegno in which he examines the obstacles that keep students from succeeding in maths. It was shared by Charlotte Hawthorne

The Cuisenaire Company B Stock which is where you will find a selection of the original maths textbooks by Caleb Gattegno available to buy at a much lower than usual price (for example *Now Johnny can do arithmetic* costs only £4). It was shared by The Cuisenaire Co UK

The Cuisenaire Company Mini Starter Pack 20% discount which includes 126 Cuisenaire® rods and a copy of *Now Johnny can do arithmetic.* It was shared by The Cuisenaire Co UK

Cuisenaire - from Early Years to Adult which is an inspiring e-book from the Association of Teachers of Mathematics (ATM) with slides by Mike Ollerton, Helen Williams and Simon Gregg. It was shared by Mary Pardoe

Fractious Fractions which is a blog by Clare Sealy in which she describes ways in which 'fractions are tricky' when you are trying to use them, or particularly when you are trying to teach or learn about them. Her explanations provide valuable insights! It was shared by Clare Sealy

Exploring Fractions which is a 2013 article from the NRICH Primary Team. It includes some discussion about why children might find fractions difficult, and links to other useful NRICH material. It was shared by Mary Pardoe

Chocolate which is a description of an extended task that students can act out, and which is briefly described in the summary above. It was shared by Mary Pardoe

Fractions ... ... which is an interesting blog by Simon Gregg illustrated with many reproductions of his young students' products that were created while they were learning about fractions and using Cuisenaire® rods. It was shared by Mary Pardoe

Visualize Equivalent Proper Fractions which is a Geogebra app which you and your students can play with in order to represent visually some fractions. It was shared by Atul Rana

Virtual manipulatives which is part of the MathsBot website where teachers and students can enjoy playing and working with a very large range of different virtual manipulatives. It was shared by Priya Shah