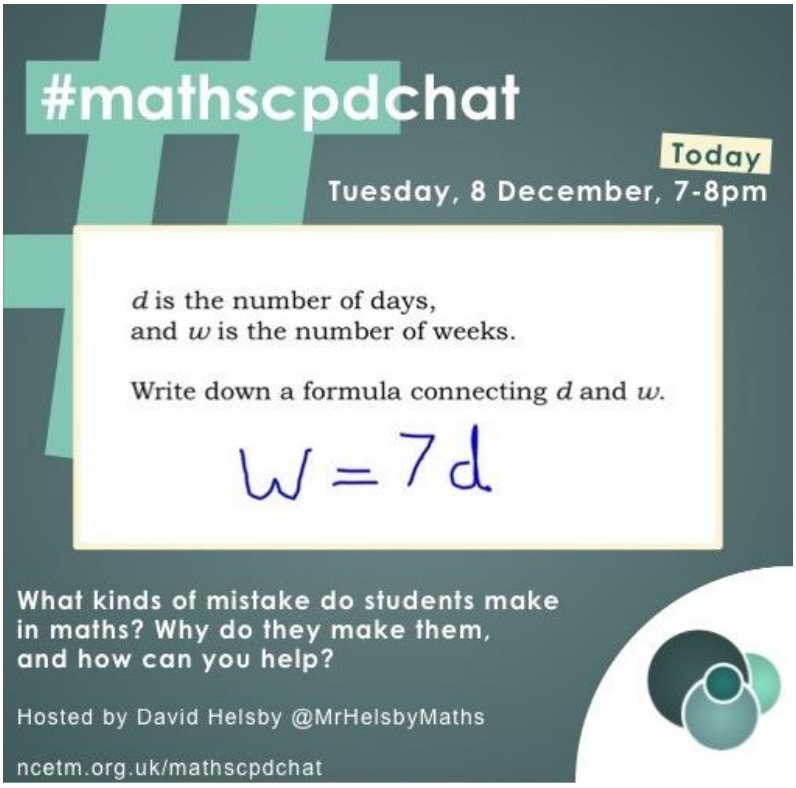
#mathscpdchat 8 December 2020

What kinds of mistake do students make in maths? Why do they make them, and how can you help?
Hosted by [David Helsby]

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focused were:

what a 'maths mistake' is, and how it differs from any other kind of mistake:

- that a 'maths mistake' may be **a careless error, a computational error or a conceptual error** … it might be the consequence of a misconception, consist of the inappropriate (or incorrect) application of a procedure, or an incorrect calculation … it might result from a reading or listening error, or from a 'lack of focus' (from a student not having their mind on what they are doing);

- that it is **not always straightforward to determine whether a student's mistake is a relatively trivial error or whether it is the consequence of a misconception** … that careful probing to prompt pupil-teacher and pupil-pupil discussion may be necessary;
- that students make mistakes owing to … **lapses in concentration … hasty reasoning … memory overload … not noticing important features of a problem**;
- that misconceptions may be based on … **alternative ways of reasoning … local generalisations from early experience**;
- that students can **make the same mistake for different reasons** … responding effectively involves identifying 'the root cause';
- that **identifying a misconception held by a student can generate a useful learning episode** … using common misconceptions in teaching … deliberately addressing the possibility that some students may acquire a particular misconception, rather than trying to avoid it … encouraging students to describe and discuss their thinking, including how they are reasoning;
- encouraging/enabling students to **regard mistakes as learning opportunities, rather than as failures**;
- encouraging students to 'look at' and **check each other's reasoning and written 'working'**;
- **whether or not it is helpful to bring up a possible misconception** when no student appears to have acquired it;
- that it is important to bear in mind that a student's **mathematical error may be the result of serious mathematical thinking** even though that thinking is based on a 'conceptualisation that is limited in some way';

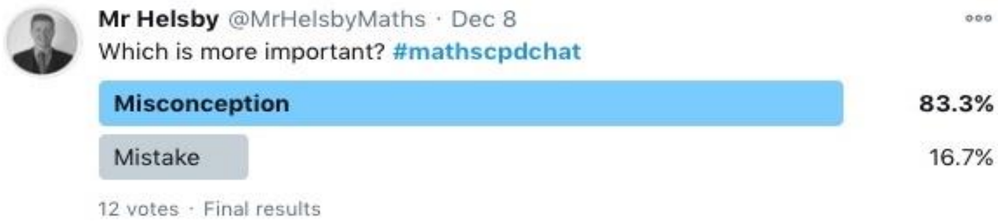
how to distinguish between a 'mistake' and a 'misconception':

- some teachers regard 'misconceptions' as **resulting from having 'understood something the wrong way',** and 'mistakes' as errors **'made out of haste or lack of proper attention'** … that misconceptions can lead to errors that may be labelled as 'mistakes';
- that a **simple mistake might possibly lead to a misconception** if a teacher fails to spot it, particularly if the same mistake is repeated without being addressed;
- there was a discussion about **how teachers would categorise a particular example of a common mistake (taking 5 × 7 to be 30)** … the person who gave that example would describe that error as 'an operand neighbour error' … the question was asked as to whether trying to decide which of the two labels, 'mistake' or 'misconception', to apply to an error **actually aids a teacher in knowing what help to offer** … that a teacher knowing why taking 5 × 7 to be 30 is not such a 'bad' mistake as taking it to be 34 may be more important (because taking that product to be 30, rather than 34, shows some awareness of the nature of multiples of 5) … that in this instance (of encountering the 5 ×

7 = 30 error) experience has shown that responding by drawing attention to 5 × 6 = 30 enables the student to self-correct … whether deciding that an error is the result of a misconception actually helps a teacher to know how to respond to a student who believes, for example, that 2n is always greater than n + 2 (because multiplication always makes numbers bigger than addition does);

- the belief that the area of any 2-D shape is calculated by applying the formula area = base × height was given by one teacher as an example of a misconception;

the host tweeted a poll:



- someone asked … **'more important for what?'**;

mathematical topics in which teachers believe that students are most likely to make mistakes:

- that pupils make **place-value mistakes** (for example by writing 0.11 for eleven tenths) … they also often make **mistakes in subtraction involving (2, 3, 4, or more)-digit numbers when 'exchanging' (e.g. 1 hundred for 10 tens) is required**;
- when **trying to solve any multi-step problem**;
- when **trying to solve equations** … and **in manipulation of algebraic expressions**, for example when changing the subject of a formula;
- when trying to find the **difference between two negative numbers**;
- **common misconceptions that are false generalisations include** … the more digits, the greater the value of a number (e.g. 0.567 > 0.8) … you always divide the larger number by the smaller number (e.g. 3 ÷ 6 = 2) … multiplication always makes bigger (e.g. 5.62 × 0.91 > 5.62);

strategies and resources that teachers have found help to reveal students' misconceptions and then generate learning to correct them:

- **concept cartoons** may be used effectively to generate thought and discussion (link provided below);
- **collaborative work** … when students share and discuss ideas … for example, when all members of a small group of students are challenged to reach a solution of a problem with which they all agree, and explain the reasoning that they jointly used;

ways of supporting colleagues in identifying students' mistakes and misconceptions, and in using them effectively to promote learning:

- a teacher suggested that a **'mistake/misconception of the week' slot** be introduced into secondary school maths-department meetings.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is a part of a conversation about what teachers believe to be the difference between the meanings of 'mistake' and 'misconception'. The discussion raised some doubt about the value of spending time (when deciding how to help a student who has made an error) trying to decide whether the error should be categorised as a simple 'mistake' or as a 'misconception'. The conversation was generated by this tweet from David Helsby:

Mr Helsby @MrHelsbyMaths · 17h
What is the difference between a mistake and a misconception? #mathscpdchat

and included these from Laura and MrHawesMaths:

Laura @mathsteacher09 · 17h
Replying to @MrHelsbyMaths
A misconception you're likely to repeat as you've understood something incorrectly. You might think area is base x height so apply that formula to any shape. That's a misconception.

MrHawesMaths @HawesMaths · 17h
Replying to @MrHelsbyMaths
One is an error through various mediums. The other is a mistaken idea like 1/10 is double 1/5 or 3/4+1/2=4/6 #mathscpdchat

these from Tom Francome, David Helsby and Lee Overy:

Tom Francome @TFrancome · 16h
Replying to @MrHelsbyMaths
How would you categorise putting 30 for 7 x 5 which we know is common? #mathscpdchat

Mr Helsby @MrHelsbyMaths · 18h
Replying to @TFrancome
Interesting question, I would want to know where this stems from- Is it a repeated computation/ recall mistake which has become a misconception? Can a misconception be broken down by mathematical reasoning but the repeated nature of it mean it's now a mistake? #mathscpdchat

Lee Overy @Lwdajo · 18h
Replying to @TFrancome and @MrHelsbyMaths
Is that regular recall of a mistake, without correction, rather than misconception? With thought, it could be corrected by the student. #mathscpdchat

Tom Francome @TFrancome · 18h
Replying to @TFrancome @Lwdajo and @MrHelsbyMaths
This is a 'better' answer than say 34 #mathscpdchat

and these from Lee Overy and Tom Francome:

Lee Overy @Lwdajo · 18h
If it can't be corrected with thought, perhaps it can be categorised as a misconception? #mathscpdchat

Tom Francome @TFrancome · 18h
I'd call it an operand neighbour error but I'm wondering how helpful the two labels are for knowing what to offer? #mathscpdchat

Lee Overy @Lwdajo · 18h
What is the reason the error is made? I understand 30 and 35 are neighbours, so is it an example of when we recall incorrectly, bypassing any processing?

Tom Francome @TFrancome · 18h
#mathscpdchat I'm not an expert on this literature but whenever you recall 5x7 you need to inhibit answers to e.g. 5x6 and 5x8 but in order to need to do that you need to have developed some awareness of multiples of 5 which is quite advanced?

Lee Overy @Lwdajo · 17h
That's interesting, thanks Tom.

Tom Francome @TFrancome · 5h
If I asked someone what's 5x7 and they responded with 30 I think I would offer "5x6 is 30" and my experience is that they self-correct.

(to read the discussion sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Standards Unit Improving learning in mathematics: challenges and strategies which is a very valuable timeless 'classic' mathematics-education document by Malcolm Swan. It exemplifies effective and enjoyable ways of teaching and learning mathematics. It was shared by David Helsby

ICCAMS which is a mathematics-education research project led by Professor Jeremy Hodgen. It investigates ways of raising students' attainment and engagement by using formative assessment to inform teaching and learning of mathematics in secondary schools. From the website you can download some very illuminating pdf documents, including *Surveying Lower Secondary Students' understandings of Algebra and Multiplicative Reasoning: to what extent do* *particular errors and incorrect strategies indicate more sophisticated understandings?* which describes interesting and useful research findings about students' maths errors. It was shared by Mary Pardoe

Exploring Common Misunderstandings in GCSE Maths which is a booklet from AQA designed for maths teachers to use with their students to explore and highlight some key misunderstandings in GCSE Maths. It was shared by Mary Pardoe

Common errors in Mathematics which is an article by Nicky Rushton from *Research Matters: A Cambridge Assessment publication*. It was shared by Mary Pardoe

Concept Cartoons in Mathematics Education which is a book by John Dabell, Brenda Keogh and Stuart Naylor containing cartoon-style drawings that pupils enjoy, and that use common mistakes and misunderstandings to generate discussion and thereby learning. It was shared by Martyn Yeo