MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.1 Comparison of quantities and measures

Teacher guide | Year 1

Teaching point 1:

Items can be compared according to attributes such as length (or height or breadth), area, volume/capacity or weight/mass.

Teaching point 2:

When comparing two sets of objects, one set can contain more objects than the other and one set can contain fewer objects than the other, or both sets can contain the same number of objects.

Teaching point 3:

The symbols <, > and = can be used to express the relative number of objects in two sets, or the relative size of two numbers.

Overview of learning

In this segment children will:

- compare items according to various attributes, and develop the appropriate vocabulary and grammar to describe these comparisons
- practise determining the quantity of objects in one or more sets
- compare the number of objects in two or more sets, and use appropriate language to describe each comparison based on the context
- learn to correctly use the appropriate mathematical symbols to compare quantities.

This segment introduces equivalence (the same as) and will therefore lay important foundations for later work on calculation and algebra. In order to deeply understand equivalence, it is important to understand what is *not* equivalent and this is done by making comparisons.

In this segment, teachers should expose children to a broad variety of measures and objects in the context of comparison. For the measures contexts, keep the comparisons qualitative – there is no need to assign a numerical value to the attributes being compared at this stage, and this also avoids the need to introduce any standard units. For example, we would say *'Bill is taller than Ben'* based on a visual comparison, rather than on quantified heights. When comparing sets of objects, children can use a visual comparison, but will progress to a comparison based on determining the quantity of objects in each set; for all of the corresponding examples, restrict the set size to a maximum of ten objects.

Throughout the segment, it is important for children to verbalise the comparisons that they make, using full sentences, and there should be a focus on accurate use of language. For example:

- countable and uncountable quantities should be described in different ways – *'There is less juice than water'*, but *'There are fewer red balloons than blue balloons'*
- measures of different types are described in different ways – a common error would be to say *'The man is longer than the boy'* instead of *'The man is taller than the boy'*.

Since there is a strong language focus in this segment, correct formulations should be modelled clearly. Key sentences should be repeated by the whole class, and children should be provided with sentence scaffolds and the appropriate vocabulary for support.

Once children have mastered the concept of comparison and can accurately describe the comparisons they make, they will progress to recording the comparison of quantities using the appropriate mathematical symbols.

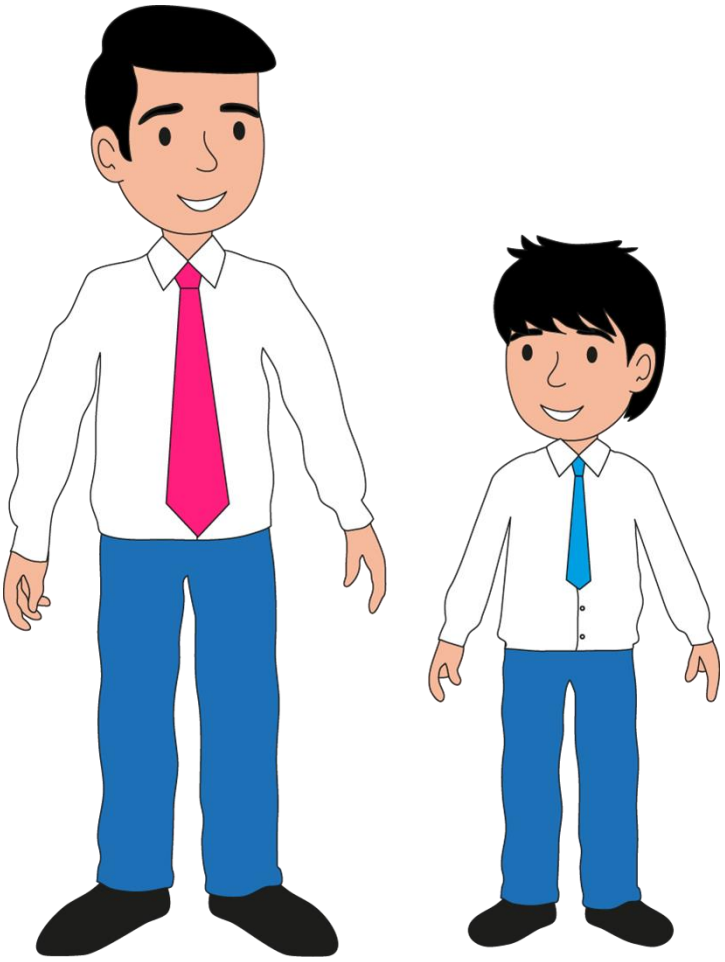
Note that it is important for children to be exposed to the variety of different measures that they will encounter throughout their time at school, but it is also important that children have the opportunity to gain a deeper understanding of these measures, and the units used to quantify them, at another time.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Items can be compared according to attributes such as length (or height or breadth), area, volume/capacity or weight/mass.

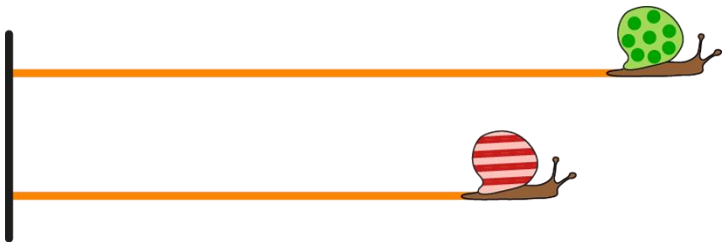
Steps in learning

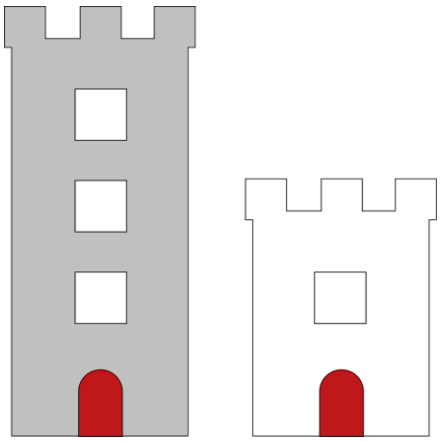
	Guidance	Representations
1:1	This teaching point will involve a lot of practical work; throughout, make sure you keep a tight focus on the concepts and language being developed. Begin by comparing attributes that children are already familiar with, such as height or weight/mass. Show a picture of an adult and a child and ask what comparisons could be made between the two people. Children should be encouraged to compare as many different things in the picture as possible – for example, height, length (of hair, tie), age, width (of tie, shoes), weight/mass and so on. At this stage the terms 'weight' and 'mass' can be used interchangeably. Children are likely to use language such as 'bigger'/'smaller', 'taller'/'shorter', 'older'/'younger' and 'longer'/'shorter', but they may not use this language accurately. An example of inaccurate use of language would be *'The man is longer than the boy'*. Make sure that you: • draw attention to the fact that the language we use to describe the length of objects varies depending on what we are describing • correct inaccurate use of language • model accurate use of language • provide children with opportunities to practise describing the variety of different contexts of length comparison using the correct language.	

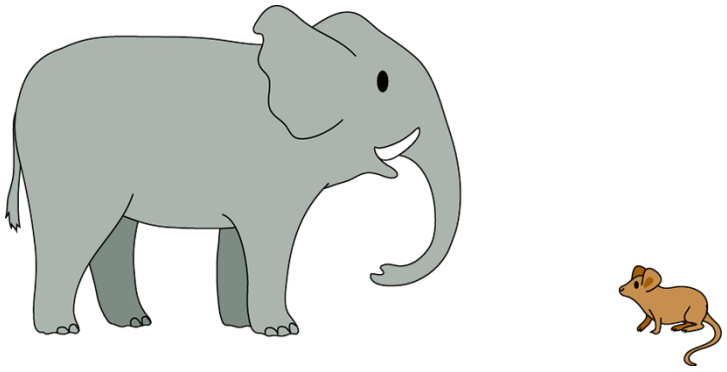
<table>
<tr>
<td>1:2</td>
<td>Now broaden the exploration to as many different contexts of measure as possible, continuing to ensure the appropriate use of comparison language.

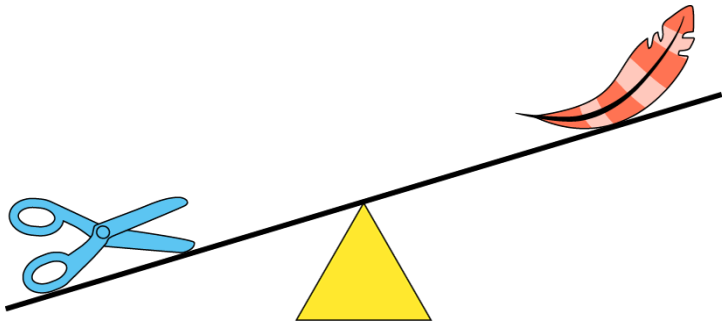
Provide different examples covering a range of contexts familiar to children, including comparing items in the classroom (for example, length of pencils) and other familiar scenarios (for example, an elephant is heavier than a mouse). Cuisenaire® rods provide an excellent opportunity for comparing lengths using their colour names or codes; for example, 'Red is longer than white'.

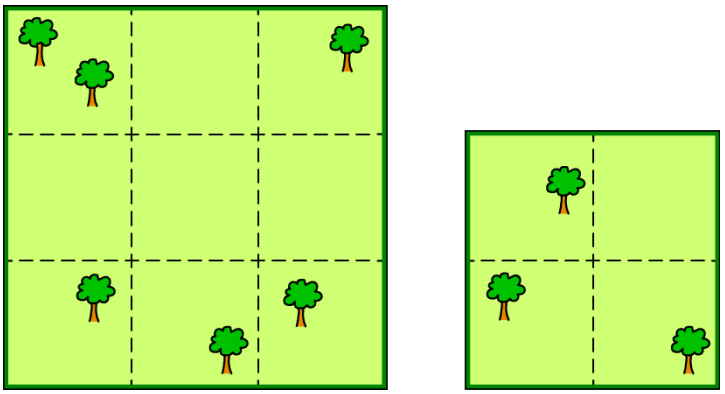
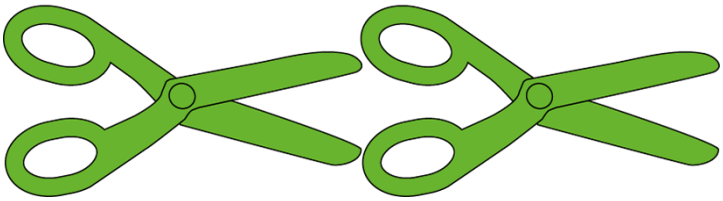
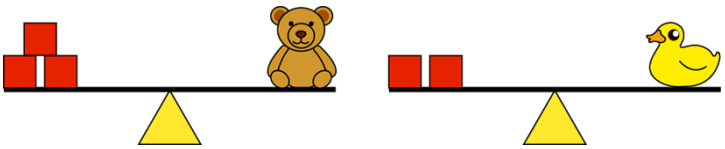
Children should work in pairs, verbalising the comparisons to one another using stem sentences such as:
• 'The ___ is heavier than the ___.'
• 'The ___ is lighter than the ___.'

Note that we say 'less' (not 'fewer') when referring to uncountable nouns; for example:
• 'There is more juice than water.'
• 'There is less water than juice.'</td>
<td>Distance:
 
• 'The spotty snail went further than the stripy snail.'
• 'The stripy snail went less far than the spotty snail.'

Height:
 
• 'The grey tower is taller than the white tower.'
• 'The white tower is shorter than the grey tower.'

Mass:
 
• 'An elephant is heavier than a mouse.'
• 'A mouse is lighter than an elephant.'</td>
</tr>
<tr>
<td>1:3</td>
<td>When providing examples, remember to include the following situations:
• Compare items that are equal to each other in length, mass, height,</td>
<td></td>
</tr>
</table>

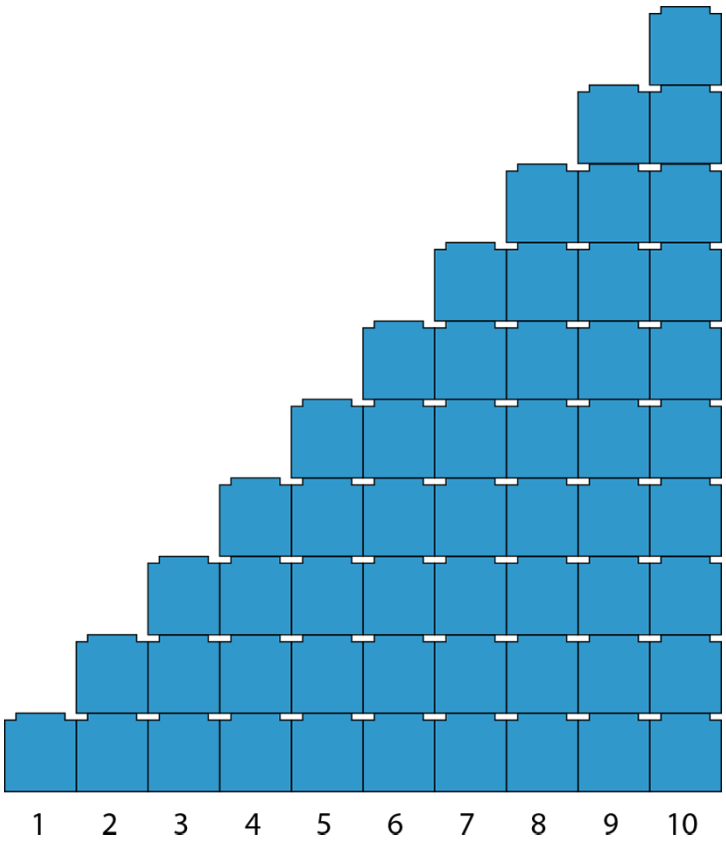
	and so on, using stem sentences such as: • ***'The ___ is the same length as the ___.'*** • ***'The ___ is the same weight as the ___.'*** • When comparing weight/mass, include pairs of objects where the children are likely to think of both as being light; for example, a balloon and a feather. • Compare the mass of lighter large objects (for example, an empty plastic bottle) with heavier small objects (for example, an apple).	Same length:   • *'The pencil is the same length as the pen.'* • *'The pen is the same length as the pencil.'*
1:4	Note that it is not necessary for children to use standard units (cm, kg, etc.) at this stage – keep the focus on descriptive comparisons, ensuring correct use of language. To avoid using standard units and measuring tools: • where possible, place objects side-by-side to facilitate length comparison • use string instead of a ruler to 'measure' objects that cannot be easily placed side-by-side • use balance scales to compare masses. Children may naturally start to notice or show an interest in how many of one object makes another, automatically treating the lesser item as a 'unit'; for example, '*Two red rods make a pink rod*'.	 • *'The scissors are heavier than the feather.'* • *'The feather is lighter than the scissors.'*
1:5	Provide varied practice, asking children to: • describe the scenarios verbally • complete written stem sentences • circle an item; for example, '*Circle the smaller field*' • draw a smaller or larger item compared to one already presented.	

	Use a dòng nǎo jīn question to provide further challenge, asking children to compare more than two objects; for example: • *'Can you find an item which is lighter than the book but heavier than the pencil?'* • *'Can you find an item which is longer than the pencil but shorter than the ruler?'*	*'Circle the smaller field.'* 
1:6	Now challenge children to compare multiple objects; for example, *'Which is longer, three rubbers or two pairs of scissors?'* Once children are confident with problems of this type, present a dòng nǎo jīn problem like the one shown here.	Comparing multiple objects:   *'Three rubbers are ___ than two pairs of scissors.'* Dòng nǎo jīn: Which of these statements is true? • *'The bear is lighter than the duck.'* • *'The duck is lighter than the bear.'* • *'The duck and the bear both weigh the same.'* Explain your reasoning. 

Teaching point 2:

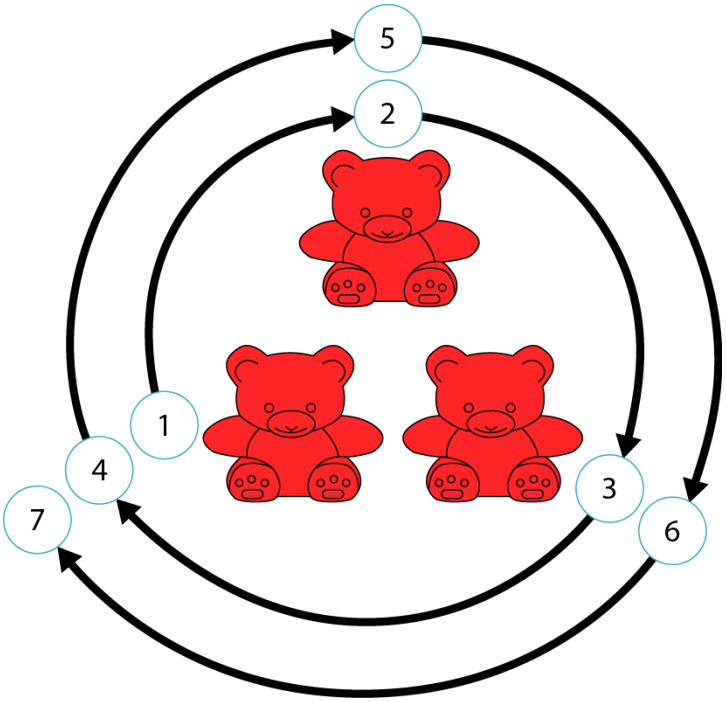
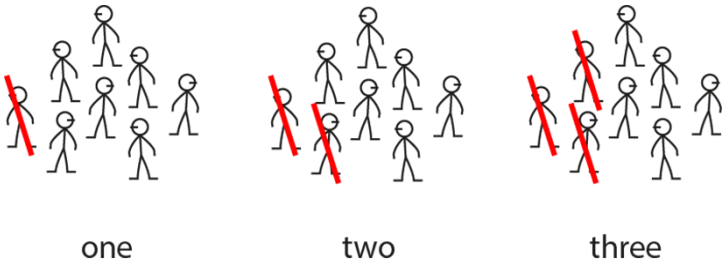
When comparing two sets of objects, one set can contain more objects than the other and one set can contain fewer objects than the other, or both sets can contain the same number of objects.

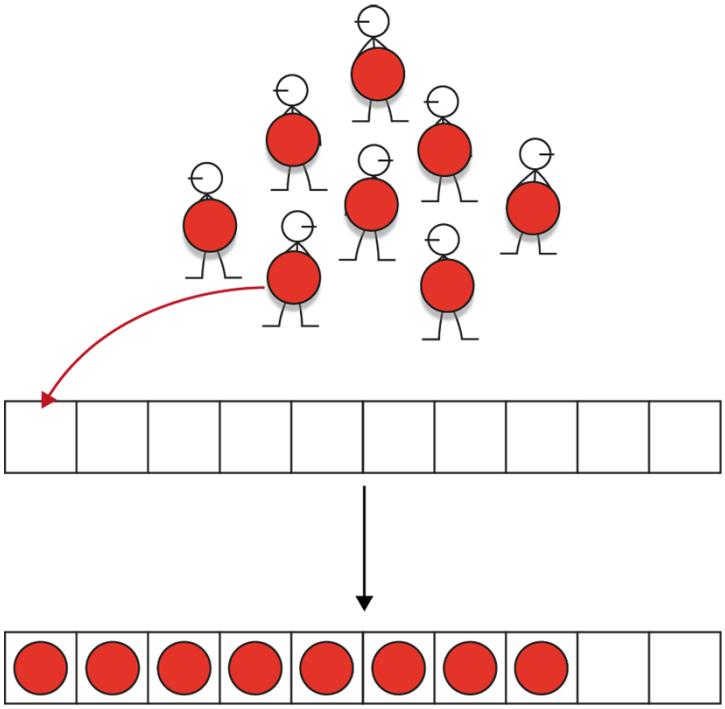
Steps in learning

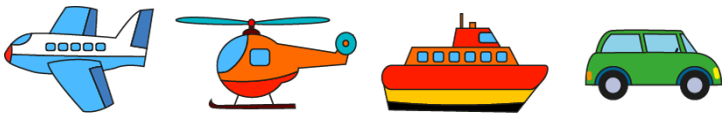
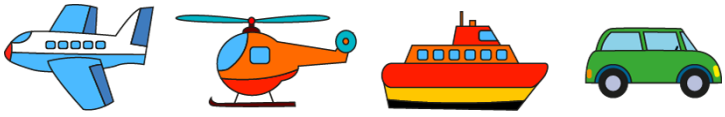
	Guidance	Representations
2:1	Before beginning work on comparing sets of objects, ensure that children can confidently count to ten. Provide children with frequent opportunities to chant/sing number-based rhymes and songs, as well as count forwards and backwards, to build a sense of sequence and synchronicity. This will allow you to see whether any children have not secured the stable-order principle (knowing that the list of words used to count must be in a specific, repeatable order). When using counting rhymes and songs, make sure you link the following aspects of number: • names (e.g. 'four') • numerals (e.g. 4). While counting, point to concrete or pictorial resources to help children make links. Examples of useful resources include: • Numberblocks images • a 'staircase' of multilink cubes • a number track. A picture of the multilink 'staircase' could be given to children to provide additional support for the next steps.	Numberblocks (1–10):  Multilink 'staircase': 

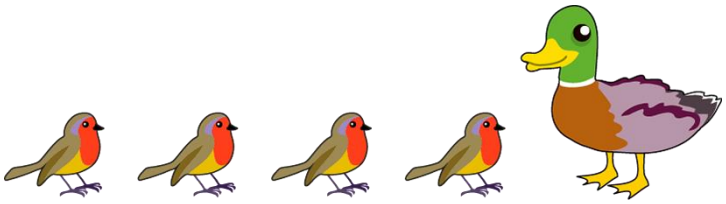
<table>
<tr><td></td><td></td><td>Number track:
<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>one</td><td>two</td><td>three</td><td>four</td><td>five</td><td>six</td><td>seven</td><td>eight</td><td>nine</td><td>ten</td></tr></table></td></tr>
<tr><td>2:2</td><td>Once you have ascertained that children can count confidently and accurately, ensure that they can:
identify the number of objects in a set (‘cardinal value’)
record the number of items in a set using the appropriate digit.
Use practical resources and demonstrate moving items from an uncounted area to a counted area as you count, to ensure items are not counted more than once.

You could emphasise the importance of only counting each item once by counting incorrectly – for example, continually counting three teddy bears; at some point children are likely to stop you and they should explain what you did wrong. Then model finding the total number correctly by moving each bear as it is counted.

To secure the cardinality principal (the last number of a count represents the number in the group), encourage children to describe the completed count as the total, for example: ‘One, two three. There are three bears.’ This will also be important later, when children begin to count two types of object in a set.</td><td>Incorrect ‘circular’ or repeated counting:
 
</td></tr>
<tr><td>2:3</td><td>Progress to correctly counting objects in a pictorial representation of irregularly arranged objects, where it is no longer possible to move the objects. Now encourage children to cross off the object when it has been counted. Alternatively each object could be matched with a counter and then the counters can be moved and counted.</td><td>Counting pictorial representations – crossing off:
 
</td></tr>
</table>

		Counting pictorial representations – using counters: 

2:4	Within your planning, include opportunities for counting where the following concepts are highlighted:
	• Order irrelevance – the order in which we count the items in a set is unimportant.
	• Conservation – the number of items in a given group is the same whether the items are spread far apart or are close together.
	• Abstraction – for example:
	  • We can count three large items or three small items; both groups have three items; the cardinal number in both cases is three.
	  • We can count four counters; the counters can represent, for example, four cats, four buttons or four sweets.
	To focus on the key concepts, present pairs of sets of objects, and use questions such as:
	• *'What's the same?'*
		Order irrelevance:
		'How many vehicles are there? Start counting with the aeroplane.'
		 
		1 2 3 4
		'How many vehicles are there? Start counting with the car.'
		 
		4 3 2 1
		Conservation:
		A 
		B 

	• *'What's different?'* • *'Set B has fewer items than set A. True or false?'* To promote depth of understanding of abstraction, you could use a 'spot the mistake' question in which only part of a set has been counted. Children will gain more experience in enumerating sets in segments *1.3 Composition of numbers: 0–5* and *1.4 Composition of numbers: 6–10* They will also gain fluency in subitising within ten (recognising the number of items without counting). Practice in these skills should also be happening outside of the main maths lesson, but steps *2:1–2:4* here will help you to assess whether children have the necessary fluency to proceed.	• 'What's the same? What's different?' • 'Set A has fewer ducks than set B – true or false?' Abstraction: A  B  • 'What's the same? What's different?' • 'Set B has fewer animals than set A. True or false?' Spot the mistake: *Jo has counted some birds. She says there are four. What mistake has she made?* 
2:5	Once children can correctly identify the number of objects in a set, you can begin to introduce contexts with two different objects within a set. Children should count the different objects separately. Continue to use a sentence to describe each completed count, linking it to the items that have been counted.	 • *'One, two. There are two red balloons.'* • *'One, two, three, four. There are four blue balloons.'*

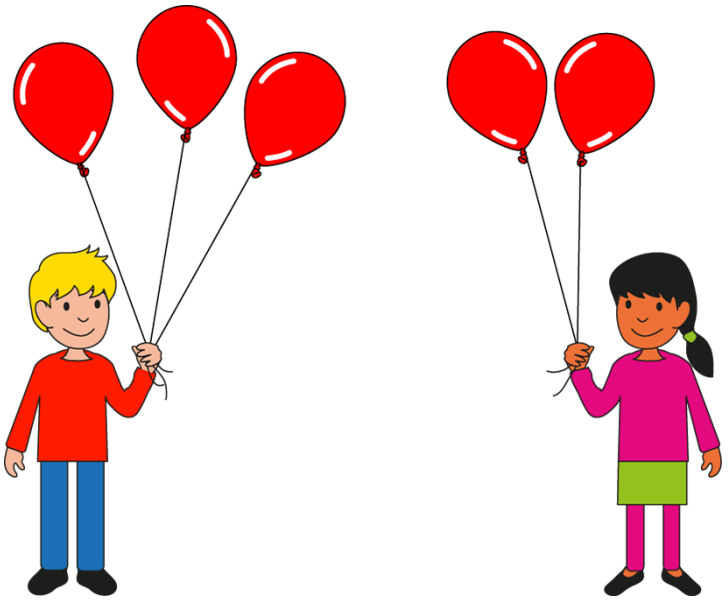
<table>
<tr>
<td>2:6</td>
<td>Once these key skills have been mastered, start to make comparisons between two sets of objects.

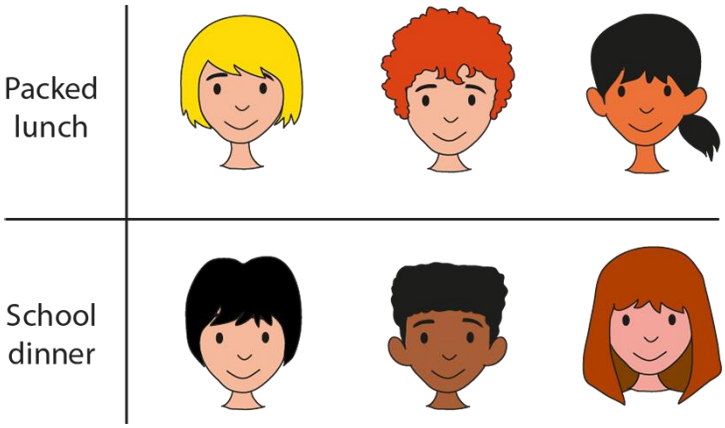
To begin with, vary the objects being compared between scenarios, but keep the objects the same within each scenario. Children should enumerate the two sets, and then describe the sets relative to one another using comparative language, as shown here.

As mentioned in Teaching point 1, the word ‘fewer’ should be used when discussing countable nouns (such as balloons), while the word ‘less’ should be used for uncountable nouns (such as volume of juice).

Children should practise saying both sentences in the scenario, for example:
• ‘The boy has more balloons than the girl.’
• ‘The girl has fewer balloons than the boy.’

This will help when children start to write symbolic expressions in Teaching point 3 (e.g. 1 < 2 and 2 > 1).

Remember to include contexts for which there are the same number of objects in each group.</td>
<td> 
• ‘One, two, three. The boy has three balloons.’
• ‘One, two. The girl has two balloons.’
• ‘The boy has more balloons than the girl.’
• ‘The girl has fewer balloons than the boy.’

• ‘One, two, three. Three children have a packed lunch.’
• ‘One, two, three. Three children have a school dinner.’
• ‘The number of children who have a packed is the same as the number of children who have a school dinner.’
• ‘The number of children who have a school dinner is the same as the number of children who have a packed lunch.’</td>
</tr>
<tr>
<td>2:7</td>
<td>Now start to compare sets of different objects. Use the following stem sentences:
• ‘There are more ___ than ___.’</td>
<td>• ‘Which are there more of, party hats or children?’
• ‘Can all of the children have a party hat?’</td>
</tr>
</table>

- ***'There are fewer ___ than ___.'***

Children may initially find it difficult to compare different objects; to support them, first provide pictures that can be rearranged into rows to facilitate comparison, then move onto using counters or cubes with pictures that cannot be rearranged. Children can then follow this process:

- Place a cube/counter of one colour on top of each picture of one type of object – for example, red counters on top of the party hats in the example shown here.
- Place a cube/counter of another colour on top of the pictures of the other type of object – for example, blue counters on top of the children.
- Rearrange the counters into two adjacent lines on a grid to see of which there are more/fewer – children can 'match' counters one-to-one with counters in the other row to establish which row has more/fewer.

When children are using this process, make sure that they have a 1:1 correspondence between the pictorially represented objects and the counters.

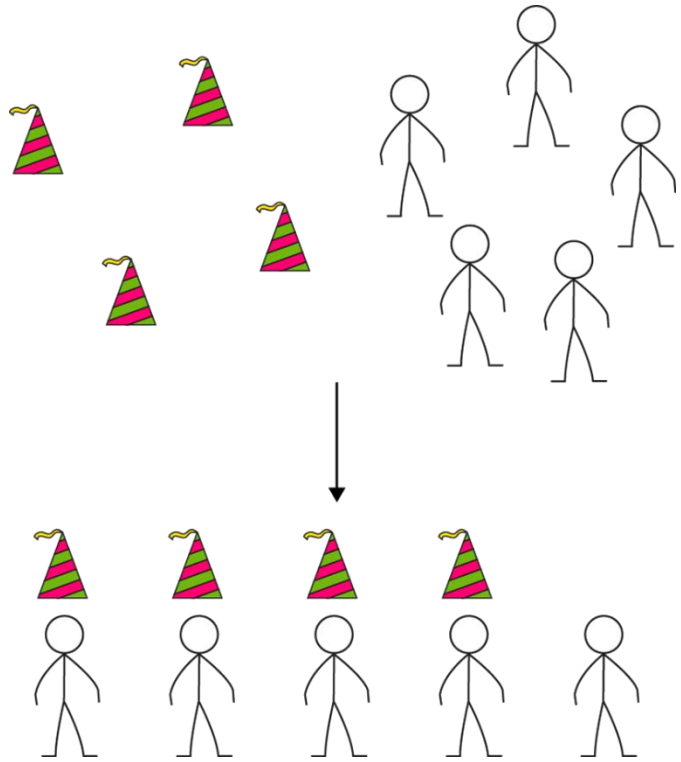
To provide further challenge, you could use a dòng nǎo jīn question:
'There are six bananas.

There are fewer monkeys than bananas.

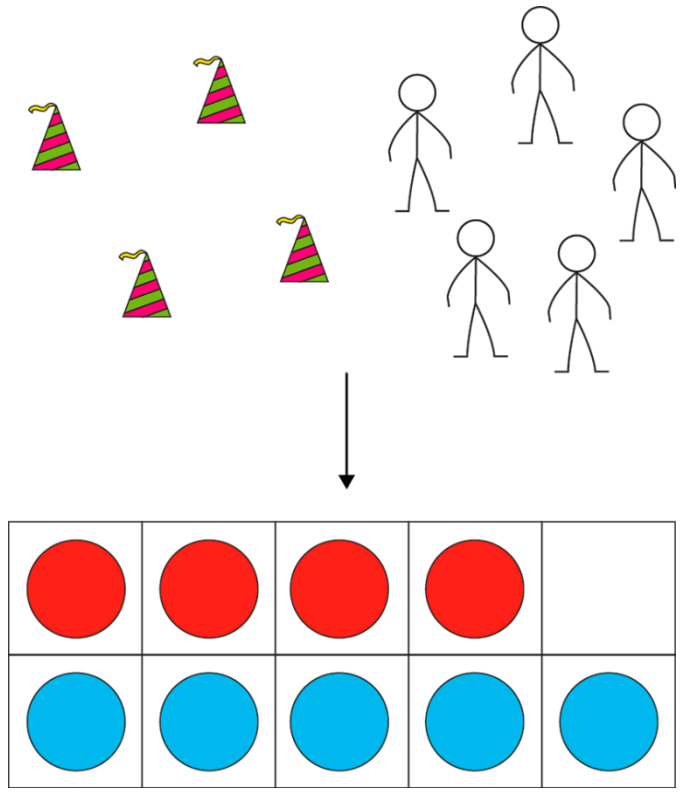
How many monkeys could there be?

Is there more than one possible answer?'

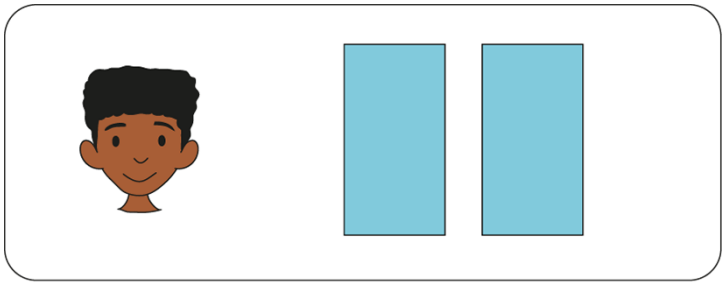
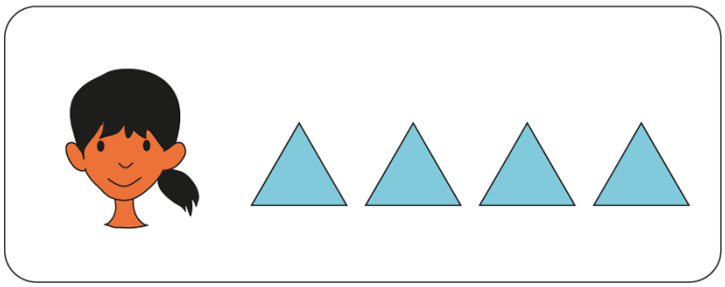
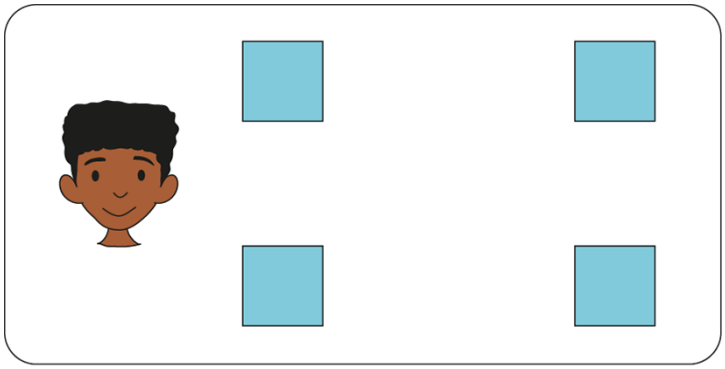
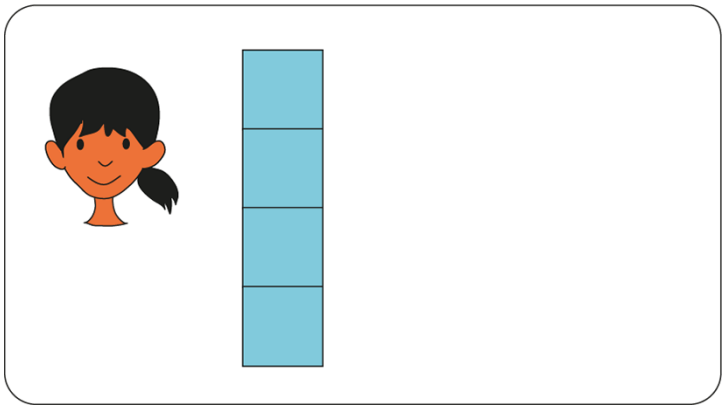
Rearranging the pictures:

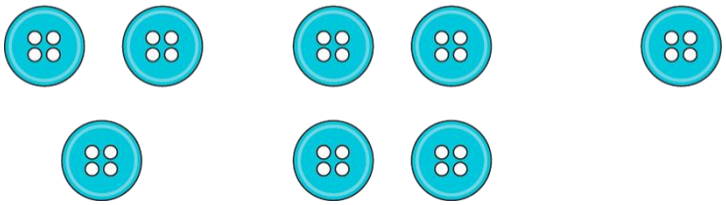
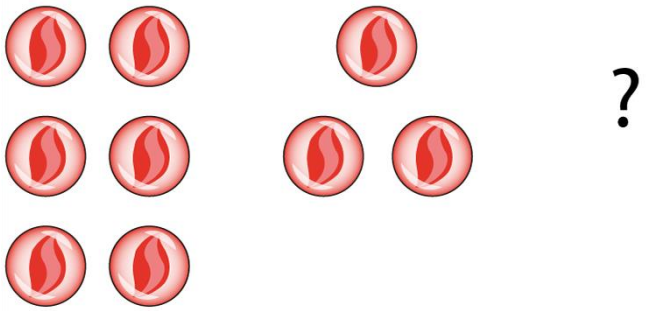


Using generalised representations:



- *'There are more children than party hats.'*
- *'There are fewer party hats than children.'*

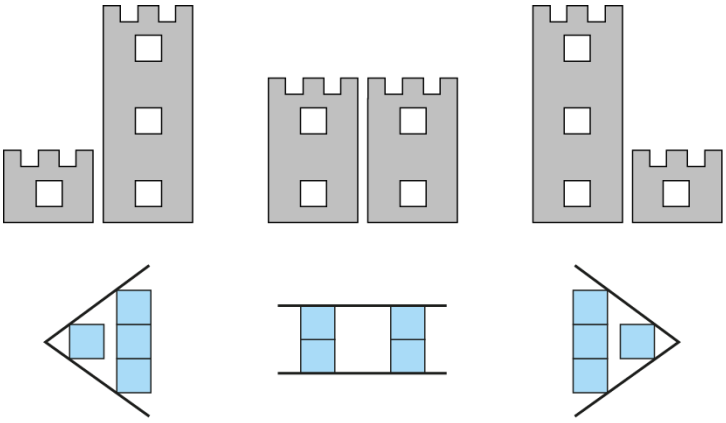
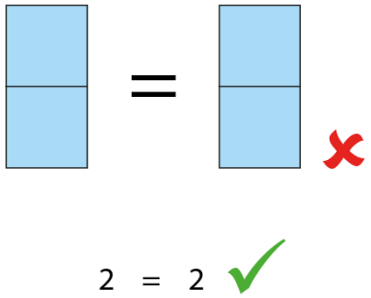
2:8	A misconception that may need to be addressed at this stage is children mistakenly thinking that the orientation, arrangement or size of the objects has a bearing on which set has more/fewer objects. These issues (conservation and abstraction) are covered in more detail in segment *1.3 Composition of numbers: 0–5*, but you may need to begin to address errors here. This also emphasises the importance of insisting children use accurate language – for example, in the first context shown here, it would be correct to say that the boy has bigger shapes than the girl, but he has fewer shapes than the girl.	*'Who has more shapes, the boy or the girl?'*   *'Who has more cubes, the boy or the girl?'*  

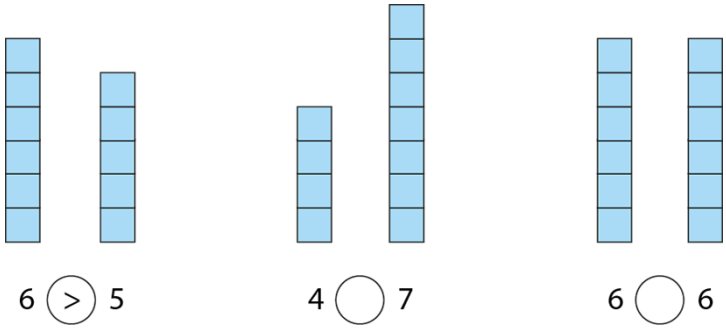
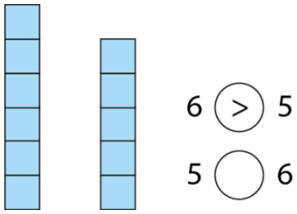
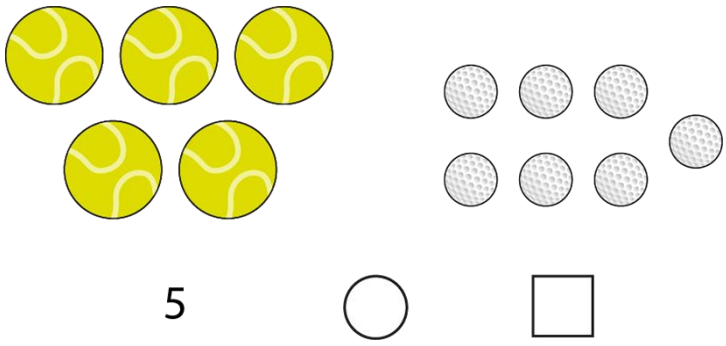
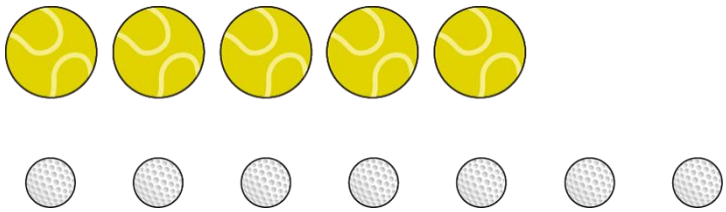
2:9	To increase the challenge, you can: • present children with three sets to compare • ask children to complete an unknown quantity based on comparative sentences.	Comparing three quantities: *'Complete the sentences.'* Adam Nick Tom  • *'___ has more buttons than ___.'* • *'___ has fewer buttons than ___.'* Identifying an unknown quantity based on comparative sentences: *'Aisha has fewer marbles than Clare, but more than Joe. How many marbles could Aisha have? Is there more than one possible answer?'* Clare Joe Aisha 

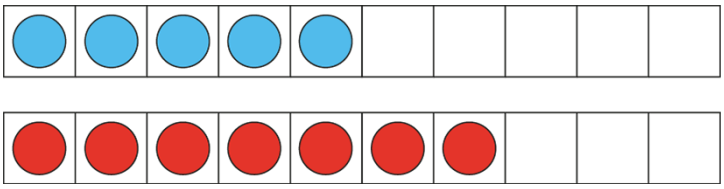
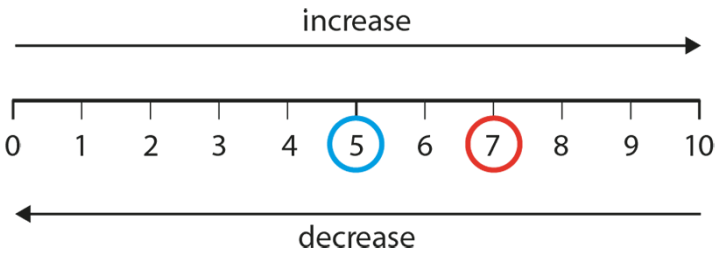
Teaching point 3:

The symbols <, > and = can be used to express the relative number of objects in two sets, or the relative size of two numbers.

Steps in learning

	Guidance	Representations
3:1	Once children have mastered comparing the number of objects in two sets, progress to using mathematical symbols to express these relationships. Begin by using pictorial representations of a context, such as the relative height of two towers. Beneath this show a generalised representation of the heights (stacks of cubes) and use this to introduce the symbols <, = and >, as shown here. Children should describe what they can see, building on the language they used in the previous teaching points: • *'One is less/fewer than three.'* • *'Two is equal to two.'* • *'Three is greater/more than one.'* At this point, make a connection between previous use of 'the same as', and use of 'is equal to'. Ask children to describe what each symbol represents in the expressions: • < represents 'is less than' • = represents 'is equal to' • > represents 'is more than'	Introducing the symbols:  1 < 3 2 = 2 3 > 1
3:2	Provide children with hands-on experience. Ask them to make two towers using multilink, then lie them down flat on a surface. They should then use two rulers/sticks lined up with the top and bottom of each tower to create the <, > or = symbol. Note that, although this activity is a very useful tool for developing depth of understanding, it is important never to use the symbols *between* concrete/pictorial representations,	Correct use of the symbols:  2 = 2

	since the symbols cannot specify what attribute of the representations is being compared. The symbols should only be used between numerals.	
3:3	Provide scaffolded practice as shown here, using a variety of pairs of towers for numbers within ten. Note the use of a circle to represent a space for a missing symbol (while a square is usually used to represent a missing numeral). Again, children should verbalise the comparisons that they are making.	*'Fill in the missing symbols.'* 
3:4	Now draw attention to the fact that if, for example, six is more than five, then five must be less than six. Extend the practice from the previous step, asking children to write the other expression in each pair.	
3:5	After children have had sufficient exposure to this concept, provide pictorial contexts showing two sets of objects that are not arranged linearly. Children can return to strategies practised earlier in the segment, including: • drawing a new version of the picture with the objects aligned • using manipulatives to represent the different types of object • relying on their sense of number alone to compare the quantities. Note that the quantity of items in each set will now always need to be enumerated to ensure correct use of the symbols, but the strategies used earlier can help with the comparison. You could also support children by using a number line to help with their sense of number. The idea of 'movement is magnitude' (moving along the number line corresponds to	*'Are there more tennis balls or more golf balls?* *'Fill in the missing symbol and number.'*  Supported using a rearranged picture: 

	an increase in value of the numbers) is covered in more detail in segment *1.3 Composition of numbers: 0–5*. As children gain confidence, you can start to present problems with missing numerals as well as missing symbols.	Supported using manipulatives:  Supported using a number line: 
3:6	Now remove the pictorial contexts, and present children with just expressions to be completed.	*'Complete the following expressions.'* 3 ◯ 4 4 > □ 2 ◯ 2 □ < 6 Dòng nǎo jīn: *'For which of the expressions is more than one answer possible? Why? Can you find all the different possible answers?'*