Secondary Magazine Issue 72

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 72 of the Secondary Magazine.

Contents

Contributors to this issue include: Dave Hewitt, Mary Pardoe, Richard Perring, Peter Ransom and Jim Thorpe.

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From the editor

We hope that you will find **time** to browse through this issue of the Secondary Magazine.

Time is always an issue in mathematics education. How often do you hear, read, or yourself use, phrases such as ‘ran out of **time**’, ‘not enough **time** to…’, ‘the **time** given to…’, ‘too **time-**consuming’, ‘investing **time** in…’, ‘the **time** it takes to …’, and so on…in connection with teaching and learning mathematics?

Quotations about ways in which **time** impacts on and influences our lives abound in history, science and literature.
For example John F. Kennedy advised:

> *We must use **time** as a tool, not as a crutch.*

Leonardo da Vinci is supposed to have said:

> ***Time** stays long enough for anyone who will use it.*

And Albert Einstein observed:

> *The only reason for **time** is so that everything doesn't happen at once!*

Here are some more.

In his interview in this issue, Dave Hewitt writes:

> *I have always been interested in the idea of economic use of **personal time** and effort, both for a learner of mathematics and for a teacher of mathematics; the idea of trying to get a lot from a little. This interest has manifested itself in many areas.*

A section about *Exposing errors and misconceptions* in Improving learning in mathematics: challenges and strategies includes the statement:

> *Sessions should include **time** for whole group discussion in which new ideas and concepts are allowed to emerge. This requires sensitivity so that learners are encouraged to share tentative ideas in a non-threatening environment.*

A report on the interesting Improving Attainment in Mathematics Project (IAMP), carried out by Anne Watson, Els De Geest and Stephanie Prestage, includes a section with the title *Giving **time** to think and learn*. It includes the information that:

> *rather than rushing through topics, the teachers gave **extended time** for learning, and often discussed this with students explicitly. They believed that **it takes time** to reach a learning goal, or make a new connection, although the learning which is going on en route may be dense and busy. **Discussion takes time**, and students would benefit from being aware of the **use of time** to explore, mull, think again, to ponder and so on.*

They describe how:

> *teachers gradually shifted learners’ perceptions of how to pace their working. One teacher put **no time limits** on any task, and explained this to students: ‘sometimes you need more **time to think**’.*

In Clock-time and personal-time in this issue, a teacher shares some thoughts about some effective ways of working that he has found take very little **time**.

Will you have the **time** to try some of the activities described in this issue – which you will find in Focus on…, An idea for the classroom, and in the Subject Leadership Diary.

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It's in the News!
Police Tweets

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but as a framework which you can personalise to fit your classroom and your learners.

On 14 October 2010 Greater Manchester Police used the social networking site Twitter to give details of every incident they dealt with over a 24-hour period. The police 'tweeted' 3 205 separate incidents in this time, including one person calling to complain that their builder had turned up two months late and another saying that their television wasn't working. These trivial distractions sit alongside much more serious crimes and illustrate the variety and intensity of the work of the force.

These 'tweets' are recorded on the police force's website, and are used in this resource to provide a context for students to develop their representing skills, one of the KS3 process skills and a part of Functional Maths.

Download this *It's in the News!* resource - in PowerPoint format

The Interview

Name: Dave Hewitt

About you: I was born in London. After a flirtation with music (I used to write songs and play in a band), I did a life-changing PGCE course under Dennis Crawforth, and taught for 11 years in and around Bristol. In 1990 I moved to the University of Birmingham where I am currently working on initial teacher education – with experienced teachers on Masters courses, and with PhD students. I have been heavily involved with the Association of Teachers of Mathematics (ATM) over many years, have been secretary of the British Society for Research into Learning Mathematics (BSRLM), an editor of Educational Review, and on the international editorial panel of For the learning of mathematics. As well as working with teachers and mathematics departments, I have given talks and presented papers at many universities and conferences throughout the world.

What got you interested in mathematics?
In 1969 I was just 13 years old when I broke my leg and ended up in hospital for two months, lying in bed with my leg in traction stuck up in the air. It was a significant time for me as up until then I had been involved in as many sports as I could. Suddenly I was physically immobile, and became bored. It was then that I started to explore my mental world more, and spent my time working on mathematical problems and puzzles, and also writing computer programmes (remember this was 1969 and access to a computer at all was rare – but I was fortunate that my school had a terminal which was connected to a computer at the University of London). It was a shift not only in my interest in mathematics but also in my sense of feeling creative – that I could make up problems of my own and that mathematics was a creative subject.

When did you start thinking about teaching?
I was fortunate to go to Warwick University for my undergraduate degree. I thoroughly enjoyed my time there and met exceptional lecturers, some of whom later became my colleagues in the mathematics education world. However, at that time I did not find every lecturer engaging. One day in my second year of undergraduate study I was sitting in a lecture room with about 200 other students. The lecturer had filled nine blackboards with lemmas, theorems and corollaries. I was approximately four blackboards behind, and writing furiously to try to keep up, when I put down my pen and asked myself what was the point of all of this? The lecturer was not interested in whether anyone understood, and I was not even trying to understand – I was just copying a collection of symbols. I had a strong sense of this being a waste of everyone's time. This was the beginning of my downfall in terms of getting a good degree as I stopped attending certain lectures. But it was also the beginning of my own thinking about what teaching might look like if I it was not like this!

Who has influenced your teaching?
Unquestionably my strongest influence has been Caleb Gattegno, who created the ATM, and made major contributions in mathematics education, the learning of languages and the teaching of reading and writing. I was lucky enough to have Dennis Crawforth, who had been a tutee of Gattegno's, as my tutor during my PGCE. This was a life-transforming year for me during which I began to re-think my whole image of what teaching might be about. Since then I have continued to learn from many people in the mathematics education world, particularly from the students I have taught, the young children of friends I have encountered, and most importantly, from my own children. Observing young children has confirmed my belief that everyone has what Gattegno calls powers of the mind, which children use to achieve so much without any formal teaching before they

enter a school. These powers can be accessed again in the mathematics classroom to help them achieve much more in less time than is currently the norm.

What are you interested in?
I have always been interested in the idea of economic use of personal time and effort, both for a learner of mathematics and for a teacher of mathematics; the idea of trying to get a lot from a little. This interest has manifested itself in many areas. As a teacher I worked hard on finding approaches to topics and ways of working with students to try to make relatively “difficult” mathematics content accessible for students – so that it felt natural and straightforward for them. I have developed computer programmes that are strongly influenced by my beliefs about teaching and learning, most notably Developing Number and Grid Algebra, which are both available from ATM. Although I feel technology can offer much for the mathematics classroom, at the end of the day it is always about people. What really matters is the way in which a teacher works with students.

I have developed a framework for looking at the mathematics curriculum in terms of those things that are names and social conventions (which I have called the *arbitrary*), and those things that are about properties and relationships (which I call the *necessary*). This framework has informed my own teaching and has seemed useful for some other people as well. It can help clarify what needs to be told in one form or another (the *arbitrary*) – with that being about assisting memory, and those things which can be noticed by students through a well designed activity (the *necessary*) – with this being about educating awareness. Recently my interest has been on teaching and learning algebra where I feel there are ways in which students can be more successful, and at an earlier age, than is often the case.

What about outside mathematics education?
I have a number of interests including playing cricket and golf when I find time. However, presently my time is primarily given to my two daughters, currently eight and 11. Fatherhood has by far been the most challenging, fascinating, difficult and rewarding thing I have ever done in my life. I am sure it will continue to be all of those in the future too.

The future?
For me – I continue loving mathematics and finding the task of teaching mathematics endlessly fascinating and challenging. There is so much to learn and I am excited about trying to learn more, seeing whether I can continue to try to become a little more effective in my own teaching each year, and to become more informed with my research activities as well.

I hope that the future might hold a climate where teachers are trusted more, and are allowed the freedom to be accountable professionals who are able to make their own educated decisions about how to work with students effectively. I am excited by the idea of more locally based communities of teachers sharing, discussing and developing ideas about teaching and learning mathematics – for example through the local branches of the ATM and the Mathematical Association. I feel this will become increasingly important following the end of the National Strategy.

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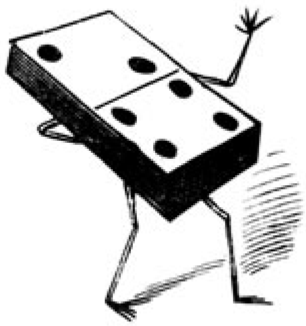
Focus on…dominoes



Domino by Franz Bonn

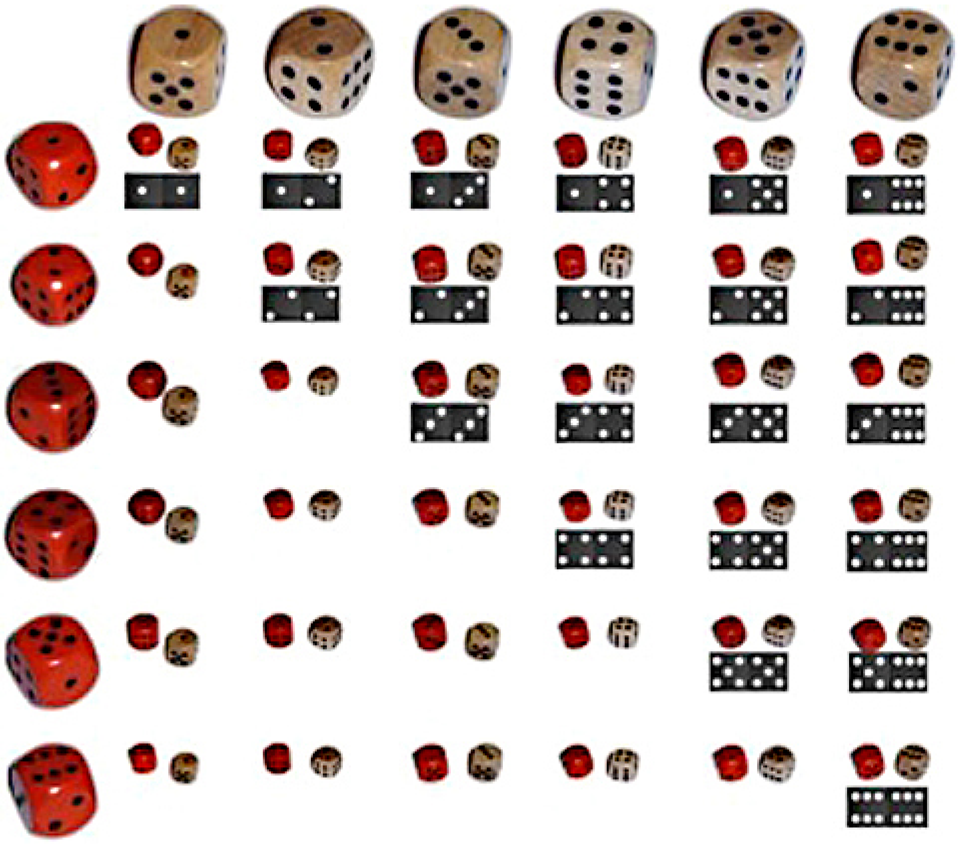
Explorations of arrangements, combinations and structures of dominoes and domino sets – in which the numbers of pips may or may not be significant – provide opportunities for students to encounter various mathematical ideas, and to act mathematically.

A shape that consists of two squares of equal size joined along complete edges to form a unit is called a domino.

During the 1950s Solomon Golomb explored the more general idea of a shape composed of *any number (n)* of squares of equal size joined along complete edges, which he called a Polyomino. Martin Gardner brought to a worldwide audience Professor Golomb's findings and puzzles about polyominoes. Polyominoes for n = 1, 2, 3, 4, 5, 6, 7 and 8 have so far been named – as *monomino*, *domino*, *triomino*, *tetronimo*, *pentomino*, *hexomino*, *heptomino*, and *octomino* respectively.

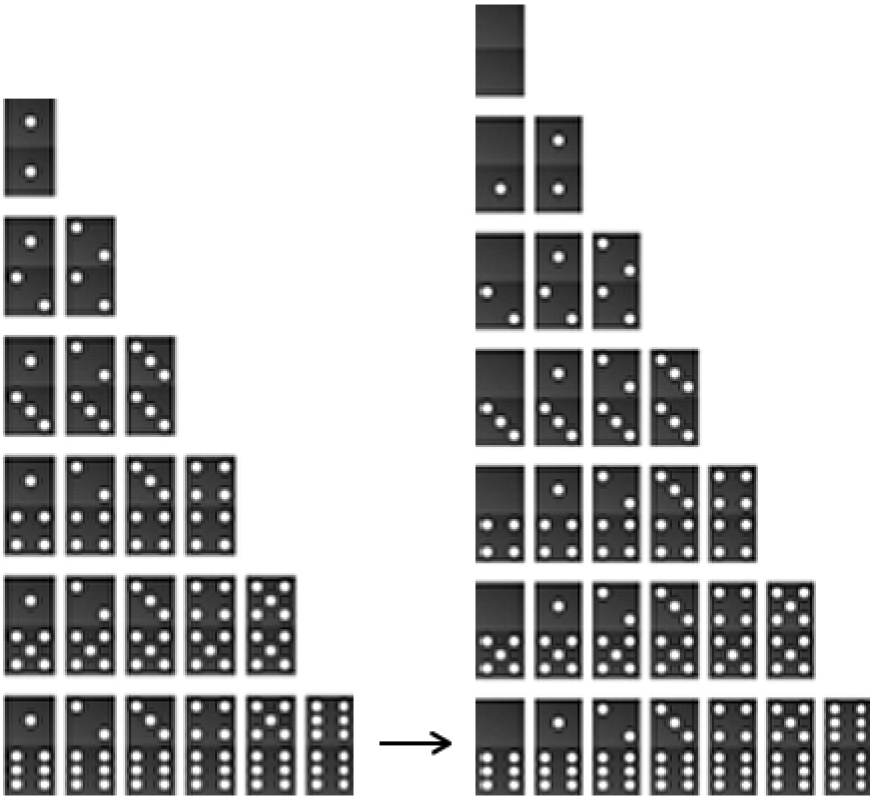
It is likely that dominoes were originally invented in China around 1120 A.D. as small bone tiles with inset round ebony pips. They seem to have been derived from cubic dice, which had been introduced into China from India. Originally each domino represented one of the possible results of throwing two dice together – the pips on one half of the domino being the pips on the top face of one die, and the pips on the other half being the pips on the top face of the other die.

Without showing students the following diagram, challenge them to explain how from the information above they can work out the number of dominoes in a full set of 12th century Chinese dominoes.



Dominoes and dice composed by the author from images of dominoes by Jelte, and dice by Alexander Dreyer

Dominoes reached Italy from China during the 18th century, and then spread throughout the rest of Europe. Europeans enlarged the Chinese set of 21 dominoes – that represented the possible combinations of two die scores – to include representation of a score of zero:

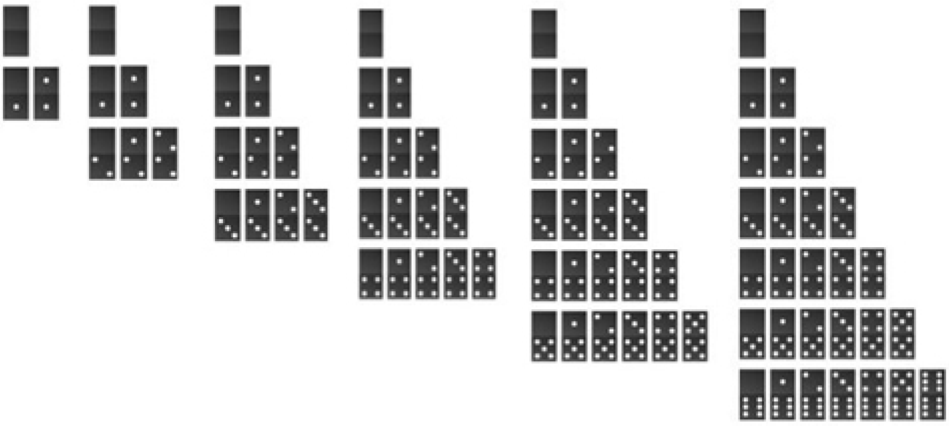


Dominoes and dice composed by the author from images of dominoes by Jelte

Ask students to explain how the European set of dominoes up to (6,6) – shown above on the right – can be thought of as containing 6 sets of 7 dominoes and yet consists of only 28 (4 x 7) dominoes rather than 42 (6 x 7) dominoes.

At the present time, in the 21st century, people everywhere in the world are familiar with dominoes. The Chilean miners who were trapped underground recently for more than two months in the San Jose mine played domino games to help maintain their good spirits!

And playing dominoes is the 'second national sport' in Cuba. As you can see in this film, Cubans play dominoes using tiles with up to 9 pips on each half – they play with double-9 domino sets. The double-9 domino set contains all the pairs taken from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Generally, the double-2 set contains all the pairs taken from 0, 1, 2, the double-3 set contains all the pairs taken from 0, 1, 2, 3, the double-4 set contains all the pairs taken from 0, 1, 2, 3, 4, and so on... This is how the sequence of double-*n* domino sets begins:



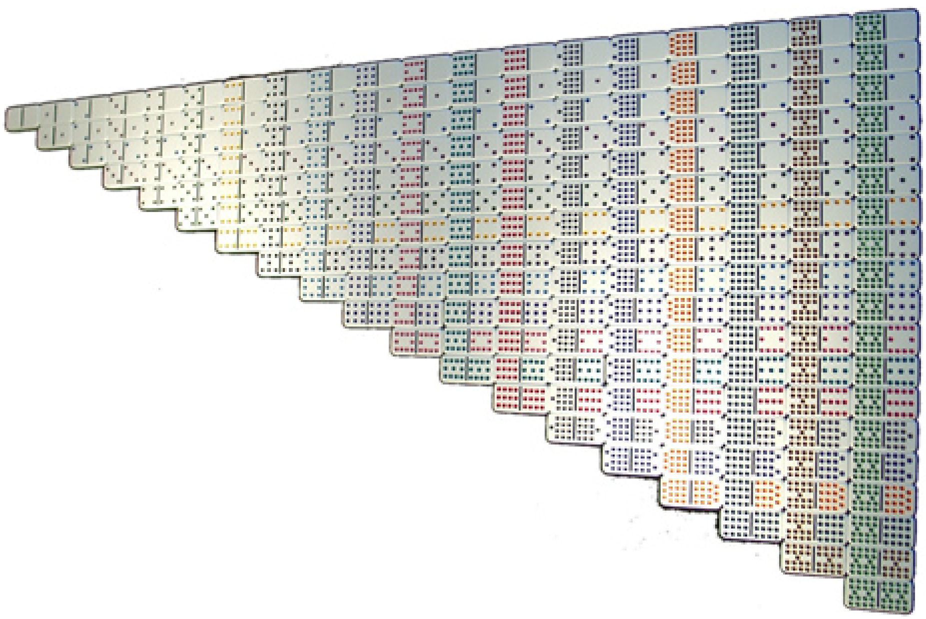
Dominoes and dice composed by the author from images of dominoes by Jelte

Can your students work out mentally the number sequence created by the total number of dominoes in each set as the sequence progresses? What is the *n*th term of the sequence?

You can help students establish mental images of the structure of domino sets by asking questions such as *'what is the probability of a domino picked at random from a double-2, double-3, double-4, … set having just one pip on at least one half of it?'*

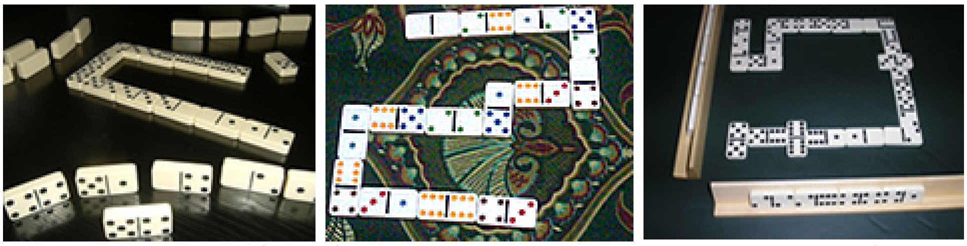
Dominoes by Honza Groh

What value of *n* has this double-*n* set of dominoes?



Dominoes by Ingo Rickmann

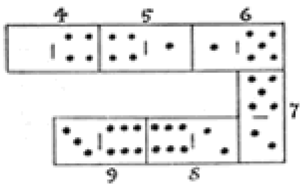
You and your students may wish to refresh your knowledge of the basic mechanics of domino games. A large variety of domino games is explained in The Game Cabinet.



Domino games, pictures from left to right by Peng, Nanami Kamimura and Roland Scheicher

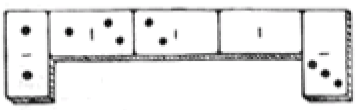
Dominoes have inspired mathematicians to devise many puzzles. Some domino puzzles are, or you can easily extend them to be, starting-points for mathematical investigation. For example Amusements in Mathematics by Henry Ernest Dudeney, published in 1917, includes these three 'investigational' puzzles:

378 Dominoes in progression



It will be seen that I have played six dominoes, in the illustration, in accordance with the ordinary rules of the game, 4 against 4, 1 against 1, and so on, and yet the sum of the spots on the successive dominoes, 4, 5, 6, 7, 8, 9, are in arithmetical progression; that is, the numbers taken in order have a common difference of 1. In how many different ways may we play six dominoes, from an ordinary box of twenty-eight, so that the numbers on them may lie in arithmetical progression? We must always play from left to right, and numbers in decreasing arithmetical progression (such as 9, 8, 7, 6, 5, 4) are not admissible.

379 The five dominoes



Here is a new little puzzle that is not difficult, but will probably be found entertaining by my readers. It will be seen that the five dominoes are so arranged in proper sequence (that is, with 1 against 1, 2 against 2, and so on), that the total number of pips on the two end dominoes is five, and the sum of the pips on the three dominoes in the middle is also five. There are just three other arrangements giving five for the additions. They are: —

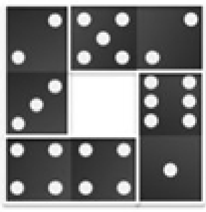
(1—0) (0—0) (0—2) (2—1) (1—3)
(4—0) (0—0) (0—2) (2—1) (1—0)
(2—0) (0—0) (0—1) (1—3) (3—0)

Now, how many similar arrangements are there of five dominoes that shall give six instead of five in the two additions?

380 The Domino Frame Puzzle

It will be seen in the illustration that the full set of twenty-eight dominoes is arranged in the form of a square frame, with 6 against 6, 2 against 2, blank against blank, and so on, as in the game. It will be found that the pips in the top row and left-hand column both add up 44. The pips in the other two sides sum to 59 and 32 respectively. The puzzle is to rearrange the dominoes in the same form so that all of the four sides shall sum to 44. Remember that the dominoes must be correctly placed one against another as in the game.

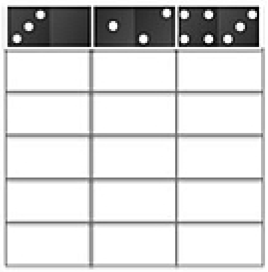
This is another well-known domino puzzle. In this ‘hollow’ square of four dominoes the sum of the dots on each of the four sides is 9:



composed by the author from images of dominoes by Jelte

Arrange the 28 dominoes of a complete double-6 set into seven hollow squares so that in each square the sum of the dots on each of the four sides is equal.

Many domino puzzles are about arranging dominoes to form ‘solid’ rectangles, as in this Dudeney ‘magic square’ puzzle. Complete this 6x6 square formed with dominoes so that the sum of the pips in every row, column and diagonal is 13.



composed by the author from images of dominoes by Jelte

This puzzle can be extended to an investigation: is there just one way of arranging 18 dominoes to make a magic square with a row, column and diagonal total of 13? Can 18 dominoes be arranged to make other 6x6 number squares in which the sum of the pips in every row, column and diagonal is the same but not 13?

Students can explore double-n (n ≤ 9) domino sets, and investigate puzzles, using the really lovely NRICH Dominoes Environment. The full screen version on the classroom IWB makes a ‘WOW’ impact!

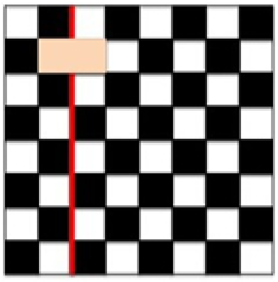
At NRICH you will also find more good ideas for activities using an ordinary double-6 set of dominoes.

Some interesting investigations and phenomena involve dominoes in which the pips are ignored.

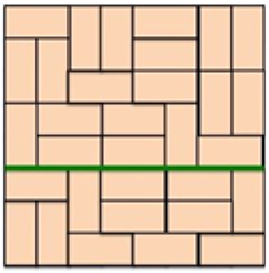
These include explorations of dominoes covering squares on grids.

Dominoes on a Chessboard

Any rectangular *n* x *m* chessboard can be covered with dominoes if, and only if, at least one of *n* and *m* is even. The squares on an *n* x *m* chessboard are created by a set of evenly spaced horizontal lines intersecting a set of equally spaced vertical lines. A domino piece that covers exactly two squares has to cross one of these lines. For example in this diagram the domino crosses the vertical line that is shown red:

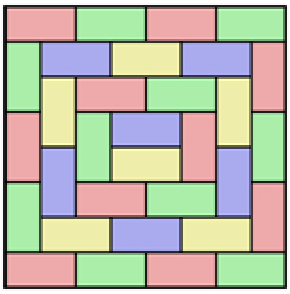


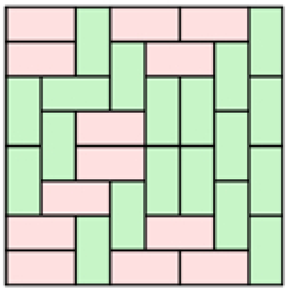
We can call a line that is not crossed by any domino a *fault line*. For example, in this covering of the board with dominoes there is just one fault line, which is shown green.



Is it possible to cover the whole board with dominoes so that every one of the lines is crossed by at least one domino – so that no fault lines are created?

Do these arrangements have fault lines?



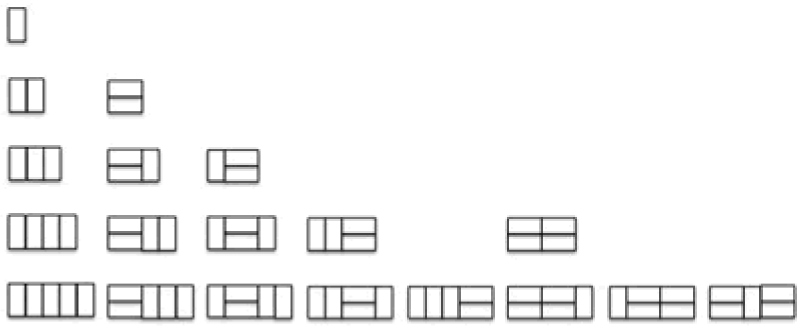


Students could explore this problem on boards with various different numbers of squares using this Cut the Knot applet. Warning – on this page, general facts are stated and explained that you will probably prefer students not to read until they have had plenty of opportunity to experiment, think for themselves and reach their own conclusions.

Jim Loy presents and explains a related Checker Board problem.

Colin Wright is available to give a mathematical talk that is based on dominoes on a chess board, and which goes on to explore pattern, possibility and proof, looking at how we can be sure that something really is impossible.

On Dr Ron Knott's multimedia website, hosted by the Mathematics Department of the University of Surrey, you will find a description of an exploration of Fibonacci numbers and Brick Wall Patterns using dominoes:

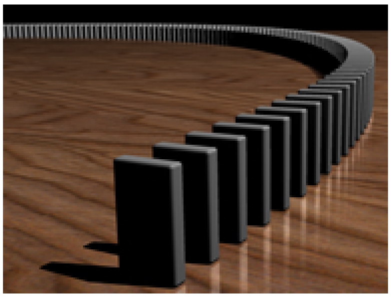


Toppling dominoes is a popular ‘sport’.

Toppling dominoes by Aussiegall

Students may enjoy watching 4 345 027 dominoes toppling spectacularly.

How do you set up thousands of dominoes so that they will all fall just by letting the first domino fall?



Dominoes by Pokipsy76

Set them up so that:

- when the first domino falls, it hits the second domino,
- make sure that when each domino hits the next domino that domino also falls.

Toppling dominoes can help students understand proof by mathematical induction.

Guillermo Bautista is a professor at the University of Waterloo in Canada. Dominoes and Mathematical Induction, which was posted recently in his Mathematics and Multimedia Blog, also relates a proof by induction to toppling dominoes.

This Cut the Knot applet may help students understand Solomon Golomb's inductive proof that if a unit square is removed from a 2^n x 2^n board the rest of the board can be tiled with L-shaped triominos.

If you give students a set of dominoes they may ‘automatically’ start trying to build **balanced structures** with them! On his web page All a matter of balancing dominoes, a senior lecturer in the School of Mathematics at the University of Birmingham, Christopher J. Sangwin, shows three interesting ways of **balancing many dominoes** one on another with no cheating – no gluing! Your students may also enjoy seeing someone balance 17 dominoes on one domino!

Dominoes also feature in art!

On the website of the Mathematical Association of America (MAA) Ivars Peterson shows how to construct domino portraits.

The Station House Opera company was the winner in 2009 of the Bank of America CREATE Art Award with their creation Dominoes 2009. Thousands of breeze block dominoes toppled their way southwards through East and South East London, finishing at dusk in Greenwich.

Out of Order is a piece of installation art by David Mach, which is known as Kingston’s phone box dominoes. These toppling domino phone boxes were ‘installed’ in 1989. Are they still there?

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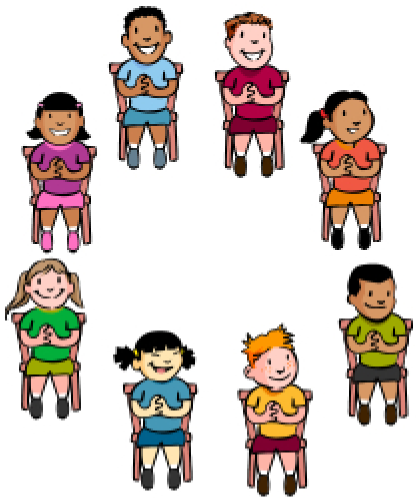
An idea for the classroom – introducing mathematical iteration

Iteration is the repeated application of a function or process in which the output of each step is used as the input for the next step.

Here are two completely different activities that students enjoy, that are interesting in themselves and that can introduce students to the idea of iteration.

Activity 1: Passing on loot

- Several students sit in a circle.

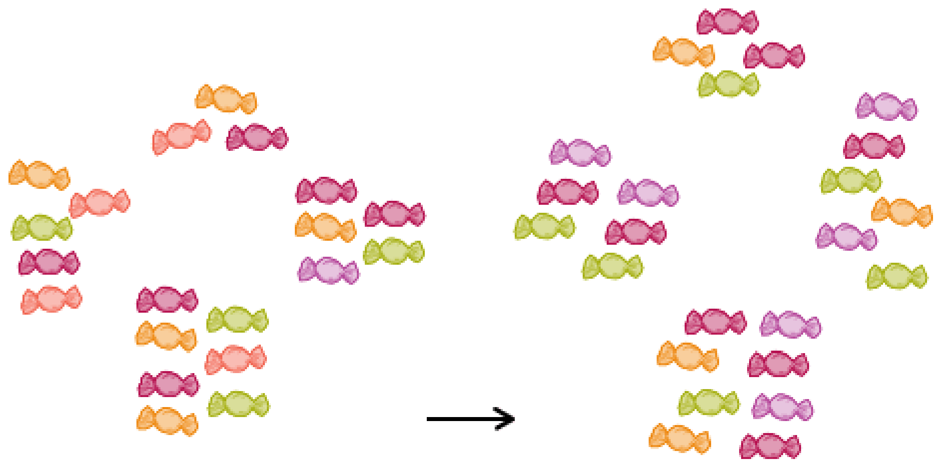


- Each student has a different even number of wrapped sweets – or small objects such as multilink cubes (their loot).
- When they are given a signal, each student passes half of his or her ‘loot’ to the student on his or her left (clockwise), and then the teacher gives any student left with an odd number of sweets an extra one to make their number of sweets even again.
- This ‘operation’ (or step) of passing-half-of-their-sweets-clockwise-and-then-adding-one-sweet-to-any-odd-numbers-of-sweets is repeated.
- It is repeated over and over again.

Example: 4 students start with 2, 4, 6, 8 sweets each respectively

Start

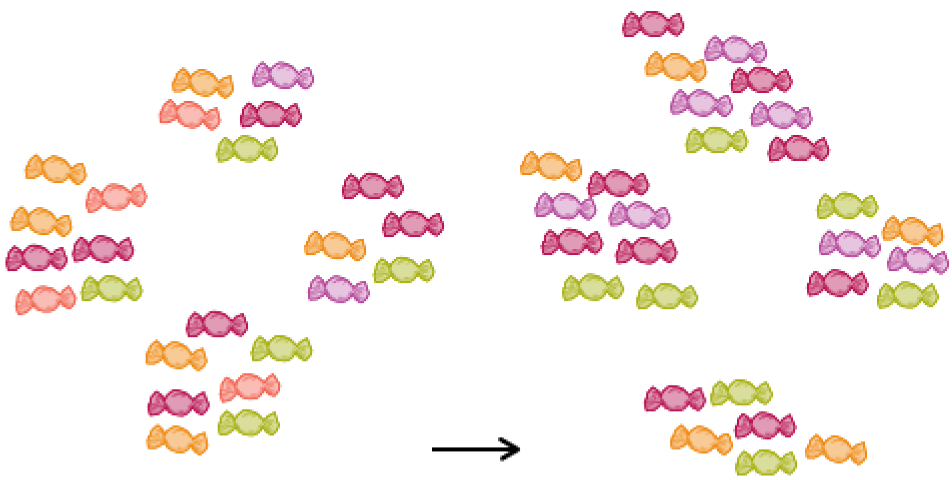
step 1



each student passes half their loot to the person on their left (clockwise)

1 sweet is added to make any odd number of sweets even

step 2



each student passes half their loot to the person on their left (clockwise)

1 sweet is added to make any odd number of sweets even

step 3 … … …

This process is simple iteration because the input for each step is the sweet distribution situation that was the output of (resulted from) the previous step.

Students can think about, discuss, and make and test conjectures about, the answers to questions such as:

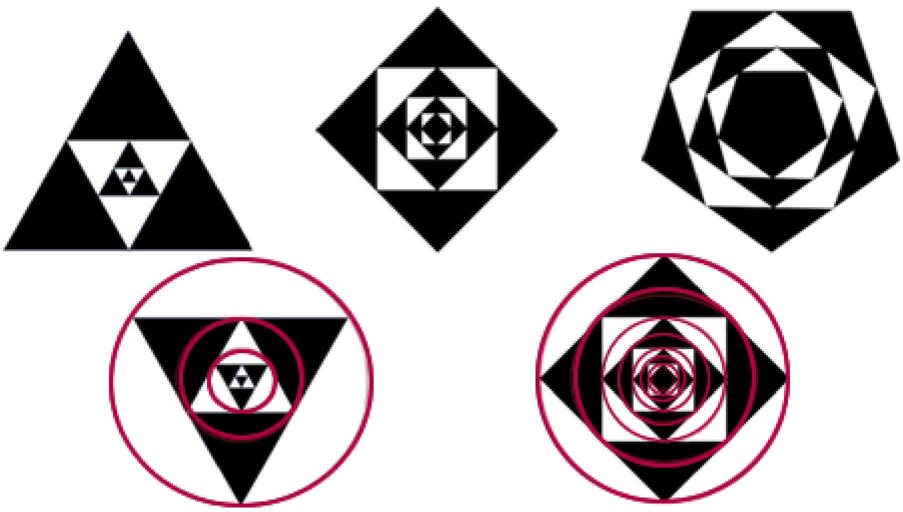
- what will happen to the distribution of sweets among the students when the ‘operation’ is performed over and over again?
- will one person end up with all the sweets?

- will everyone's 'booty' grow larger and larger as the teacher gives out more and more extra sweets?
- will the number of sweets 'stabilise', eventually evening out among the students?
- perhaps an oscillating pattern will occur, with 'clumps' of sweets moving around the circle?
- does what happens depend on the number of students in the circle?
- does what happens depend on the initial distribution of sweets?

Activity 2: Nested polygons

At WolframMathWorld you can see that 'beautiful patterns can be created by drawing sets of nested polygons such that the incircle of the n^{th} polygon is the circumcircle of the $(n + 1)^{st}$ polygon'.

Explain that in a set of nested polygons each 'new' polygon is mathematically similar to the previous polygon, and its vertices are at the mid-points of the sides of the previous polygon. Then challenge students to create, possibly using dynamic geometry software, sets of nested regular polygons such as these:



Each 'step' is the creation of a 'new' polygon that nests in the previous polygon. The output of each step is the input for the next step. So this is another simple example of an iterative process.

Creating accurately each set of nested polygons is a good mathematical challenge in itself. Some students could also explore relationships between the lengths of corresponding sides, and between the areas of corresponding regions in each set of nested polygons. They might try to answer their own questions, such as:

- *what proportion of each diagram is black?*
- *in each set of nested polygons what are the angles between each side of a polygon and the two sides of the previous polygon that it meets?*

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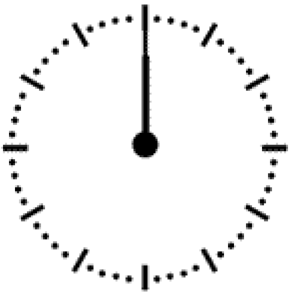
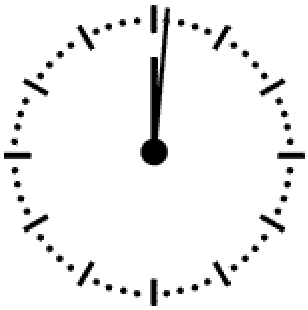
National Centre for Excellence in the Teaching of Mathematics

Clock-time and personal-time



clock by Micthev

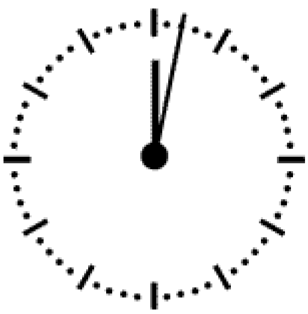
It took the same incident, in a school in which I was working as a supply teacher, repeated several times to awaken my awareness of how time relates to the significance of events. Such was my belief about people that it took someone else to force me to learn a lesson about how we use time. In response to my requests for guidance, the head of department arranged to meet me one lunch break. He didn't turn up, and knowing how busy he was I left our meeting place without offence. However, this request and failure to meet occurred twice more, and, when a colleague said, "Perhaps he doesn't *want* to meet you", I protested, ever willing to believe in the good will of the head of department.



clock by Micthev

In thinking about time in mathematics classrooms we need to move beyond the notion of mere clock-time – the 'tick-tock' of the incessant linear breakdown of lived experience into metered chunks. What other kind is there? How about the more elastic concept that is associated with particular events – time rushes by when you're having fun, but slows when you're heading for a crash, they say?

On a week's holiday I find that the first few days go slowly, say Saturday to Tuesday. Then something happens – suddenly it's Saturday again and time to drive home. I'm not sure whether this phenomenon is controllable but I recall that Edward de Bono suggested that one way to slow down the perceived speed of a holiday is take along with you the most boring person you can find. Next Saturday would then retreat towards eternity!



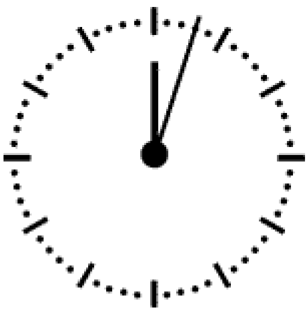
clock by Micthev

I will call my two versions of time *clock-time* and *personal-time*.

Assuming that personal-time is composed from irrational moments squeezed between the rational 'tick-tocks' of *clock-time* (as irrational numbers are squeezed between the rationals), what sort of things might be done in *personal-time*, and how may we plan to employ it? I suppose some might think it best ignored – life is, after all, hard enough already! But don't we make rods for our backs by acting as if learning proceeds smoothly, linearly, in *clock-time* chunks? It's only teaching that often proceeds like that: learning happens in fits and starts. Perhaps only pure training can be 'shoe-horned' into clock-time – I imagine WWII navigators learned the mathematics of their craft that way.

In distinguishing between two senses of time – each sense of time capable of being in harmony or in conflict with the other – I link the way that we allocate time to how we value things. For example, I question the response to advocating classroom discussion that goes "Oh, I don't have time for all that." If, as I believe, values determine available time, that response is equivalent to "I don't value classroom discussion."

The well-researched finding that mathematics teachers typically leave no more than about three seconds between asking a question and getting (or giving) an answer points to our living as much in personal as in clock-time. Nobody should consider a mere three seconds as a decent allowance of thinking time for a mathematics question that is worth asking.

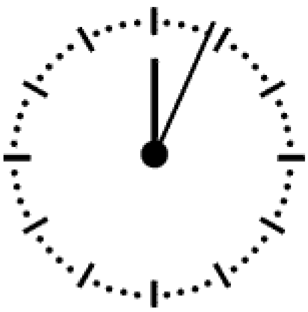


clock by Micthev

We can relate time allocation to decision-making, and note how events are comprised of micro-decisions.

> *Oliver says breathlessly to his father, "I've just heard that the bunch I was due to drive to Paris with yesterday were hit by a lorry. It was a bad crash, Tom and Penny were killed." After a pause, Oliver quietly reflects, "I'm glad I didn't go." "But if you had gone", his father replies, "the accident wouldn't have happened." Silence.*

This story brings to mind the thought that events are a result of a series of tiny decisions. Can that be stated in your scheme of work?

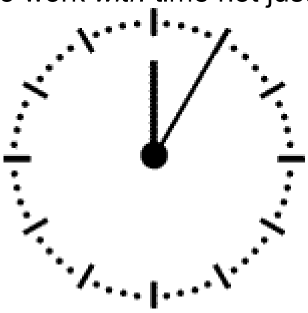


clock by Micthev

Sometimes a micro-decision breaks into clock-time producing an effect way out of proportion to the *clock-time* spent on making and carrying out the micro-decision – it happens in almost zero *clock-time* and yet it is extremely significant. Such unplanned events figure large in *personal-time*.

For example, on one occasion I observed a teacher using the space between ‘tick-tocks’ tellingly. She was walking past a table at which students were working when she suddenly stopped by one student, pointed at her work, and said, “You’re wrong!” Fierce scribbling and ‘head-scratching’ followed. Then the student rushed up to the teacher and exclaimed “No, I’m *not*, because…!”. Knowing that particular student well, the teacher made a micro-decision to intervene in a way that positively resulted in the student thinking hard about what she was doing – on another occasion, with another student, the same intervention might have had a negative effect.

‘If you want something done’, the saying goes, ‘ask a busy person’. It’s not that they know how to work harder, just that they know how to work *with* time not just *in* time.



clock by Micthev

Here are some observations about pedagogical events that are large in personal-time - owing to the significance of their effects - yet brief in clock-time. They are presented in the style of Wittgenstein’s Philosophical Investigations because they are intended to provoke reflection – a little effort may be needed to interpret them.

1.0 Manipulation both of algebraic and arithmetical expressions is partly structured by the ways we put things on paper and partly by associated mental action – ‘in our heads’. On a bad day, for me, they don’t co-operate, and I can be surprised by what I write down. Fleeting thoughts trigger micro-decisions that may elucidate or may lead to mistakes.

1.1 Recently, for a reason I have not been told, in school textbooks the distinction between the binary operation of subtraction, and the additive inverse element, which is indicated by a unary operator, has been removed. So where we once wrote, for example, -

$^{-}7 - {}^{-}7$, we now write –7 – (–7)

I presume this reversion to a much earlier practice is in the interest of fluency, i.e. unconsciously efficient symbol manipulation. In which case the brackets which hint at one of the ‘–’ signs being used as additive inverse are superfluous, as the aim is to react to mathematical signs not interpret them[1]. Perhaps we would do well, when say going through on the whiteboard a rehearsed routine of teaching signed numbers, to insert between the tick-tocks of the current formulation a light-handed exploration using '$^{-}$' and '–'. Such a light “by-the-way …” might be significantly illuminating, thus retrospectively looming large in students’ *personal-time*. We don’t want to make everything a matter of explicit rules.

1.11 Years ago it was the fashion to teach about mathematical objects called sets, as in ‘sets, relations and functions’. As part of *clock-time* textbook exercises about ‘sets’ students had to do trivial things that made little sense. What if, instead, the language of sets – union, intersection, complement, empty set, … had been part of the non-formal communication of mathematics, along with gesticulating and pointing? Wouldn't that have been a good way of relating clock-time communication to *personal-time*? Mischievous forays into set theory might include for example, 'is a shed that is empty of cows the same as a shed that is empty of pigs?'

2.0 The National Strategies go beyond arithmetic as mere computation:

The laws of arithmetic
Primary children need to understand how the laws of arithmetic work in practice if they are to multiply and divide successfully (but they do not need to know the names of the laws, or see them expressed algebraically).

> *commutative law of multiplication: a × b = b × a*
> *associative law of multiplication: (a × b) × c = a × (b × c)*
> *distributive law of multiplication over addition: (a + b) × c = (a × c) + (b × c)*
> *distributive law of multiplication over subtraction: (a − b) × c = (a × c) − (b × c).*

Written formally like this the laws can look daunting, but anyone who does a multiplication calculation probably uses them unconsciously.

I wonder if many teachers give class-time to these laws. Perhaps their proper place is as hinted generalisations that slip almost unnoticed into spaces between clock-time practice of arithmetic.

3.0 Efficiency isn't in every way good. Sometimes we need to slow things up to allow students to employ personal-time. For example, a teacher writes on the board:

> *When a polynomial, f(x) is divided by (x – a), the remainder is f(a).*

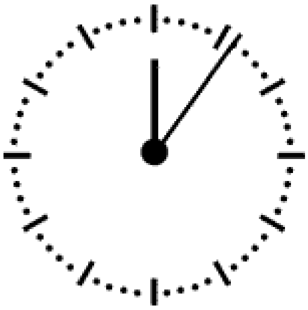
Then the teacher sets exercises. I can imagine circumstances that would justify for the teacher that decision, but wouldn't it more often be better to enable student exploration of quotients and remainders of polynomial division in order to allow students reflection in their *personal-time*?
It might be argued that students should be shown a proof, perhaps along the lines of

> *f(x) = (x – a).g(x) + R; when x = a*
> *f(a) = R*

Student belief might be thereby coerced, but how would such swift brevity chime with their existing knowledge of polynomials and their factors?

3.1 Similarly, rather than presenting 'completing the square' as a pre-packaged formula – with or without proof – might not the formulation emerge naturally from trying to control coefficients b, c in quadratics of the form x^2 + bx + c by experimenting with examples of the form $(x + p)^2 + q$?

THEOREM 1.0
Discussion promotes use of personal-time.



clock by Micthev

tick-tock!

[1] *Language and Mathematics*, pub ATM, ISBN 0 90095 31 8

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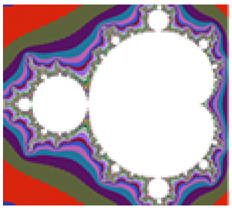

5 things to do this fortnight

- Maths Inspiration, aiming to inspire the next generation of mathematicians, engineers and other mathematical and numerate professionals, are now taking bookings for their spring shows, the dates of which are as follows:
    - Newcastle, Friday 4 March (pm only)
    - Leeds, Tuesday 8 March (pm only)
    - Bristol, Tuesday 15 March (pm only)
    - Nottingham, Thursday 17 March
    - Liverpool, Tuesday 22 March
    - Portsmouth, Tuesday 29 March (pm only)

  You can see Maths Inspiration featured on BBC Breakfast Television.

- Benoît Mandelbrot would have been 86 on 24 November. Sadly he died on 15 October. You can read an obituary in the Daily Telegraph, and you can watch him giving a talk last February about the extreme complexity of roughness, and fractal mathematics.

- The new *AQA Certificate in Use of Mathematics* is an exciting, and long overdue, new qualification.
  Students who, from September 2011, work towards it will develop knowledge, skills and understanding of mathematical methods and concepts, develop mathematical reasoning skills, and acquire the ability to solve open-ended problems. This new qualification is equivalent to GCSE Mathematics. It is built from Free-Standing Mathematics Qualifications, of which students will choose two, focussing on mathematics in the topics of Finance, Data, Shape or Algebra.
  To find out more, look at the AQA leaflet or go to the NCETM news item.
  You can attend one of the launch meetings being held on these dates in these locations:
    - 23 November, Warrington
    - 24 November, Leeds
    - 1 December, Exeter
    - 2 December, London
    - 7 December, Birmingham

  To book your place at one of these meetings, go to the AQA website and search for ‘Use of Mathematics (L1/L2)’.

- Friday 3 December is NSPCC Number Day. Your school should have received a Number Day pack – there is still time to find out where it is! Why not register your school – which can either take part in the NSPCC’s own Guinness World Record attempt, or you can devise an original fundraising event to celebrate numbers!

- The ATM/MA Secondary Maths Roadshow, on Saturday 20 November, at Our Lady’s Catholic High School in Preston, is organised by the Mathematical Association and the Association of Teachers of Mathematics. It will be an opportunity for KS3 and KS4 mathematics teachers to find new ideas. Dr David Acheson, Emeritus Fellow at Jesus College, Oxford, will give the keynote address, *Proof, Pizza and Playing the Guitar*. If you take part you will have a choice of sessions, and will receive a DVD of resources.

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National Centre for Excellence in the Teaching of Mathematics

Subject Leadership Diary

The countdown to 0.5-term has begun. Somehow decimals just don't have the same ring as fractions!

In the last fortnight, collaborating together as a faculty, we have been doing quite a bit of STEM (Science, Technology, Engineering and Mathematics) work. This has been themed around Brunel and his tremendous achievements of the early 19th century. This Isambard Kingdom Brunel video makes a nice introduction to any work related to Brunel. We are building on the 'new' KS3 curriculum, taking this into some KS4 work. When I visited the Brunel archive at Bristol University library back in September to see what they had, I discovered his Clifton Suspension Bridge calculations – which provided me with some applications of mathematics that are very useful. There are examples of the application of simultaneous equations, Pythagoras' Theorem, area calculations involving circles and some integration.

We have been working with bridge mathematics. The Clifton Suspension Bridge provides us with inspiration – we suspend a chain (easily obtainable from hardware stores in a variety of types) between two supports, measuring the distance horizontally and vertically so that we can insert data into suitable graph-plotting software.

Then we fit an equation to the scatterplot. For younger students we look at a quadratic fit, older students use the basic catenary curve, $y=\cosh(x)$ – although this is beyond secondary level, students can be very motivated by being challenged to work at the frontiers of their knowledge. They see where the catenary appears on the graph page then think about how they can transform it to fit the scatterplot. This is good revision for Y11 reminding them that $f(x – k)$ translates a curve k units to the right, $f(x) + k$ translates k units up and $f(kx)$ stretches by a factor 1/k in the horizontal sense.

The work has also allowed us to use plenty of ICT. We have used Texas Instruments nspire handheld technology through the wireless Navigator system. This allows the teacher to send files and short questions instantaneously to all the class and collect their answers. The fact that all students' screens can be seen on the IWB allows me to pinpoint any problems, or to bring a student's good work to the attention of others. Students who wish to use other ICT such as Autograph, Geometer's Sketchpad, Cabri, or Geogebra can do so. This of course can create some unease with teachers who might not be familiar with all the software available, but generally our younger digital natives cope far better than we sometimes imagine! It took me a while to realise I do not have to be an expert in all software since there always seems to be someone in the class who can sort out any unforeseen challenges.

We have also used a temperature probe to explore what happens when you mix salt with ice. Full lesson notes can be downloaded from Nspiring Learning. It is fascinating to see the temperature of the water in which ice and salt have been mixed drop to -18°C, and then get no lower. This is of course why freezers bring the temperature down to -18°C because some foods contain salt and so need to be brought down to this temperature to ensure that they are properly frozen. STEM work is very informative as well as interesting!

How do we find time to fit in this work? This is a question that we have asked ourselves from time to time because we know that we have a responsibility to our students and our school to obtain the best results possible – and work like this means less time for bookwork. Well, we reckon that this work generates a lot of the understanding needed to succeed in mathematics, and that 'drill and practice' needs far less emphasis when students have seen, for example, how graphs are transformed in a practical sense. Our results have risen over the past few years – so we continue to enthuse our students and PGCE students, hoping that we are setting an example for the future by inspiring a love of science, technology, engineering and mathematics in them all.