Primary
& Early Years
Magazine 82

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for over 80 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the Primary Magazine Archive.

Contents

In this issue we have the second of three articles which look at assessing the aims of the National Curriculum; this month the focus is on reasoning.

Maths in the Staff Room suggests ways in which collective teacher discussions - both formal and informal - can form part of the ongoing process of professional learning, and help increase the effectiveness of maths teaching across the school. This month’s article, 'Does order matter?' looks at understanding a key aspect of mathematical structure.

Seen and Heard provides a specific example of a child’s response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. This month a Year 4 pupil prompts us to think about what children understand about mathematically structured resources - in this case, a cube - and how they support understanding of the number system.

If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please email it to us. If we publish your suggestion, we’ll put a £20 voucher in the post.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

Image credit

News

ACME has issued a report on maths teacher training, Beginning teaching: best in class? The report looks at what high-quality initial teacher education (ITE) for teachers of mathematics should look like, setting out principles about the standards required in order to equip teachers of mathematics with the skills they need. This will be of interest to all primary and secondary schools involved in teacher training as well as ITE providers.

The British Educational Research Association (BERA) has published a paper, Effectiveness of mathematics teaching: the truth about China and England, which includes a number of recommendations, based on decades of teaching effectiveness research findings and the Effective Mathematics Teaching (EMT) project findings, which will be of interest to all teachers.

Tim Oates, from Cambridge Assessment, has produced a second video blog, Beyond a world of targets, to support schools with ‘life after levels'. It builds on, and adds to, a similar one he made last year.

A new set of NCETM videos looking in detail at a maths lesson in Year 3, which follows elements of the teaching for mastery approach, is now available. The school involved is one of 47 primary schools across England leading the development of teaching for mastery approaches as part of the England-China project within the Maths Hubs programme, coordinated by the NCETM and funded by the DfE. The videos include interviews with the class teacher and the subject leader for mathematics.

A study, Intergenerational Effects of Parents’ Math Anxiety on Children’s Math Achievement and Anxiety, appears in Psychological Science. A team of researchers led by Uchicago psychologists Sian Beilock and Susan Levine found that children of maths-anxious parents learned less maths over the school year and were more likely to be maths-anxious themselves - but only when these parents provided frequent help with the child’s maths homework.

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National Curriculum in Focus

***National Curriculum in Focus** is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series here*

Assessing the Aims: Part Two - Reasoning

This is the second of three articles focused on assessment of the aims of the National Curriculum

The National Curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

As stated in the NCETM Teaching for Mastery booklets:

> *"Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills. Key measures of this are the abilities to reason mathematically and to solve increasingly complex problems, doing so with fluency, as described in the aims of the National Curriculum."*

As teachers and schools grapple with decisions about assessment it will be important for them to consider how the aims are reflected in their:

- assessment principles
- assessment criteria, and
- assessment practice.

The second of the aims, reasoning, is at the heart of mathematical thinking. It includes working systematically so that it is possible to notice patterns, recognising what changes and what stays the same when considering different examples, and using this understanding to identify mathematical structures and to generalise. Reasoning should be part of all decision making in mathematics.

In order to assess reasoning, children will need to communicate their thinking, and this will include using mathematical images, pictures and symbols, as well as explanation. Mathematical talk is indicated by the wording of this aim in the National Curriculum, but this talk is more than using mathematical vocabulary. It requires structuring mathematically and grammatically accurate sentences that communicate the connections that have been made, and convince others that the reasoning is sound.

In his article Private Talk, Public Conversation (adapted from *Transforming Primary Maths*) Mike Askew explains the role of talk in mathematics as:

> *"It is not simply that children are talking about mathematics, but that they are talking mathematics. There is truth to the adage that mathematics is a language and just as there is a difference between talking about Italian and talking Italian, so the vocabulary to talk mathematics becomes part of the classroom discourse – it's not a list of words that you select from to talk about and describe something else, it's becoming immersed in the mathematics talk. The skill of the teaching of talking mathematics is giving children something mathematically worthwhile to talk about, accepting what children say, and then supporting them in crafting the talk."*

Ofsted has identified reasoning as, currently, the least well-developed of the aims, and has identified that:

- Not all classrooms have a positive ethos that encourages learning from mistakes
- Tasks are not used well enough to develop reasoning
- Talk often focuses on the 'how' rather than the 'why', 'why not', and 'what if' in:
    - Teachers' explanations and questions
    - Pupils' responses [Mathematics mastery primary conference, Ofsted 2015].

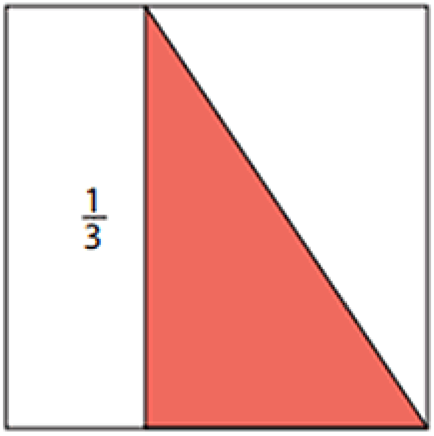
To support both the development and assessment of reasoning, the NCETM's Progression Maps with reasoning contain useful questions and prompts including:

- Odd one out
- True or false?
- Always, sometimes, never
- What comes next?
- Make up an example
- Working backwards
- What do you notice?
- What else do you know?

It is possible for children to get correct solutions to problems without reasoning, but this often involves less efficient and less elegant methods which may not be generalisable. For example, responding to the following question by cutting out and fitting shapes on top of each other could lead a child to identifying that the two triangles are the same size and that they have the same area as the rectangle, but this is not reasoning and does not tell you something about other shapes and shadings:

What fraction of the square is shaded?

Explain your reasoning.



If a child reasons that the shaded area is one third because the whole square is three thirds, meaning the two triangles equal two thirds of the square and the triangles are the same because the diagonal line cuts the rectangle they form in half, then this thinking can be used for other shapes and shadings.

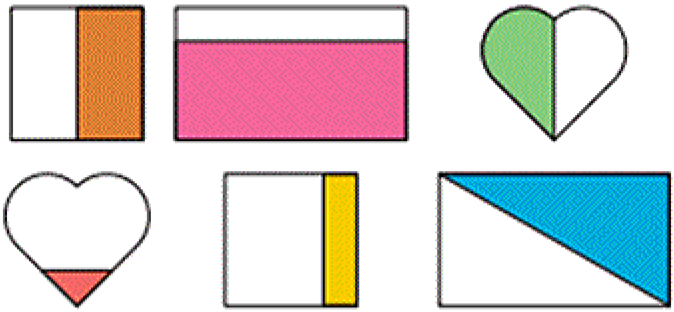
Assessment of reasoning must therefore include assessing **how** children have thought about the mathematics, using what they know to make connections and find solutions, rather than just assessing **if** they have been successful in getting correct answers. This will often necessitate listening to the children, using probing questions to get underneath their thinking and expecting them to demonstrate their understanding using different representations; evidence of reasoning is often not contained within maths books, but comes from interaction with the children. This does not mean children are not able to write about their thinking; as the children move through the primary years, expecting them to capture some of their thinking in writing will deepen their understanding. Good examples of children's writing about their thinking can be found on the NRICH website, which provides opportunities for children to write about their mathematical thinking for others.

The Teaching for Mastery booklets are full of examples of questions which require reasoning including:

- **Using examples and non-examples**

Which of these show half of each whole shape?
Explain your reasoning.

Children should talk about the two parts needing to be equal parts of the whole.

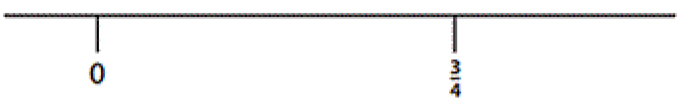


Year 1

With this example, children should be able to explain that splitting a shape into two parts is not enough for the pieces to be halves; the pieces need to be the same size. They should also understand that the same shape can be halved in different ways, with the pieces possibly looking quite different as a result. For example, half of a rectangle could look like a rectangle or a triangle or a trapezium (etc) which would all have the same area.

- **Generating own examples**

Mark another fraction on this line.
And another, and another.



Year 2

Generating your own examples and non-examples of some mathematics requires reasoning and depth of understanding. It is much harder to come up with examples of a particular bit of mathematics than to respond to examples provided by the teacher.

- **Using statements**

Captain Conjecture says,
'If you add together six 0s the answer is 6.'
Do you agree?

Explain your reasoning.

Year 1

Captain Conjecture says, 'If you add 6 to a number ending in 7 you will always get a number ending in 3.' Is he correct?

Explain your answer.

Year 3

Statements are useful for exposing and challenging common misconceptions, as with the example from Y1, and for attending to structure and connections (the example from Y3). In both cases, explaining the thinking requires reasoning.

- **Using different representations**

Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.

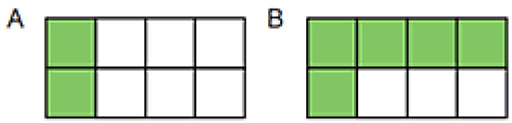
Explain your reasoning.

Year 6

This example provides a non-standard experience of two unit fractions, where the two fractions are different but of equal length. The standard experience for children is starting from a whole and partitioning to find a fraction. Making sense of this situation will reveal whether or not children have over-generalised about fractions.

- **Making sense of other people's thinking**

  Each bar of toffee is the same. On Monday, Sam ate the amount of toffee shown shaded in A. On Tuesday, Sam ate the amount of toffee shown shaded in B.

  Sam says he ate $\frac{7}{8}$ of a bar of toffee.
  Jo says Sam ate $\frac{7}{16}$ of the toffee.
  Explain why Sam and Jo are both correct.

  Year 5

  Here the same situation is viewed differently by two different children; it is important that children experience different ways of seeing the same mathematics. Interpreting the thinking of other people involves connecting their explanations to your own understanding of the mathematics.

Assessing reasoning will, therefore, focus on assessing mathematical thinking and will be underpinned by talking with children, listening to and probing their thinking. Embedding this in assessment systems and practices is one of the challenges for schools at the moment. Next month we will look at assessing problem-solving.

Image Credits

Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room *provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series* *here*

Understanding key mathematical structures. Part three: Does order matter?

In the last two magazines, Maths in the Staff Room focused on on looking at a key structure in mathematics, 'Doing and undoing'. The first part of this session follows the pattern of the first part of the session in *Issue 80**. If this has been used with staff, then adapt the introduction to this session and explain that you are now exploring a second structure*

Meeting aims

- To consider the importance of structure in mathematics
- To explore understanding of a key mathematical structure, which has relevance across the curriculum
- To make explicit opportunities to embed the aims of the National Curriculum.

Timings

- Ten minutes initial input
- Ten minutes, thirty minutes, sixty minutes or ninety minutes follow up after two weeks.

Resources

- Asking Mathematical Questions Mathematically by John Mason
- Cards with × 0, + 5, ÷ 2, × 1,and blank cards (or sticky notes)
- Large sheet of paper for display in the staffroom with 'Does order matter?' in the middle.

Ten minute introduction

1. Explain that understanding structure is an important part of mathematics, and that asking children to attend to structure is part of the role of the teacher in mathematics lessons. Say that John Mason explains this as the difference between 'working-through' some maths and 'working-on' some maths, and share the following extract from his paper:

> ***working through exercises and working on exercises***
>
> *The first describes the student who does a few questions, takes a break, does a bit more on the bus, copies a bit from a friend, and ends up with no overall sense of the exercises as examples of anything or what they are about. Contrast this with the student who in doing the exercises asks themself what is similar about the questions and what different, what it is about the context which enables the technique to work, what sorts of difficulties might the technique encounter in different situations, etc. That student is working-on the exercises.*
>
> *The two states of working-through and working-on are completely different, and in particular they involve different energies. Working-through minimises effort through minimum involvement. It is unreflective and unmathematical. Working-on minimises effort mathematically, by trying to locate underlying structure and so reduce memory demands.*

Explain that this notion of 'working on' rather than 'working through' fits with the intention of variation: "where the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity". (Gu 2004)[1]

2. Explain that you are going to explore an example, Does order matter? Understanding when order does matter and when it does not matter in mathematics requires a deep understanding of mathematical relationships and how the mathematics works. 'Does order matter?' includes things such as the relationship between the addition and subtraction, function machines, and order of operations.

   Use a function machine as the context. Explain that you have a machine that can process two steps. Show the pair of cards ÷ 2 and × 1. Say that you put a number into the machine and then you can decide the order of the steps. Demonstrate putting in 6 and working out 6 × 1 ÷ 2 and 6 ÷ 2 × 1. Ask everyone to try putting other numbers into the function machine with the same two steps and to explain what happens when they change the order of the steps, and why. Ask people to use an image or to draw something to show what is happening.

3. Introduce two new function cards × 0 and +5, and ask everyone to explore using these two steps in a different order in the function machine and to explain what they notice and why this happens. Ask them to use an image or to draw something to show what is happening.

4. On a large sheet of paper with 'Does order matter? in the centre, split the sheet into Yes and No. Write 6 × 1 ÷ 2 and 6 ÷ 2 × 1 on the 'doesn't matter' side and and 6 × 0 + 5 and 6 + 5 × 0 on the 'does matter' side as the first idea on the sheet.

5. Ask everyone to make up another example of each; two steps for the function machine where the order doesn't matter and two steps where the order does matter. Add these to the sheet.

6. Use the different examples to see if it is possible to make any general statements about when, in calculation, order does matter. Capture general statements on the sheet.

7. Invite everyone to think about this theme across the maths curriculum over the next two weeks, and whenever they think of an example of where order does or does not matter in maths, to add it to the sheet which you will come back to during a future meeting.

Follow-up meeting two weeks later (you may need to prompt people to add to the sheet and model this by adding ideas during the two weeks). Have the large sheet which has ideas connected to 'Does order matter?'.

- Look at the mathematical ideas. Ask: *Can you think of any other examples of where order does or does not matter in other areas of mathematics?* Ideas might include:
    - Different combinations of operations, for example:
        - multiplying and dividing
        - adding and subtracting
        - multiplying and adding
        - dividing and subtracting
        - percentage increases and decreases.

- Different combinations of transformations of shapes, for example
    - rotating and reflecting
    - translating and reflecting
    - enlarging and translating.

- Converting measures, for example
    - cm to m to mm.

- Choose one idea relevant to your children and discuss how to support understanding.
  Consider:
    - What contexts make sense of the relationship?
    - How could it be modelled with different resources/pictures/drawings?
    - How could it be recorded symbolically?
    - What would you want the children to notice and be able to explain?
    - What sort of questions would show that the children have understood?

    Asking children to do things physically and modelling with resources and drawings will prompt different explanations and support the children in focusingon what has changed and what has stayed the same.

- Ask: *Can you make any general statements about when order does or doesn't matter?*
  Add the general statements to the sheet.

[1] Gu, L., Huang, R., & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In Lianghuo, F., Ngai-Ying, W., Jinfa, C., & Shiqi, L. (Eds.) How Chinese learn mathematics: Perspectives from insiders. Singapore: World Scientific Publishing Co. Pte. Ltd. page 315. 6

Seen and Heard

Seen and Heard *will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

(Thanks to Hilary Southwell for this *Seen and Heard* example: Hilary is a Maths SLE working in a large primary school in Nottinghamshire. She currently teaches Year 4)

During a lesson on place value in Y4, the children were exploring base ten equipment. One child looked at the large cube and suggested it had the value of 600.

- What would you think if a child said this?
- What do they understand about the structure of the equipment?
- How do they think this connects with the structure of the number system?
- How would you use the equipment to explore the relationship between both the different pieces and the parts of the number system?
- What other maths resources are structured in a way that the children might misinterpret?
- How is it best to introduce these resources if they are to support development of understanding?

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.

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