Primary
& Early Years
Magazine 84

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for over 80 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the Primary Magazine Archive.

Contents

This month we have the first of a series of articles, which looks at developing and demonstrating understanding of mathematics in the National Curriculum; this month the focus is on describing and explaining.

Maths in the Staff Room suggests ways in which collective teacher discussions - both formal and informal - can form part of the ongoing process of professional learning, and help increase the effectiveness of maths teaching across the school. This month's article looks at how teaching for misconceptions is part of teaching for understanding.

Seen and Heard provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. This month children in a mixed-age Year 4/5/6 group respond to an elicitation task on fractions, prompting us to think about what children understand about the structure of fractions, and how fractions fit into the number system.

If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please email it to us. If we publish your suggestion, we'll put a £20 voucher in the post.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

Information and guidance relating to this summer's National Curriculum tests (SATs) continues to flow from the Standards and Testing Agency (STA), an arm of the DfE:

- at the end of January the STA published what's called teacher assessment exemplification for both KS1 and KS2. This contains samples of pupils' work matched to the standards (sentences explaining what pupils should be able to do)
- in addition to this, the STA has been emailing a number of 'assessment updates' to schools, covering matters such as what equipment Year 2 pupils can and can't use in the tests, and informing schools that, on test papers this year, numbers of a thousand or more will contain a comma after the digit in the thousands position (for example 4,642) rather than a space, which has been practice so far. You can find detail on all this by following the assessment updates link on this page.

If you're interested in a detailed rationale for, and explanation of, teaching for mastery you may be interested in a new video just posted in the mastery section of the NCETM website. It is a recording of a keynote address given by the NCETM's Director for Primary, Debbie Morgan, to a teachers' conference in Lincolnshire in December. The video lasts about an hour, but it is also broken down into shorter sections, covering depth of understanding, using the bar model, and variation theory, among other aspects.

The NCETM, in conjunction with the Maths Hubs programme, has begun work with researchers at Oxford University on a project funded by the Education Endowment Foundation (EEF) centred on developing mathematical reasoning in Year 2 pupils. Eight Maths Hubs will be recruiting schools to take part in the project next school year. Details of how primary schools can apply to participate will be published on the NCETM website before the end of this term.

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National Curriculum in Focus

National Curriculum in Focus *is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series here*

Developing and demonstrating understanding: Part One - Describing and explaining

This is the first of a series of articles focused on how teaching can support the development of understanding and how pupils can demonstrate understanding of the mathematics within the National Curriculum

In the NCETM Teaching for Mastery booklets it states that:

> *"Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills."*

One of the phrases within this statement, with which many teachers are currently grappling, is 'a deep understanding', provoking the questions: 'What does it mean to have a deep understanding of the mathematics?' and 'How do I know if a child has a deep understanding of the mathematics?' If teachers are to focus on supporting children to develop a deep understanding, then finding answers to these questions is crucial.

The Teaching for Mastery booklets provide some support with this, with the inclusion of the following section in the introduction:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)[1];
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;
- make use of it in various ways, including in new situations[2].

Here we will focus on the first and third of these, 'describe it in his or her own words' and 'explain it to someone else'. These highlight the importance of talk in learning mathematics and in demonstrating understanding of mathematics.

For description to be useful it needs to go beyond simple recount. Ofsted has identified that often this is not the case in classrooms:

- *Talk often focuses on the 'how' rather than the 'why', 'why not', and 'what if' in:*
  - *teachers' explanations and questions*
  - *pupils' responses.*[3]

When it is done well, describing mathematical thinking involves:

- making sense of what you have done
- situating the current mathematics in relation to existing understanding (linking it to what you already know)
- making clear mathematical structures which have been identified
- using mathematical and contextual language to both represent the mathematics and communicate thinking.

One way to support children with describing their thinking in this way is to use simple sentence starters which give an indication of what it is important to talk about, for example:

- I noticed that…
- I think that…
- I wonder if…

'Describe it in his or her words' and 'Explain it to someone else' might sound like the same idea and they are connected, but implicit in the second statement is the idea that the person listening will be able to understand what has been explained and are then able to explain it themselves. This is one way of providing challenge for children who seem to have grasped a mathematical idea which other children are still struggling to make sense of; explaining the mathematics to one of the children who have not yet understood, so that they understand.

Teachers, and parents, sometimes worry that in this situation the children doing the explaining are no longer working on their own mathematics. However, explaining to someone else so that they understand puts a greater demand on the person explaining in terms of their own understanding and as a result it will deepen the understanding of the person explaining. In her book 'What's math got to do with it?' Jo Boaler explains this as follows:

> *"When parents ask me why their children should 'waste' time in class explain in their words when they know the answers, I tell them that they need to explain their work because it is the most mathematical of acts… Whenever students offer a solution to a math problem, they should know whether the solution is appropriate, and they should draw from mathematical rules and principles when they justify the solution rather than just saying that a text-book or a teacher told them it was right. Reasoning and justifying are both critical acts, and it is very difficult to engage with them without talking."*[4]

In order to demonstrate understanding or convince someone else about the mathematics, a child's description or explanation will often need to be accompanied by pictures, images, symbols and contexts. In next month's article we will explore how to support the development of these different forms of representation and how these have a part to play in children demonstrating understanding.

[1] The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner's conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since the early 1980s. See here for an introduction to this approach
[2] Adapted from a list in 'How Children Fail', John Holt, 1964
[3] Slide 26 from presentation given Jane Jones HMI National Lead for Mathematics 2015
[4] Boaler J 2015 What's math got to do with it? Penguin: New York.

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Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room *provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series here*

Learning from Misconceptions

Meeting aims

- To consider the importance of exposing misconceptions in teaching for understanding
- To explore possible misconceptions a particular question can provoke
- To plan questions to expose misconceptions.

Timings

- Thirty minutes initial input
- Thirty or sixty minutes minutes follow-up after three weeks.

Resources

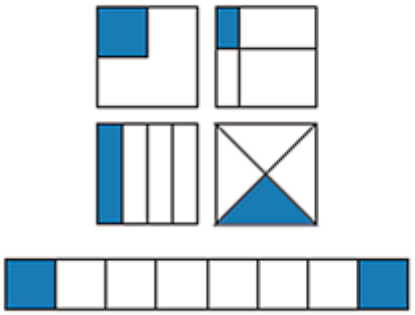
- Copy of the Y2 question below from the Teaching for Mastery Y2 booklet
- Teaching for Mastery booklets for each year group
- Teachers will need to bring planning for their current teaching sequence.

Ten minute introduction

1. Explain that one of the key aspects of teaching for understanding is planning and teaching for misconceptions. This means identifying possible misconceptions and putting children in situations where these misconceptions are likely to arise, so that lessons and teaching sequences focus on the children developing conceptual understanding rather than just 'doing' some mathematics

2. Say that many of the questions in the Teaching for Mastery booklets have been written to expose misconceptions. Show everyone the following example from the Y2 booklet:

Which of these diagrams have $\frac{1}{4}$ of the whole shaded?



Explain your reasoning.

3. Ask everyone to discuss the possible misconceptions this question could expose and then come back together and share the misconceptions identified. These might include:

   - The top left picture is not a quarter because there are only two parts
   - The top right is a quarter because there are four parts
   - Neither of the two in the second row are quarters because they are not divided into four squares
   - The bottom one is not a quarter because there are eight parts not four
   - The bottom one is not a quarter because the two shaded parts are not next to each other.

4. Ask everyone to think about the mathematical focus of their current teaching sequence and to discuss common misconceptions associated with the mathematics still to be

   taught. Ask them to identify a question which could be used to expose misconceptions; this question might come from the relevant Teaching for Mastery booklet, other texts/sources or might be created by the teachers.

5. Share the questions identified and revise in response to suggestions from other teachers.

6. Ask everyone to use their question as part of their teaching sequence and to observe which misconceptions arise, how other children respond to these misconceptions and how the children were supported to make sense of the mathematics. Suggest that people collect children's work, and photographs to share at the next meeting.

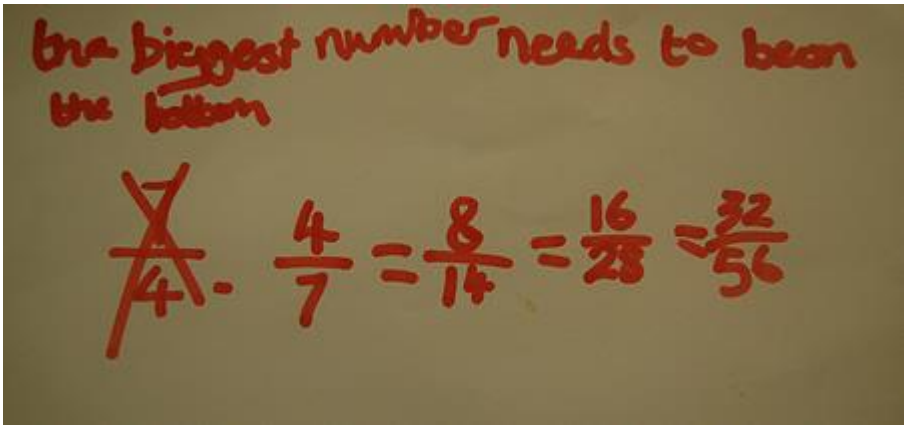
Follow-up meeting three weeks later to share what happened when using the questions. Focus on the following:

- Which questions were really effective at exposing misconceptions? Why was this?
- How were children able to explain and show why someone else's thinking was not correct?
- Did any other misconceptions arise during the teaching sequence for which you had not planned? How might you plan for these in the future?
- What misconceptions do you need to plan for in your next teaching sequence?

Seen and Heard

Seen and Heard *will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

A teacher was using an elicitation task with a group of Y4, Y5 and Y6 children, at the start of a teaching sequence on fractions. The task started with the question 'What do you know about fractions?'. Responding to this question, one child, Joel *, wrote:



Later Joel said that $^{3}/_{7}$ can be simplified because 'you can divide both three and seven by one'. Another child, Kara, responded to this saying: 'I don't agree, you can't simplify $^{3}/_{7}$ because they are odd numbers.'

- How can Joel be supported to understand that fractions can be bigger than one? Which images would work best? What contexts might help him to make sense of this?
- What does Joel understand about equivalent fractions? Do you think he would recognise that $^{12}/_{21}$ Is equivalent to the other fractions he has written? If not, what images and contexts would help to support this understanding?
- What do Joel and Kara understand about simplifying fractions? Which fractions would be interesting for them to explore in order to gain a full understanding of simplifying?
- What does the teacher need to plan for in her teaching sequence?

** both children's names have been changed.*

With thanks to Angela Sullivan from Willowbank Primary School, Cullompton, Devon, for sending us this example.

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.