NCETM
NATIONAL CENTRE FOR EXCELLENCE
IN THE TEACHING OF MATHEMATICS

Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 6

How confident are you that you understand and can support children to identify the principles of quotitive and partitive structures in division?
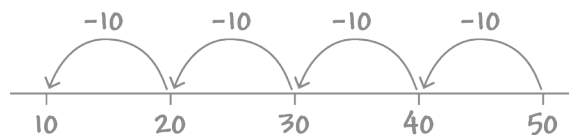
1 ☐   2 ☐   3 ☐   4 ☐

How would you respond …?

a. **Look at the diagram. Jake wrote this in his book. Is he correct? Explain your answer, linking your explanation to the structure of quotitive division.**

50 is divided into groups of 10.

There are 4 groups.

b. **Sort these word problems into quotitive or partitive division.**

- There are 35 oranges. I put 5 oranges in each bag. How many bags will I need?
- Fourteen children are put into two equal teams. How many children are there in each team?
- There are 14 shoes. The shoes are put into pairs. How many pairs of shoes are there?
- Two children can fit into each carriage of a fairground ride. How many carriages are needed for 20 people?
- Ten people can fit in a minibus. How many minibuses are needed for 50 people?
- There are 5 monkeys, 35 bananas are shared equally between them. How many bananas does each monkey get?
- If 70 tins of beans are shared equally between 10 boxes, how many tins will there be in each box?

Responses

Note your responses to the questions here before you engage with the rest of this section:

a. Jake is not correct as he has not continued his number line to 0 and therefore has not found the right number of groups. There should be five groups.

b.

Quotitive division
- There are 35 oranges. I put 5 oranges in each bag. How many bags will I need?
- There are 14 shoes. The shoes are put into pairs. How many pairs of shoes are there?
- Two children can fit into each carriage of a fairground ride. How many carriages are needed for 20 people?
- Ten people can fit in a minibus. How many minibuses are needed for 50 people?

Partitive division
- Fourteen children are put into two equal teams. How many children are there in each team?
- There are 5 monkeys, 35 bananas are shared equally between them. How many bananas does each monkey get?
- If 70 tins of beans are shared equally between 10 boxes, how many tins will there be in each box?

Structures of division

This section will explore two structures of division: quotitive and partitive. Quotitive division is sometimes referred to as 'division as grouping', while partitive division is referred to as 'division as sharing'. To support understanding of these structures, the use of language will be discussed.

Quotitive division

Quotitive division problems are those where the total quantity and the group size are both known, while the number of groups is unknown.

There are fifteen biscuits. If I put them into bags of five, how many bags will I need?

It is important that the forming of the groups is made clear, counting out a group of five and articulating that '*one bag of five is five, two bags of five is ten, three bags of five is fifteen.*'

Fifteen is divided into groups of five. There are three groups so there are three groups of five in fifteen.

When children are confident with grouping, they should then consider how the problem could be written as an equation.

- '*We can represent this as fifteen divided into groups of five: 15 ÷ 5.*'
- '*The "15" represents the total number of biscuits.*' '*The "5" represents the number of biscuits in each group/bag.*'
- '*So 15 ÷ 5 = 3.*'
- '*Fifteen divided into groups of five is equal to three: 15 ÷ 5 = 3. So, we need three bags.*'
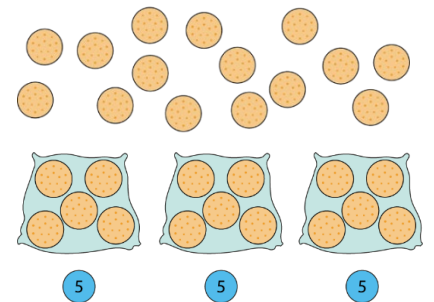
Quotitive division can be linked to skip counting, using a number line to show the repeated addition and subtraction respectively. By skip counting using the group size, the number of groups can be found. For example, '*I have eight balloons. I give two balloons to each child. How many children get balloons?*' Children know to skip count using the divisor (the number of balloons each child gets).

$2 + 2 + 2 + 2 = 8$

4 twos

$8 - 2 - 2 - 2 - 2 = 0$
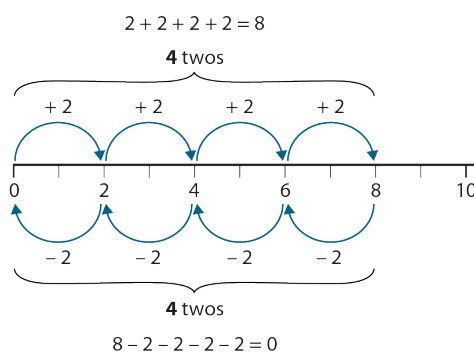
As children become fluent with multiplication facts, they will be able to apply these.

Children should be introduced to the language of division. Avoid generalising that the divisor is the group size and the quotient is the number of groups, as this is only the case in quotitive division.

$$8 \div 2 = 4$$
$$\text{dividend} \div \text{divisor} = \text{quotient}$$

It is worth highlighting that in quotitive division the context of the dividend is different to the context of the quotient. For example, *'There are thirty-five oranges (dividend). I put five oranges in each bag. How many bags (quotient) will I need?'*

Partitive division

In partitive division, the total quantity is partitioned/divided into an unknown number of equal shares as indicated by the divisor.

Children may have some everyday experience of sharing. For example, sharing items equally between friends, using the 'one for you, one for me' approach until there are none left.
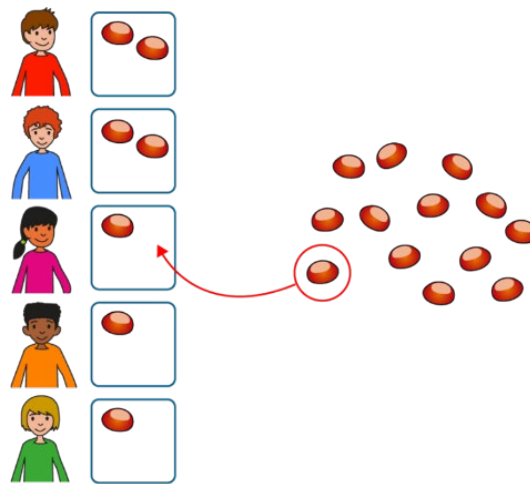
In partitive division, the language of grouping is avoided and there is a focus on the language of sharing or dividing between. For example, if there are 20 conkers shared between 5 children, we have 20 divided between 5. Each child gets an equal share; the size of the equal shares is four.

Provide opportunities for children to use the one-by-one sharing process when dividing between groups and discuss the effectiveness of this strategy.

When introducing the structure, keep the divisor to numbers they are confident skip counting in, such as 2, 5 or 10, so they can focus on the structure and not be distracted by the numbers.

You can build understanding through the dividend being divided into five equal shares. For example:

- *'If there were 5 children and 5 conkers, how many conkers would each child have if they were shared equally?'*
- *'If there were 10 conkers, how many would each child get?'*
- *'If there were 15 conkers how many would each child get?'*

Ensure that children can see that it is more efficient to skip count than share one at a time. They can then relate this understanding to the question.

- *'One five is one each. That's five.'*
- *'Two fives is two each. That's ten.'*
- *'Three fives is three each. That's fifteen. '*
- *'Four fives is four each. That's twenty.'*
- *20 ÷ 5 = 4. 'Twenty divided between five is equal to four each.'*
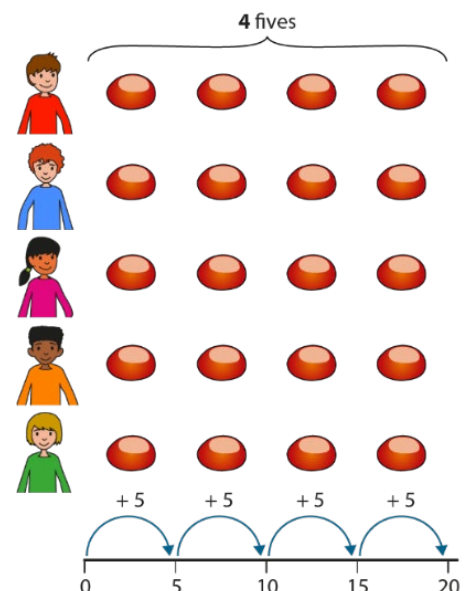
Common errors in this area may include:

- children confusing the language of quotitive and partitive division; when developing practice questions, ensure the question exemplifies the appropriate division structure.

What to look for

Can a child:

- use skip counting to solve division problems?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:
- Topic 2.6: Structures: quotitive and partitive division

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: