

Mastery Professional Development

Multiplication and Division



2.29 Decimal place-value knowledge, multiplication and division

Teacher guide | Year 6

Teaching point 1:

To multiply a number by 10/100/1,000, move the digits one/two/three places to the left; to divide a number by 10/100/1,000, move the digits one/two/three places to the right.

Teaching point 2:

Measures can be converted from one unit to another using knowledge of multiplication and division by 10/100/1,000.

Overview of learning

In this segment children will:

- review their knowledge of multiplying whole-number multiplicands by 10 or 100, and extend to multiplication by 1,000, generalising about moving the digits of the multiplicand one/two/three places to the left, respectively
- review their knowledge of:
 - dividing multiples of 10 by 10
 - dividing multiples of 100 by 100
 and extend to division of multiples of 1,000 by 1,000, generalising about moving the digits of the dividend one/two/three places to the right, respectively
- review the equivalence of:
 - dividing by 10 and multiplying by 0.1
 - dividing by 100 and multiplying by 0.01
 and extend to an understanding of the equivalence of dividing by 1,000 and multiplying by 0.001
- multiply and divide by 10, 100 and 1,000 for calculations involving decimal numbers with up to three decimal places (crossing the '1' boundary), e.g.:
 - $2 \div 100 = 2 \times 0.01 = 0.02$
 - $0.02 \times 1,000 = 20$
 - $25 \div 10 = 25 \times 0.1 = 2.5$
 - $0.092 \times 1,000 = 92$
- convert between metric units of measure.

Teaching point 1 explores strategies for multiplying and dividing by 10, 100 and 1,000. Place-value charts and the Gattegno chart are used to draw attention to the difference in value, and the position of the digits, before and after multiplication/division. The focus is on developing efficient calculation strategies by bringing together and extending learning from the following segments:

- *Spine 1: Number, Addition and Subtraction*, segments 1.23 and 1.24
- segment 2.13 *Calculation: multiplying and dividing by 10 or 100*
- segment 2.19 *Calculation: \times/\div decimal fractions by whole numbers.*

In *Teaching point 2*, these calculation strategies are applied in the context of converting between metric units of measure, including length, mass and capacity.

As discussed in segment 2.19, *Overview of learning*, when, for example, a multiplicand is multiplied by ten (e.g. $1.5 \times 10 = 15$), we can interpret the effect on the multiplicand in two possible ways. We can say that the digits have moved one place to the left, or we can say that the decimal point has moved one place to the right. These actions are equivalent. Throughout *Spine 2*, when multiplying or dividing by a power of ten, we refer to movement of the digits, not to movement of the decimal point.

2.29 Extending decimal place value to \times/\div

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

To multiply a number by 10/100/1,000, move the digits one/two/three places to the left; to divide a number by 10/100/1,000, move the digits one/two/three places to the right.

Steps in learning

1:1 Children have already applied their understanding of place value to multiply whole numbers by 10 or by 100, and to divide multiples of 10/100 by 10/100 (segment 2.13 *Calculation: multiplying and dividing by 10 or 100*). They have also learnt the equivalence of:

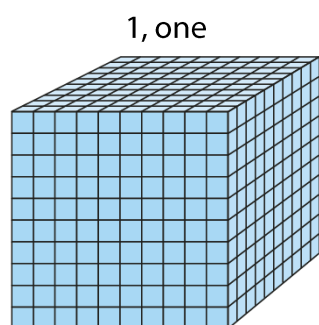
- multiplying by 0.1 and dividing by 10
- multiplying by 0.01 and dividing by 100

(segment 2.19 *Calculation: \times/\div decimal fractions by whole numbers*).

In this teaching point children will draw on this knowledge, alongside their understanding of tenths, hundredths and thousandths (from *Spine 1: Number, Addition and Subtraction*, segments 1.23 and 1.24) to multiply and divide *any* number by 10, 100 or 1,000, including 'bridging 1' (e.g. $30 \div 1,000 = 0.03$).

Begin by briefly reviewing children's understanding of tenths, hundredths and thousandths, including how they are represented on a place-value chart, and how they are related to one another (for more guidance, see *Spine 1*, segments 1.23 and 1.24). Also recap the idea of 'movement is magnitude' on the place-value chart (see *Spine 1*, segment 1.24, step 2:1), and review the relationships between the different powers of ten using the Gattegno chart.

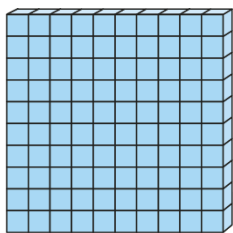
Write division equations, as shown below, connecting each quantity (0.1, 0.01, 0.001) to the whole (1). Then use children's understanding that dividing by 10/100 is equivalent to multiplying by 0.1/0.01 to write the equivalent multiplication equations, extending this understanding to multiplying by 0.001/dividing by 1,000.



1,000s	100s	10s	1s	0.1s	0.01s	0.001s
			1			

- 'One is the whole.'

0.1, one tenth



1,000s	100s	10s	1s	0.1s	0.01s	0.001s
				1		

- 'When a whole is divided into ten equal parts, each part is one tenth of the whole.'
- 'There are ten tenths in one whole.'

$$1 \div 10 = 0.1$$

$$1 \times 0.1 = 0.1$$

0.01, one hundredth



1,000s	100s	10s	1s	0.1s	0.01s	0.001s
					1	

- 'When a whole is divided into one hundred equal parts, each part is one hundredth of the whole.'
- 'There are one hundred hundredths in one whole.'

$$1 \div 100 = 0.01$$

$$1 \times 0.01 = 0.01$$

- 'When one tenth of the whole is divided into ten equal parts, each part is one hundredth of the whole.'
- 'There are ten hundredths in one tenth.'

$$0.1 \div 10 = 0.01$$

$$0.1 \times 0.1 = 0.01$$

0.001, one thousandth



1,000s	100s	10s	1s	0.1s	0.01s	0.001s
						1

- 'When a whole is divided into one thousand equal parts, each part is one thousandth of the whole.'
- 'There are one thousand thousandths in one whole.'

$$1 \div 1,000 = 0.001$$

$$1 \times 0.001 = 0.001$$

- 'When one hundredth of the whole is divided into ten equal parts, each part is one thousandth of the whole.'
 - 'There are ten thousandths in one hundredth.'
- $$0.01 \div 10 = 0.001 \qquad 0.01 \times 0.1 = 0.001$$
- 'When one tenth of the whole is divided into one hundred equal parts, each part is one thousandth of the whole.'
 - 'There are one hundred thousandths in one tenth.'
- $$0.1 \div 100 = 0.001 \qquad 0.1 \times 0.01 = 0.001$$

Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

1:2 Now remind children that when we multiply by 10/100 we move the digits one/two places to the left, and when we divide by 10/100 (or multiply by 0.1/0.01) we move the digits one/two places to the right; you can use whole-number calculations to remind children of this, e.g. 6×100 , $600 \div 100$ and 600×0.01 (for more guidance, see segment 2.13 *Calculation: multiplying and dividing by 10 or 100*).

Then extend this understanding to:

- multiplication of a single-digit number by 1,000
- division of a multiple-of-1,000 by 1,000, and the equivalent multiplication of the single-digit number by 0.001.

Generalise:

- **'When a number is multiplied by one thousand, the digits move three places to the left.'**
- **'When a number is divided by one thousand, the digits move three places to the right.'**
- **'Dividing by one thousand is equivalent to multiplying by one thousandth.'**
- **'When a number is multiplied by 0.001/one thousandth, the digits move three places to the right.'**

You can also review the calculations on the Gattegno chart.

Multiplying by 1,000 – place-value chart:

1,000s	100s	10s	1s	0.1s	0.01s	0.001s
			6			
6	0	0	0			

$\downarrow \times 1,000$

$$6 \times 1,000 = 6,000$$

- 'What is the value of the "6" in six?'

- 'six'

6

- 'What is the value of the "6" in six thousand?'

- 'six thousand'

6,000

'We had six ones. We now have six thousands.'

Dividing by 1,000 / multiplying by 0.001 – place-value chart:

1,000s	100s	10s	1s	0.1s	0.01s	0.001s
6	0	0	0			
			6			

$\div 1,000 \downarrow$

$\downarrow \times 0.001$

$$6,000 \div 1,000 = 6$$

$$6,000 \times 0.001 = 6$$

- 'What is the value of the "6" in six thousand?'

- 'six thousand'

6,000

- 'What is the value of the "6" in six?'

- 'six'

6

'We had six thousands. We now have six ones.'

2.29 Extending decimal place value to \times/\div

Multiplying and dividing by 1,000 – Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

1:3

Now explore multiplication and division by 10/100/1,000 for calculations that ‘bridge 1’.
Begin with:

- division of a single-digit number by 10/100/1,000 and the equivalent multiplication of the single-digit number by 0.1/0.01/0.001; i.e.:

- $1 \div 10 = 1 \times 0.1 = 0.1$
 $2 \div 10 = 2 \times 0.1 = 0.2$
 \vdots

- $1 \div 100 = 1 \times 0.01 = 0.01$
 $2 \div 100 = 2 \times 0.01 = 0.02$
 \vdots

- $1 \div 1,000 = 1 \times 0.001 = 0.001$
 $2 \div 1,000 = 2 \times 0.001 = 0.002$
 \vdots

(as indicated, in each case it is useful to start with a dividend/multiplicand of ‘1’ before moving to division/multiplication of other single-digit numbers)

- multiplication of a whole number of tenths/hundredths/thousandths (from one to nine tenths/hundredths/thousandths) by 10/100/1,000; i.e.:

- 0.001×10
 0.002×10
 \vdots

- 0.01×10
 0.02×10
 \vdots

- 0.1×10
 0.2×10
 \vdots

- 0.001×100
 0.002×100
 \vdots

- 0.01×100
 0.02×100
 \vdots

- 0.1×100
 0.2×100
 \vdots

- $0.001 \times 1,000$
 $0.002 \times 1,000$
 \vdots

- $0.01 \times 1,000$
 $0.02 \times 1,000$
 \vdots

- $0.1 \times 1,000$
 $0.2 \times 1,000$
 \vdots

(as indicated, in each case it is useful to start with a multiplicand of one tenth/hundredth/ thousandth before moving to other whole numbers of tenths/hundredths/thousandths).

Keep using the generalisation from the previous step, and the equivalent generalisations for multiplying and dividing by 10 and 100:

- **'When a number is multiplied by ten, the digits move one place to the left.'**
- **'When a number is divided by ten, the digits move one place to the right.'**
- **'Dividing by ten is equivalent to multiplying by one tenth.'**
- **'When a number is multiplied by 0.1/one tenth, the digits move one place to the right.'**

- **'When a number is multiplied by one hundred, the digits move two places to the left.'**
- **'When a number is divided by one hundred, the digits move two places to the right.'**
- **'Dividing by one hundred is equivalent to multiplying by one hundredth.'**
- **'When a number is multiplied by 0.01/one hundredth, the digits move two places to the right.'**

Initially use place-value charts, so children can clearly see how the digits are moving, but progress to working without this support. If any children struggle with left and right, encourage them to focus on the position of the digits; ask *'Are the digits moving so that the number becomes larger or smaller?'* Draw attention to the placement of the digits relative to the decimal point, and the value of the digits, before and after each calculation has been carried out. Ensure children understand when they need to include zeros as place-value holders.

Continue to review the calculations on the Gattegno chart.

Example 1 – division of a single-digit number by 100 / multiplication by 0.01:

'We are dividing by one hundred / multiplying by 0.01, so we need to move the digits two places to the right.'

	1,000s	100s	10s	1s	0.1s	0.01s	0.001s	
$\div 100 \downarrow$				8				
				0	0	8		$\downarrow \times 0.01$

8	\div	100	=	0.08
8	\times	0.01	=	0.08
<ul style="list-style-type: none"> • 'What is the value of the "8" in eight?' • 'eight' <p>8</p>				<ul style="list-style-type: none"> • 'What is the value of the "8" in zero-point-zero-eight?' • 'eight hundredths/ zero-point-zero-eight' <p>8</p>
<i>'We had eight <u>ones</u>. We now have eight <u>hundredths</u>.'</i>				

2.29 Extending decimal place value to \times/\div

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Example 2 – multiplication of a whole number of hundredths by 1,000:

'We are multiplying by one thousand, so we need to move the digits three places to the left.'

1,000s	100s	10s	1s	0.1s	0.01s	0.001s
					5	
		5	0	0	0	

$\downarrow \times 1,000$

$$0.05 \quad \times \quad 1,000 \quad = \quad 50$$

- 'What is the value of the "5" in zero-point-zero-five?'
- 'five hundredths/ zero-point-zero-five'
- 0.05
- 'What is the value of the "5" in fifty?'
- 'five tens/fifty'
- 50

'We had five hundredths. We now have five tens.'

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
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0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

2.29 Extending decimal place value to \times/\div

1:4	<p>Spend some time writing equations that reinforce the links between multiplication, division and fractions, e.g. $2 \div 10 = 2 \times \frac{1}{10} = \frac{2}{10} = 2 \times 0.1 = 0.2$</p>																																							
1:5	<p>Now progress to similar calculations where the 'starting number' (multiplicand/dividend) has two or more non-zero digits; for example:</p> <ul style="list-style-type: none"> • $25 \div 10$ and 25×0.1 • $25 \div 100$ and 25×0.01 • $25 \div 1,000$ and 25×0.001 <ul style="list-style-type: none"> • 0.37×10 • 0.37×100 • $0.37 \times 1,000$ <p>As before, initially use place-value charts, so children can clearly see how the digits are moving, but progress to working without this support. Continue to review the calculations on the Gattegno chart.</p> <p>Example 1 – two-digit number divided by 100/multiplied by 0.01: <i>'We are dividing by one hundred / multiplying by 0.01, so we need to move the digits <u>two places</u> to the <u>right</u>.'</i></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>1,000s</th> <th>100s</th> <th>10s</th> <th>1s</th> <th>0.1s</th> <th>0.01s</th> <th>0.001s</th> <th></th> </tr> </thead> <tbody> <tr> <td style="text-align: right;">$\div 100 \downarrow$</td> <td></td> <td></td> <td>2</td> <td>5</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>0</td> <td>2</td> <td>5</td> <td></td> <td style="text-align: left;">$\downarrow \times 0.01$</td> </tr> </tbody> </table> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>25</td> <td>\div</td> <td>100</td> <td>=</td> <td>0.25</td> </tr> <tr> <td>25</td> <td>\times</td> <td>0.01</td> <td>=</td> <td>0.25</td> </tr> </table> <table border="0" style="width: 100%;"> <tr> <td style="vertical-align: top; width: 50%;"> <ul style="list-style-type: none"> • 'What is the value of the "2" in twenty-five?' <ul style="list-style-type: none"> • 'two tens/twenty' 20 • 'What is the value of the "5" in twenty-five?' <ul style="list-style-type: none"> • 'five' 5 </td> <td style="vertical-align: top; width: 50%;"> <ul style="list-style-type: none"> • 'What is the value of the "2" in zero-point-two-five?' <ul style="list-style-type: none"> • 'two tenths/zero-point-two' 0.2 • 'What is the value of the "5" in zero-point-two-five?' <ul style="list-style-type: none"> • 'five hundredths/zero-point-zero-five' 0.05 </td> </tr> </table> <p style="text-align: center;"><i>'We had twenty-five <u>ones</u>. We now have twenty-five <u>hundredths</u>.'</i></p>		1,000s	100s	10s	1s	0.1s	0.01s	0.001s		$\div 100 \downarrow$			2	5									0	2	5		$\downarrow \times 0.01$	25	\div	100	=	0.25	25	\times	0.01	=	0.25	<ul style="list-style-type: none"> • 'What is the value of the "2" in twenty-five?' <ul style="list-style-type: none"> • 'two tens/twenty' 20 • 'What is the value of the "5" in twenty-five?' <ul style="list-style-type: none"> • 'five' 5 	<ul style="list-style-type: none"> • 'What is the value of the "2" in zero-point-two-five?' <ul style="list-style-type: none"> • 'two tenths/zero-point-two' 0.2 • 'What is the value of the "5" in zero-point-two-five?' <ul style="list-style-type: none"> • 'five hundredths/zero-point-zero-five' 0.05
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2.29 Extending decimal place value to \times/\div

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Example 2 – multiplying a decimal fraction by 1,000:

'We are multiplying by one thousand, so we need to move the digits three places to the left.'

1,000s	100s	10s	1s	0.1s	0.01s	0.001s
			0	3	7	
0	3	7	0			

$\downarrow \times 1,000$

$$0.37 \quad \times \quad 1,000 \quad = \quad 370$$

- 'What is the value of the "3" in zero-point-three-seven?'

- 'three tenths/
zero-point-three'
0.3

- 'What is the value of the "7" in zero-point-three-seven?'

- 'seven hundredths/
zero-point-zero-seven'
0.07

- 'What is the value of the "3" in three hundred and seventy?'

- 'three hundred'
300

- 'What is the value of the "7" in three hundred and seventy?'

- 'seven tens/seventy'
70

'We had thirty-seven hundredths. We now have thirty-seven tens.'

2.29 Extending decimal place value to \times/\div

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

1:6 To complete this teaching point, provide children with practice, including:

- sets of missing-number problems (use intelligent practice to draw out the relationship between different calculations and between the place value of different digits; encourage children to notice patterns and describe relationships)
- pattern-spotting/explaining
- true/false style problems and error-spotting.

Missing-number problems:

'Fill in the missing numbers.'

$$0.04 \div \square = 0.004$$

$$0.4 \div \square = 0.004$$

$$0.04 \times \square = 0.4$$

$$0.04 \times \square = 4$$

$$0.2 \times 1,000 = \square$$

$$0.2 \times 100 = \square$$

$$0.5 \times 1,000 = \square$$

$$0.5 \times 100 = \square$$

$$\square \times 1,000 = 170$$

$$\square \times 100 = 170$$

$$170 \times 0.1 = \square$$

$$240 \times \square = 24$$

$$170 \times 0.01 = \square$$

$$240 \times \square = 2.4$$

$$170 \times 0.001 = \square$$

$$240 \times \square = 0.24$$

True/false-style and error-spotting problems:

• *'Decide whether each of these calculations and their explanations are true or false.'*

- $0.45 \div 10 = .45$
because I have removed a '0' to divide by 10.
- $90 \div 10 = 9$
because I have removed a '0' to divide by 10.
- $0.09 \div 10 = 0.9$
because I have removed a '0' to divide by 10.
- $25.34 \times 10 = 25.340$
because I have placed a '0' at the end to multiply by 10.

2.29 Extending decimal place value to \times/\div

$0.092 \times 1,000 = \square$

$92 \div 1,000 = \square$

$0.92 \times 1,000 = \square$

$920 \div 1,000 = \square$

$9.2 \times 1,000 = \square$

$9,200 \div 1,000 = \square$

$92 \times 1,000 = \square$

$92,000 \div 1,000 = \square$

	$\times 1,000$	$\div 1,000$
46		
60.2		
	562,000	0.562

- Which calculations are incorrect? Why?

$3020 \div 10 = 302$

$302 \div 10 = 30.2$

$30.2 \div 10 = 3.2$

$3.02 \div 10 = 0.302$

$0.32 \div 10 = 0.032$

$405 \times 10 = 4050$

$4.05 \times 10 = 4.050$

$40.5 \times 10 = 405$

$4.05 \times 10 = 40.5$

$0.405 \times 10 = 4.05$

Dòng nào jìn:

$0.02 \times 1,000 = \square$

$0.02 \times 10,000 = \square$

$0.02 \times 100,000 = \square$

2.29 Extending decimal place value to \times/\div

Missing-number problems and pattern-spotting:

'Fill in the missing numbers. For each set of calculations describe what is the same and what is different, and explain the pattern.'

$370 \times 1 = \square$	$\square \div 1 = 370$
$37 \times 10 = \square$	$\square \div 10 = 37$
$3.7 \times 100 = \square$	$\square \div 100 = 3.7$
$0.37 \times 1,000 = \square$	$\square \div 1,000 = 0.37$
$37 \times \square = 37$	$37 \div \square = 37$
$3.7 \times \square = 37$	$37 \div \square = 3.7$
$0.37 \times \square = 37$	$37 \div \square = 0.37$
$0.037 \times \square = 37$	$37 \div \square = 0.037$
$\square \times 1 = 0.37$	$0.37 \div 1 = \square$
$\square \times 10 = 3.7$	$3.7 \div 10 = \square$
$\square \times 100 = 37$	$37 \div 100 = \square$
$\square \times 1,000 = 370$	$370 \div 1,000 = \square$
$\square \times 1 = 0.037$	$0.037 \div 1 = \square$
$\square \times 10 = 0.37$	$0.37 \div 10 = \square$
$\square \times 100 = 3.7$	$3.7 \div 100 = \square$
$\square \times 1,000 = 37$	$37 \div 1,000 = \square$

Teaching point 2:

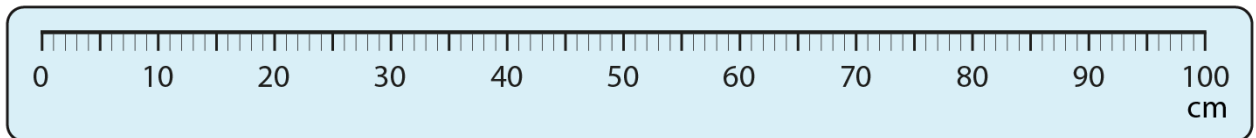
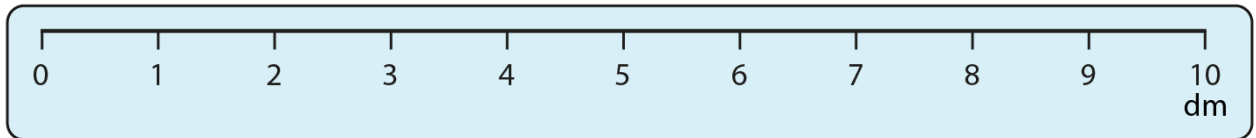
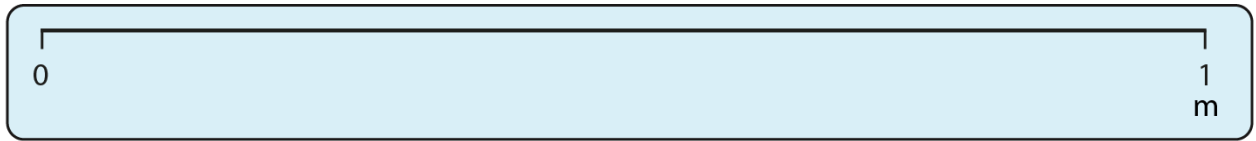
Measures can be converted from one unit to another using knowledge of multiplication and division by 10/100/1,000.

Steps in learning

- 2:1** In segment 2.19 *Calculation: \times/\div decimal fractions by whole numbers*, step 1:13, children briefly applied their understanding of multiplying by decimal fractions to convert units of measure. Now explore conversions between metric units in more detail, using the learning from *Teaching point 1*, above.
- Begin by reviewing units of length, since children will be familiar with using a ruler and making connections between millimetres, centimetres and metres. Using actual rulers, or visualisers, marked in centimetres and millimetres, ask children what is the same and what is different about each scale used. Compare a given length (such as 30 cm / 300 mm). Draw attention to the fact that both quantities are the same length, and ask children to explain why this is the case, using their understanding that 1 cm = 10 mm. Highlight the fact that the length is the same, but the unit that is being used to describe length is different for each scale; there are ten times as many millimetres as there are centimetres, in the given length.
- Provide children with a variety of objects/lines to measure recording the lengths in both centimetres and millimetres. Then provide children with a measuring strip/ruler that is marked only in millimetres; provide more objects/lines and ask children to measure the lengths in millimetres, then convert each to centimetres. Then work the other way round, with children measuring some object/line-lengths with a strip/ruler marked only in centimetres, and converting the measurements to millimetres.
- Now ask children to examine metre sticks, marked in metres, decimetres, centimetres and millimetres. Again, ask them to compare the scales. Children may not have encountered decimetres before, so draw attention to the fact that 1 dm = 10 cm (and that the 'deci' prefix refers to one tenth, so a decimetre is one-tenth of a metre). Compare a given length (e.g. 0.3 m) in the different scales, again asking children to explain the relationship between the different units of measure used to describe the length (e.g. 0.3 m = 3 dm = 30 cm = 300 mm).
- Finally, ask children to convert the object/line lengths they measured earlier into both decimetres and metres; children can use the metre sticks to check their answers. Also provide some objects/lines for them to measure in:
- metres, then ask them to convert each into decimetres, centimetres and millimetres
 - or
 - decimetres, then ask them to convert each into metres, centimetres and millimetres.
- Include some objects/lines that are greater than one metre in length.
- Throughout, draw attention to the fact that a given object/line is the same length, irrespective of the unit the length is expressed in. You can use the example problem below to check understanding. It is useful to work out some reference conversions to display in the classroom, or set children the task of completing a conversion chart like the one shown below.
- Dòng nào jīn:
- 'Katie says a line that measures 120 mm is longer than a line that measures 10 cm because 120 is greater than 10. Can you improve her explanation?'*

Comparing a given length:

- 'What's the same?'
- 'What's different?'



Not to scale.

Summary of measures conversions – mm, cm, dm and m:

$$1 \text{ m} = 10 \text{ dm}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m} = 1,000 \text{ mm}$$

		Convert from			
		mm	cm	dm	m
to	mm		$\times 10$	$\times 100$	$\times 1,000$
	cm	$\div 10$ or $\times 0.1$		$\times 10$	$\times 100$
	dm	$\div 100$ or $\times 0.01$	$\div 10$ or $\times 0.1$		$\times 10$
	m	$\div 1,000$ or $\times 0.001$	$\div 100$ or $\times 0.01$	$\div 10$ or $\times 0.1$	

<p>2:2</p>	<p>Before moving on to different types of measure (mass and capacity), review the relationship between metres and kilometres, and ensure that children are able to convert distances from one to the other.</p> <p>When converting distances, encourage children to explain their reasoning, as exemplified opposite. Continue to encourage children to sense-check their answers; for example, when converting 1.75 km to metres children can reason whether the number will get larger or smaller: <i>'Metres are smaller than kilometres, so there will be more metres in 1.75 km than there are kilometres.'</i></p> <p>It is useful to work out some reference conversions to display in the classroom:</p> <ul style="list-style-type: none"> • 1 km = 1,000 m • 0.1 km = 100 m • 0.01 km = 10 m • 0.001 km = 1 m <p>Also draw children's attention to the movement of the digits, when converting. Look at the ones digit (when it is non-zero) and notice how, when converting 1.75 km into metres, for example, this digit moves to the thousands place:</p> <ul style="list-style-type: none"> • <i>'In "1.75", the '1' represents one <u>one</u>; in "1750" the '1' represents one <u>thousand</u>.'</i> • <i>'We had "1.75" <u>ones</u>; we now have "1.75" <u>thousands</u>.'</i> <p>This is a useful strategy for checking that answers are sensible when converting from one metric unit to another, so encourage children to use this in the subsequent steps.</p>	<p>Measures conversions – m and km:</p> $1 \text{ km} = 1,000 \text{ m}$ $\frac{1}{1,000} \text{ km} = 1 \text{ m}$ $0.001 \text{ km} = 1 \text{ m}$ <p>Converting from kilometres to metres: <i>'Convert 1.75 km into metres.'</i></p> <ul style="list-style-type: none"> • <i>'One kilometre is equal to one thousand metres.'</i> $1 \text{ km} = 1,000 \text{ m}$ <ul style="list-style-type: none"> • <i>'So, to convert one-point-seven-five kilometres into metres, we need to multiply by one thousand.'</i> $\begin{aligned} \text{distance in m} &= \text{distance in km} \times 1,000 \\ &= 1.75 \times 1,000 \end{aligned}$ <ul style="list-style-type: none"> • <i>'When a number is multiplied by one thousand, the digits move three places to the left.'</i> $1.75 \times 1,000 = 1,750 \text{ m}$ <p>Converting from metres to kilometres: <i>'Convert 346 m into kilometres.'</i></p> <ul style="list-style-type: none"> • <i>'One metre is equal to one thousandth of one kilometre.'</i> $1 \text{ m} = \frac{1}{1,000} \text{ km} \qquad 1 \text{ m} = 0.001 \text{ km}$ <ul style="list-style-type: none"> • <i>'So, to convert three hundred and forty-six metres into kilometres, we need to divide by one thousand (or multiply by 0.001).'</i> $\begin{aligned} \text{distance in km} &= \text{distance in m} \div 1,000 \\ &= 346 \div 1,000 \end{aligned}$ <ul style="list-style-type: none"> • <i>'When a number is divided by one thousand, the digits move three places to the right.'</i> $346 \div 1,000 = 0.346$ <p>so</p> $346 \text{ m} = 0.346 \text{ km}$
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<p>2:3</p>	<p>Now, in a similar way, review the relationship between grams and kilograms. You could work practically, by weighing a given item and recording the mass in different units; for example, you could place a 500 g bag of sugar on a balance, and toggle between grams and kilograms, drawing attention to the fact that the mass has remained the same but the unit has changed.</p> <p>As a class work out some reference conversions to display in the classroom:</p> <ul style="list-style-type: none"> • 1 kg = 1,000 g • 0.1 kg = 100 g • 0.01 kg = 10 g • 0.001 kg = 1 g <p>Draw attention to the similarity with the reference conversions for metres and kilometres in the previous step.</p> <p>Then practise converting masses from grams to kilograms and vice versa. As in the previous steps, continue to encourage children to sense check their answers; for example, <i>'Kilograms are larger than grams, so there will be fewer kilograms in 725 g than there are grams.'</i></p>	<p>Measures conversions – g and kg:</p> $1 \text{ kg} = 1,000 \text{ g}$ $\frac{1}{1,000} \text{ kg} = 1 \text{ g}$ $0.001 \text{ kg} = 1 \text{ g}$ <p>Converting from kilograms to grams: <i>'Convert 2.5 kg into grams.'</i></p> $1 \text{ kg} = 1,000 \text{ g}$ $\begin{aligned} \text{mass in g} &= \text{mass in kg} \times 1,000 \\ &= 2.5 \times 1,000 \\ &= 2,500 \end{aligned} \quad \downarrow \text{'Move the digits three places to the left.'}$ <p>So, 2.5 kg = 2,500 g</p> <p>Converting from grams to kilograms: <i>'Convert 725 g into kilograms.'</i></p> $1 \text{ g} = \frac{1}{1,000} \text{ kg} \qquad 1 \text{ g} = 0.001 \text{ kg}$ $\begin{aligned} \text{mass in kg} &= \text{mass in g} \div 1,000 \\ &= 725 \div 1,000 \\ &= 0.725 \end{aligned} \quad \downarrow \text{'Move the digits three places to the right.'}$ <p>So, 725 g = 0.725 kg</p>
<p>2:4</p>	<p>Repeat the process for litres, decilitres, centilitres and millilitres. You could work practically by transferring a given volume of water between different containers, each marked in different units.</p> <p>Children may not have encountered decilitres and centilitres before, so make sure you clearly define these units. Draw attention to the meaning of the prefixes <i>'deci'</i> (one-tenth) and <i>'centi'</i> (one-hundredth), comparing with how these prefixes were used before <i>'metre'</i> in step 2:1.</p>	

2.29 Extending decimal place value to \times/\div

Throughout, draw attention to the fact that a given quantity of liquid has the same capacity, irrespective of the unit it is expressed in; you can use the example problem about measuring cylinders, on the next page, to check understanding.

As in step 2:1, set children the task of completing a conversion chart like the one shown opposite. See if children can notice/explain the fact that this chart is identical to the one in step 2:1, except the 'm' representing metres has been changed to an 'l', representing litres, throughout.

Summary of measures conversions – ml , cl , dl and l :

$$1\ l = 10\ dl$$

$$1\ l = 100\ cl$$

$$1\ l = 1,000\ ml$$

		Convert from			
		ml	cl	dl	l
to	ml		$\times 10$	$\times 100$	$\times 1,000$
	cl	$\div 10$ or $\times 0.1$		$\times 10$	$\times 100$
	dl	$\div 100$ or $\times 0.01$	$\div 10$ or $\times 0.1$		$\times 10$
	l	$\div 1,000$ or $\times 0.001$	$\div 100$ or $\times 0.01$	$\div 10$ or $\times 0.1$	

Converting from litres to millilitres:

'Convert 1.575 l into millilitres.'

$$1\ l = 1,000\ ml$$

$$\begin{aligned} \text{capacity in } ml &= \text{capacity in } l \times 1,000 \\ &= 1.575 \times 1,000 \\ &= 1,575 \end{aligned}$$

↓
'Move the digits three places to the left.'

So, 1.575 l = 1,575 ml

2.29 Extending decimal place value to \times/\div

Converting from millilitres to litres:

'Convert 175 ml into litres.'

$$1 \text{ ml} = \frac{1}{1000} \ell$$

$$1 \text{ ml} = 0.001 \ell$$

$$\text{capacity in } \ell = \text{capacity in ml} \div 1,000$$

$$= 175 \div 1,000$$

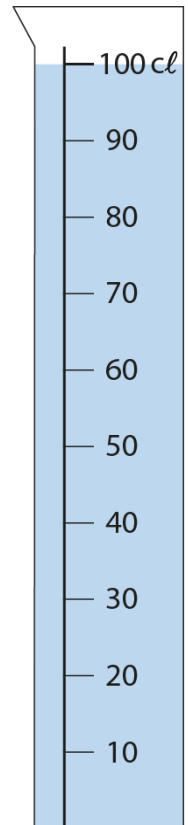
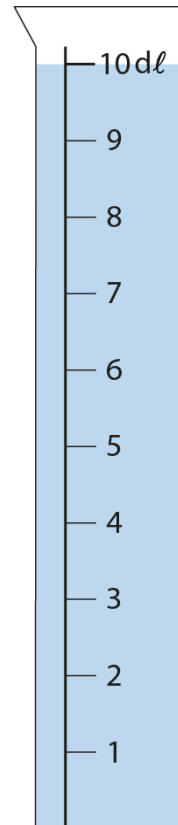
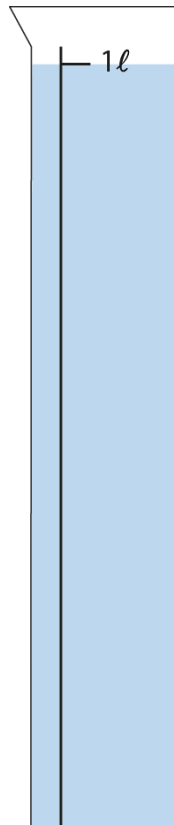
$$= 0.175$$

↓ 'Move the digits three places to the right.'

So, 175 ml = 0.175 ℓ

Comparing a given volume of liquid:

- 'What's the same?'
- 'What's different?'



2:5

To complete this teaching point, provide children with practice, including:

- comparing and ordering measurements given in different units, encouraging children to make sensible choices about which common unit to convert to before comparing
- contextual problems where children need to convert measures to a common unit before calculating, for example:
 - *'I need 10 kg of flour. I already have 3,200 g. How much more flour do I need?'*
 - *'Some children drank 3 l of water between them. Each child drank a 250 ml glass of water. How many children drank?'*
 - *'Sean bought 850 kg of sand to build a wall. He used 75,000 g on Monday and 250,000 g on Tuesday. How much sand was left at the end of Tuesday?'*
 - *'An aeroplane travels 150 m in one second. How many kilometres will it travel in one hour?'*

Comparing and ordering measures in different units:

- *'Which is more, 505 ml or 0.5 l?'*
- *'Year 6 have grown some sunflowers. These are the heights of their plants:'*

Plant	Height
A	286 cm
B	3.40 m
C	3.14 m
D	260 cm

- *'Put the plants in order from shortest to tallest.'*
- *'What is the difference in height between the tallest and the shortest plant?'*
- *'What is the average (mean) height of the sunflowers?'*

Dòng nào jīn:

'A bus company has made a sticker showing the dimensions of their buses.'

HEIGHT 3,475 mm	WIDTH 250 mm
	LENGTH 9,600 mm

'There is a mistake on the sticker. Which dimension do you think is incorrect and why?'