

Mastery Professional Development

Multiplication and Division



2.26 Mean average and equal shares

Teacher guide | Year 6

Teaching point 1:

The mean is the size of each part when a quantity is shared equally.

Teaching point 2:

The mean is defined as the sum of all the numbers in a set of data divided by the number of numbers/values that make up the set of data. If we know the mean of a set of data and the number of numbers/values in that set, we can calculate the total of the set. The mean of a set changes if the total value of the set changes or if the number of numbers/values in the set changes.

Teaching point 3:

The mean can be used to compare data.

Teaching point 4:

The mean is not always an appropriate representation of a set of data.

Overview of learning

In this segment children will:

- be introduced to the concept of the 'mean average'
- learn how to calculate the mean from a set of data
- explore how to use the mean to solve a variety of problems
- use the mean to compare sets of data
- learn when it is/is not appropriate to use the mean to represent a set of data.

This segment begins by focusing on the concept of the 'mean average'. The 'mean' is defined as the sum of all the numbers in a set of data divided by the number of numbers/values that make up the set of data. The concept is abstract in the sense that it represents a set of numbers, and it is therefore important that children are confident with the concept before moving on.

The median and mode averages are not addressed in the primary curriculum. The median is the middle value in a set of data where all the numbers/values are lined up in numeric order, and the mode is the number/value that appears the most often.

The word 'set' is introduced to represent a group of numbers/values. Within the context of the mean average the following terms are distinguished:

- the total value of the set – all the numbers/values in the set added together
- the number of numbers/values in the set.

For example, if there are 32 books shared between four children:

- the total value of the set is 32 – the number of books added together
- the number of numbers/values is four – the four children.

The mean represents the number of books that each child would have if the books were distributed equally. This is calculated by dividing the total number of books by the number of values: $32 \div 4 = 8$. So the mean is eight.

Teaching point 1 uses the familiar topics of equal and unequal sharing (segment 2.2 *Structures: multiplication representing equal groups*) and partitive division (segment 2.6 *Structures: quotitive and partitive division*) to introduce the concept of the mean. Counters and bar models are used to help with visualisation. Simple numbers should be used in the early stages so that children can focus on the structure and concept. By the end of this teaching point they should be confident with the generalisation: **'The mean is the size of each part when a quantity is shared equally.'**

Once the concept of the mean has been established, *Teaching point 2* explores how to calculate the mean, working with the generalisation: **'The mean is the total of the numbers divided by how many numbers there are.'** Stem sentences are used to ensure that children understand what each number represents.

Next children will work with the mean in various ways:

- calculating the total quantity when the mean and the number of values is known
- exploring how the mean is affected when the total quantity or the number of values changes
- calculating the mean when one of the numbers in the set is zero
- using the mean to find missing information in sets of data.

In *Teaching points 3* and *4* children learn that the mean can be used to compare data, but that there are contexts where the mean is not a suitable representation due to outliers in the set of data.

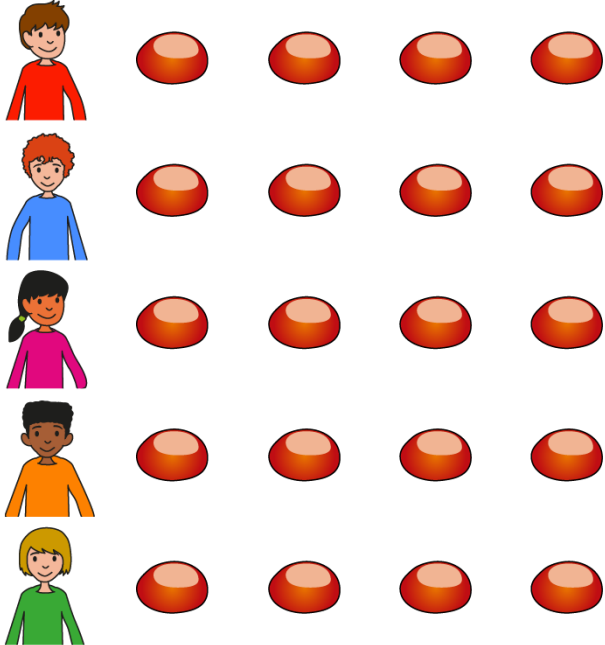
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

The mean is the size of each part when a quantity is shared equally.

Steps in learning

	Guidance	Representations																																																																																																				
1:1	<p>To introduce the topic of mean average (referred to as 'mean' from here onwards), this teaching point considers how to move from unequal sharing to equal sharing. Children will build on their knowledge of equal and unequal groups covered in segment 2.2</p> <p><i>Structures: multiplication representing equal groups.</i></p> <p>Begin with a context where objects are being shared unequally, for example a group of children who have some football cards. You may want to use actual football cards or represent the cards using counters or multilink cubes. Arrange them in a way that is easy to compare, as shown opposite.</p> <p>Present the problem and look at how the cards could be redistributed so that they are shared equally.</p> <p>For the example opposite, you could start with Dave, who has the most cards. He could give two of his cards to Cath and two of his cards to Evie. Ask children <i>'Is this now fair?'</i></p> <p>Three of the children now have the same number of cards, but Amy and Evie still have fewer cards than Ben, Cath and Dave. If Ben, Cath and Dave each give one card to Amy she will have four. Ask children <i>'Is this now fair?'</i></p> <p>Yes, they all have four cards; <i>'The cards have been shared equally.'</i></p>	<p><i>'Amy has one card, Ben has five cards, Cath has three cards, Dave has nine cards and Evie has two cards. How many do they have each if they share them equally?'</i></p> <table border="1" data-bbox="762 589 1481 1167"> <tbody> <tr><td></td><td></td><td></td><td>●</td><td></td></tr> <tr><td></td><td></td><td></td><td>●</td><td></td></tr> <tr><td></td><td></td><td></td><td>●</td><td></td></tr> <tr><td></td><td></td><td></td><td>●</td><td></td></tr> <tr><td></td><td>●</td><td></td><td>●</td><td></td></tr> <tr><td></td><td>●</td><td></td><td>●</td><td></td></tr> <tr><td></td><td>●</td><td>●</td><td>●</td><td></td></tr> <tr><td></td><td>●</td><td>●</td><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr> <tr><td>Amy</td><td>Ben</td><td>Cath</td><td>Dave</td><td>Evie</td></tr> </tbody> </table> <ul style="list-style-type: none"> <i>'Dave could give two of his cards to Cath and two of his cards to Evie.'</i> <table border="1" data-bbox="762 1328 1481 1906"> <tbody> <tr><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>●</td><td>●</td><td>●</td><td></td></tr> <tr><td></td><td>●</td><td>●</td><td>●</td><td>●</td></tr> <tr><td></td><td>●</td><td>●</td><td>●</td><td>●</td></tr> <tr><td></td><td>●</td><td>●</td><td>●</td><td>●</td></tr> <tr><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr> <tr><td>Amy</td><td>Ben</td><td>Cath</td><td>Dave</td><td>Evie</td></tr> </tbody> </table>				●					●					●					●			●		●			●		●			●	●	●			●	●	●	●	●	●	●	●	●	Amy	Ben	Cath	Dave	Evie																						●	●	●			●	●	●	●		●	●	●	●		●	●	●	●	●	●	●	●	●	Amy	Ben	Cath	Dave	Evie
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<p>1:2</p>	<p>Now look at how children can use their knowledge of the partitive structure of division (see segment 2.6 <i>Structures: quotitive and partitive division</i>) for support when working with the mean.</p> <p>First revisit a sharing context such as the conkers used in segment 2.6. Remind children that the total quantity is divided into a known number of equal shares, as indicated by the divisor (partitive division). For example, with the conkers, the total quantity (20) is divided into a known number of equal shares (five).</p> <p>Next return to the football cards used in step 1:1, this time starting with the total number of cards.</p> <p>'Amy, Ben, Cath, Dave and Evie have twenty football cards between them. How many do they have each if they share them equally?'</p> <ul style="list-style-type: none"> 'One five is one each. That's five...' 'Four fives is four each. That's twenty.' 'They get four cards each.' 	<ul style="list-style-type: none"> 'I have twenty conkers and I share them equally between five children. How many conkers does each child get?'  <p style="text-align: center;">4 each</p> <ul style="list-style-type: none"> 'One five is one each. That's five...' 'Four fives is four each. That's twenty.' 																																																		

Now consider the problem using the original wording, where the children each have a different number of cards to start with and they choose to share them out equally. You could use a table such as the one shown opposite.

In both cases ask children before and after the cards are shared:

- 'What's the same?'
(The total number of cards remains the same. There are 20 in total for both distributions.)

'What's different?'
(The number of cards each child has.)

- 'Amy has one card, Ben has five cards, Cath has three cards, Dave has nine cards and Evie has two cards. How many do they have each if they share them equally?'

	Number of cards to start with	Number of cards after sharing equally
Amy	1	4
Ben	5	4
Cath	3	4
Dave	9	4
Evie	2	4
Total number of cards	20	20

- 'They have four cards each after they have shared them equally.'

1:3

Next, use bar models to represent the problem. Use problems that have been discussed previously, such as the football card problem in step 1:2, so that children can focus on the structure rather than calculating the answer. Ask children to complete bar models representing both interpretations:

- when the total number and the group size are known
- when the answer can be calculated by redistributing unequal shares.

Total number known:

'Amy, Ben, Cath, Dave and Evie have twenty football cards between them. How many do they have each if they share them equally?'

20				
?	?	?	?	?

Redistributing unequal shares:

'Amy has one card, Ben has five cards, Cath has three cards, Dave has nine cards and Evie has two cards. How many do they have each if they share them equally?'

?				
1	5	3	9	2
?	?	?	?	?

1:4

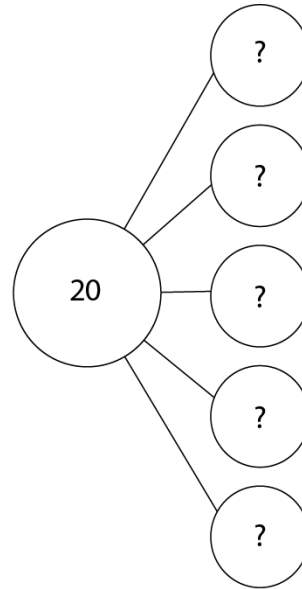
Represent the same problems using part-part-whole models. Make sure children are confident in their understanding that all of the parts must be the same size if things have been shared equally.

Provide children with practice dividing quantities into equal or unequal shares using part-part-whole models.

Finish this teaching point by linking back to the concept of the mean with the following generalisation: **'The mean is the size of each part when a quantity is shared equally.'**

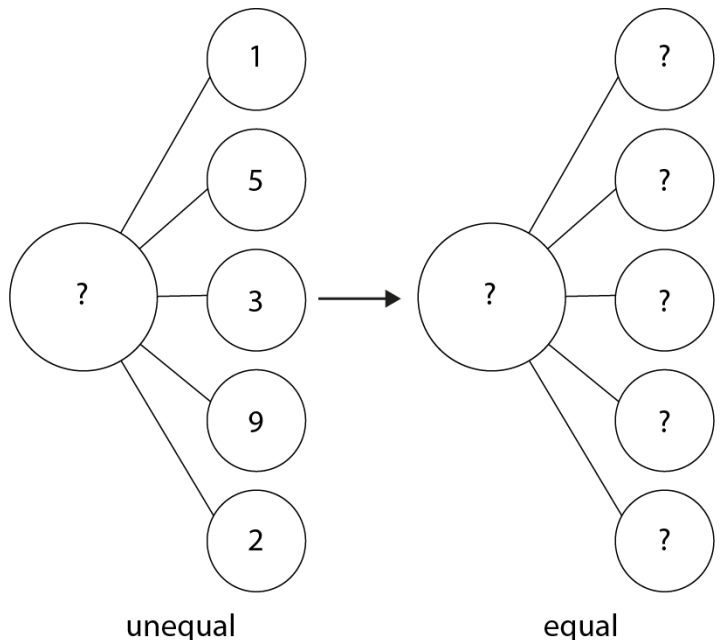
Total number known:

'Amy, Ben, Cath, Dave and Evie have twenty football cards between them. How many do they have each if they share them equally?'



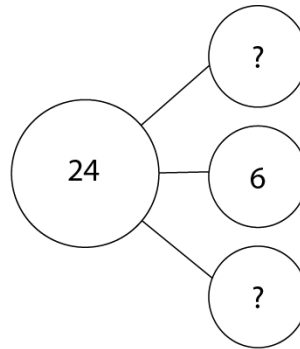
Redistributing unequal shares:

'Amy has one card, Ben has five cards, Cath has three cards, Dave has nine cards and Evie has two cards. How many do they have each if they share them equally?'

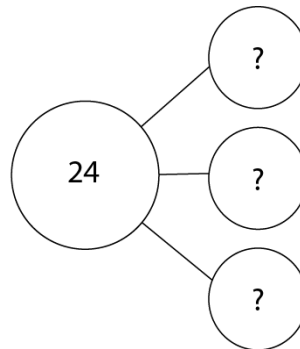


Dividing quantities into equal or unequal shares:

- 'Complete this part-part-part-whole model using unequal shares. Is there more than one possibility?'



- 'Now complete it using equal shares.'



Teaching point 2:

The mean is defined as the sum of all the numbers in a set of data divided by the number of numbers/values that make up the set of data. If we know the mean of a set of data and the number of numbers/values in that set, we can calculate the total of the set. The mean of a set changes if the total value of the set changes or if the number of numbers/values in the set changes.

Steps in learning

	Guidance	Representations
2:1	<p>Return to the football cards problem from <i>Teaching point 1</i>, using a division equation to show how to find the mean (the size of each equal part). First use the version of the problem where the total number of cards is provided. Use the following stem sentence to ensure children understand how the equation is formed: 'The ___ represents the ___.'</p> <p>Now build on this to calculate the mean when the total quantity is not provided. We must first work out the total quantity by adding together the unequal shares.</p> <p>Work towards the following generalised statement: 'The mean is the total of the numbers divided by how many numbers there are.'</p>	<p>Total number known:</p> <p><i>'Amy, Ben, Cath, Dave and Evie have twenty football cards between them. How many do they have each if they share them equally?'</i></p> <ul style="list-style-type: none"> <i>'The dividend is "20".'</i> <i>'The "20" represents the total number of cards.'</i> <i>'The divisor is "5".'</i> <i>'The "5" represents the five children.'</i> $20 \div 5 = 4$ <i>'The quotient of "4" represents the mean.'</i> <p>Redistributing unequal shares:</p> <p><i>'Amy has one card, Ben has five cards, Cath has three cards, Dave has nine cards and Evie has two cards. How many do they have each if they share them equally?'</i></p> <ul style="list-style-type: none"> <i>'The dividend is $1 + 5 + 3 + 9 + 2 = 20$'</i> <i>'The "20" represents the total number of cards.'</i> <i>'The divisor is "5".'</i> <i>'The "5" represents the five children.'</i> $20 \div 5 = 4$ <i>'The quotient of "4" represents the mean.'</i>
2:2	<p>Build on this knowledge by calculating the mean using discrete data in a variety of contexts. Explore two kinds of problems: those where the total quantity is known, and those where the total quantity must be worked out. Ensure the mean is always a whole number.</p> <p>Use the following stem sentences:</p> <ul style="list-style-type: none"> <i>'The dividend is ___.'</i> <i>'The divisor is ___ because ___.'</i> <i>'The mean is $__ \div __ = __$.'</i> 	<p>Total number known:</p> <ul style="list-style-type: none"> <i>'Eight children read thirty-two books over the summer term. What is the mean number of books read?'</i> <i>'The dividend is "32".'</i> <i>'The divisor is "8" because there are eight children.'</i> <i>'The mean is $32 \div 8 = 4$.'</i>

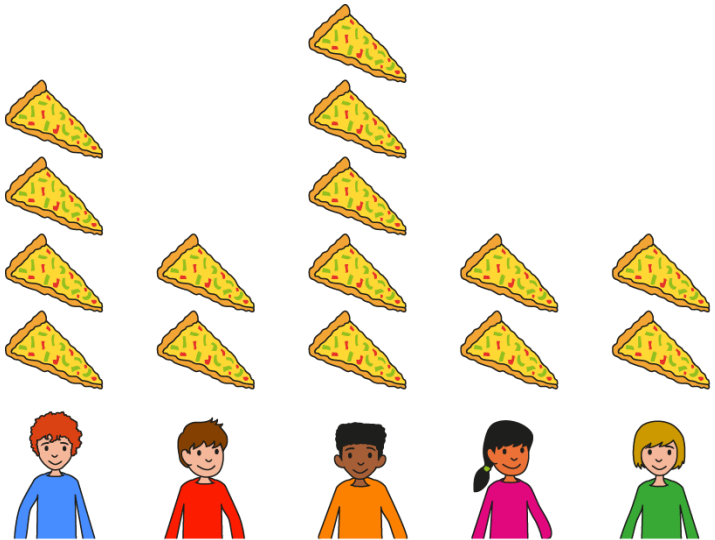
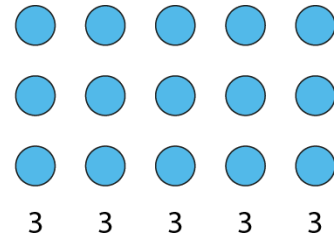
- 'Twenty pencils are divided between four pencil cases. What is the mean number of pencils?'
- 'The dividend is "20".'
- 'The divisor is "4" because there are four pencil cases.'
- 'The mean is $20 \div 4 = 5$.'

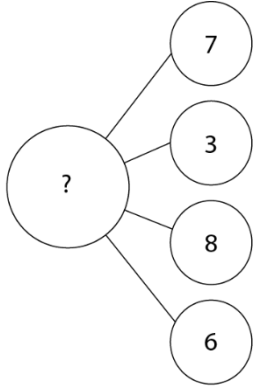
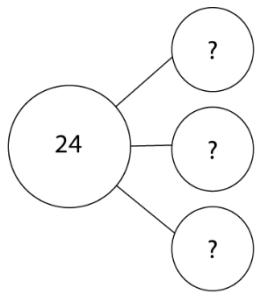
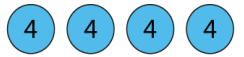
Redistributing unequal shares:

- 'What is the mean number of books read in the summer term?'

Name	Number of books read in the summer term
Fred	5
Grey	4
Hari	1
Indigo	2
James	5
Kia	3
Liz	8
Mohamed	4

- 'The dividend is "32" ($5 + 4 + 1 + 2 + 5 + 3 + 8 + 4$).'
- 'The divisor is "8" because there are eight children.'
- 'The mean is $32 \div 8 = 4$.'

		<ul style="list-style-type: none"> • 'Some children are given slices of pizza. What is the mean number of slices?'  <ul style="list-style-type: none"> • 'The dividend is "15" (4 + 2 + 5 + 2 + 2).' • 'The divisor is "5" because there are five children.' • 'The mean is $15 \div 5 = 3$.'
<p>2:3</p>	<p>Now extend this knowledge using multiplication to calculate the total quantity when we know the mean and the number of numbers/values in the set. Ensure children are confident with the distinction between the sum of all of the numbers in the set, and the number of numbers/values that make up the set. Continue to use the stem sentences from step 2:2 if needed.</p> <p>Explore one of the contexts covered in a previous teaching point, this time providing the mean and the number of values in the set. You could use counters to help initially.</p>	<p>'Five children are given slices of pizza. The mean number of slices is three. What is the total number of slices?'</p>  <ul style="list-style-type: none"> • 'There are five groups of three.' $5 \times 3 = 15$ <ul style="list-style-type: none"> • 'The total number of slices is fifteen.'
<p>2:4</p>	<p>At this point, provide children with practice calculating the mean and finding the total quantity when the mean and number of values in the set is known. For example:</p> <ul style="list-style-type: none"> • 'There are thirty sheep shared out between six fields. What is the mean number of sheep in each field?' 	

<ul style="list-style-type: none"> Dòng não jīn: <i>'George has twelve pens. Jess has half as many pens as George. Andy has three times as many pens as Jess. What is the mean number of pens?'</i> 	<ul style="list-style-type: none"> <i>'Draw lines to join the information on the left to the matching information on the right.'</i> <div data-bbox="762 322 1050 501"> <p><i>'The mean number of pieces of fruit eaten is 4 per person.'</i></p> </div> <div data-bbox="762 510 1050 920"> <p>$24 \div 3 = 8$</p> </div> <div data-bbox="762 929 1050 1108"> <p><i>'The mean is 8.'</i></p> </div> <div data-bbox="762 1120 1050 1433"> <p><i>'The mean is a square number and the total is a square number.'</i></p> </div> <div data-bbox="762 1444 1050 1624"> <p><i>'The mean number of pieces of fruit eaten is 3 per person.'</i></p> </div> <div data-bbox="762 1635 1050 1814"> <p><i>'The mean is 6.'</i></p> </div> <div data-bbox="1193 322 1485 501"> <p><i>'8 people eat some fruit. The total number of pieces of fruit eaten is 24.'</i></p> </div> <div data-bbox="1193 510 1485 920">  </div> <div data-bbox="1193 929 1485 1108"> <p><i>'6 people eat some fruit. The total number of pieces of fruit eaten is 24.'</i></p> </div> <div data-bbox="1193 1120 1485 1433">  </div> <div data-bbox="1193 1444 1485 1624">  </div> <div data-bbox="1193 1635 1485 1814"> <p><i>'Dan eats 10 grapes, Jon eats 9 cherries and Imra eats 5 plums.'</i></p> </div>
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		<ul style="list-style-type: none"> 'This table shows the number of children in each year group. What is the mean number of children?' <table border="1" data-bbox="932 295 1315 640"> <thead> <tr> <th>Year group</th> <th>Number of children</th> </tr> </thead> <tbody> <tr> <td>Year 1</td> <td>15</td> </tr> <tr> <td>Year 2</td> <td>17</td> </tr> <tr> <td>Year 3</td> <td>12</td> </tr> <tr> <td>Year 4</td> <td>12</td> </tr> </tbody> </table>	Year group	Number of children	Year 1	15	Year 2	17	Year 3	12	Year 4	12														
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Year 3	12																									
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<p>2:5</p>	<p>Next explore how the mean changes when the total quantity changes. To begin with, ensure the mean is always a whole number so that children can easily see the comparison. Start by presenting a problem such as this: 'On Monday four friends shared twenty-four sweets. What is the mean number of sweets per friend?'</p> <p>Using the stem sentences from step 2:2 state that:</p> <ul style="list-style-type: none"> 'The dividend is "24".' 'The divisor is "4" because there are four friends.' 'The mean is $24 \div 4 = 6$.' <p>Then present a similar problem in which the dividend has changed: 'On Tuesday four friends shared twenty-eight sweets. What is the mean number of sweets per friend?'</p> <p>Before working out the answer, ask children to predict whether the mean for Tuesday will be larger or smaller than the mean for Monday. Ask children to explain their answers.</p> <p>Then work out the mean using the stem sentences:</p> <ul style="list-style-type: none"> 'The dividend is "28".' 'The divisor is "4" because there are four friends.' 'The mean is $28 \div 4 = 7$.' <p>Draw children's attention to the fact</p>	<p>Unknown total:</p> <ul style="list-style-type: none"> 'This table shows the weights of some dogs in kilograms. What is the mean weight?' <table border="1" data-bbox="908 804 1335 1173"> <thead> <tr> <th>Name of dog</th> <th>Weight (kg)</th> </tr> </thead> <tbody> <tr> <td>Molly</td> <td>14</td> </tr> <tr> <td>Boss</td> <td>8</td> </tr> <tr> <td>Tucker</td> <td>4</td> </tr> <tr> <td>Chester</td> <td>3</td> </tr> <tr> <td>Maggie</td> <td>6</td> </tr> </tbody> </table> <ul style="list-style-type: none"> 'The dividend is $14 + 8 + 4 + 3 + 6 = 35$.' 'The divisor is "5" because there are five dogs.' 'The mean is $35 \text{ kg} \div 5 = 7 \text{ kg}$.' <ul style="list-style-type: none"> 'This table shows the weights of the same dogs a year later. What is the mean weight now?' <table border="1" data-bbox="908 1453 1335 1823"> <thead> <tr> <th>Name of dog</th> <th>Weight (kg)</th> </tr> </thead> <tbody> <tr> <td>Molly</td> <td>15</td> </tr> <tr> <td>Boss</td> <td>10</td> </tr> <tr> <td>Tucker</td> <td>4</td> </tr> <tr> <td>Chester</td> <td>4</td> </tr> <tr> <td>Maggie</td> <td>7</td> </tr> </tbody> </table> <ul style="list-style-type: none"> 'The dividend is $15 + 10 + 4 + 4 + 7 = 40$.' 'The divisor is "5" because there are five dogs.' 'The mean is $40 \text{ kg} \div 5 = 8 \text{ kg}$.' <ul style="list-style-type: none"> 'This table shows the weights of the same dogs two 	Name of dog	Weight (kg)	Molly	14	Boss	8	Tucker	4	Chester	3	Maggie	6	Name of dog	Weight (kg)	Molly	15	Boss	10	Tucker	4	Chester	4	Maggie	7
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that the mean has increased. Discuss that the number of numbers/values has stayed the same ('4' – represented by the four friends), but the number of sweets that each friend gets has changed: each friend gets more sweets because the total has increased and so there are more sweets to share. Therefore, the mean has also increased.

Repeat the above process, increasing the dividend by multiples of four, until children are confident with the concept.

Now use an example where the total quantity is not given but must be worked out, such as the example of comparing the weights of dogs opposite. Continue to discuss that when the number of values remains the same but the total increases, the mean also increases.

Next ask children what they think happens to the mean when the total decreases. Repeat the examples above and opposite, this time reducing the total: *'On Wednesday four friends shared twenty sweets. What is the mean number of sweets per friend?'*

Before working out the answer, ask children to predict whether the mean for Wednesday will be larger or smaller than the mean for Tuesday. Ask children to explain their answers.

Then work out the mean using the stem sentences:

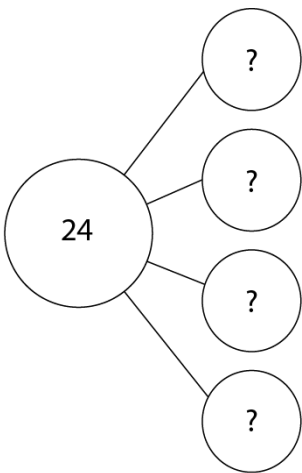
- *'The dividend is "20".'*
- *'The divisor is "4" because there are four friends.'*
- *'The mean is $20 \div 4 = 5$.'*

Discuss that the number of numbers/values has stayed the same ('4' – represented by the four friends), but the number of sweets that each child gets has changed: each child gets fewer sweets because the total has

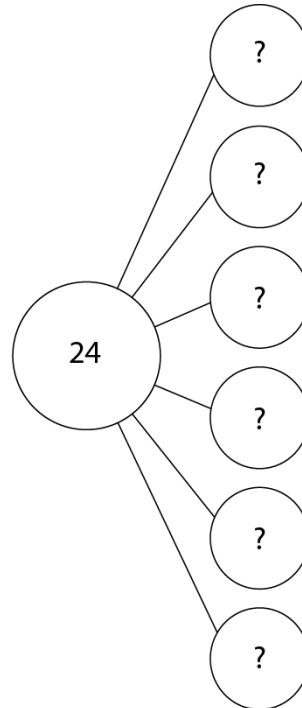
year later. What is the mean weight now?'

Name of dog	Weight (kg)
Molly	12
Boss	8
Tucker	3
Chester	2
Maggie	5

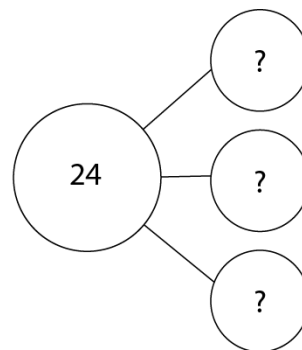
- *'The dividend is $12 + 8 + 3 + 2 + 5 = 30$.'*
- *'The divisor is "5" because there are five dogs.'*
- *'The mean is $30 \text{ kg} \div 5 = 6 \text{ kg}$.'*

	<p>decreased and so there are fewer sweets to share. Therefore, the mean has also decreased.</p> <p>By the end of this step children should be confident with the following generalisations:</p> <ul style="list-style-type: none"> • <i>'If the number of values in the set stays the same and the total increases, the mean also increases.'</i> • <i>'If the number of values in the set stays the same and the total decreases, the mean also decreases.'</i> 	
2:6	<p>Now build on this to see what happens to the mean if you keep the total quantity the same but increase or decrease the number of values in the set. Use similar problems to those used in the previous step, working through examples of both an increased and a decreased number of values in the set. Ensure children understand what is staying the same (the total quantity) and what is changing (the number of values in the set).</p> <p>You could use part-part-whole models to help children visualise what is changing each time.</p>	<ul style="list-style-type: none"> • <i>'On Monday four friends shared twenty-four sweets. What is the mean number of sweets per friend?'</i> 

- 'On Tuesday six friends shared twenty-four sweets. What is the mean number of sweets per friend?'



- 'On Wednesday three friends shared twenty-four sweets. What is the mean number of sweets per friend?'



2:7 This step explores how to calculate the mean when one of the numbers in the set is zero. Show children a problem such as the one opposite, where one of the numbers in the set is zero. Explore two different solutions, one that includes the zero value and one that does not. Ask children which of the two methods is correct, and to explain their reasons.

'The table shows the number of children who went to the library at break time this week. What is the mean number of children per day?'

Day of the week	Mon	Tue	Wed	Thu	Fri
Number of children	6	0	7	2	5

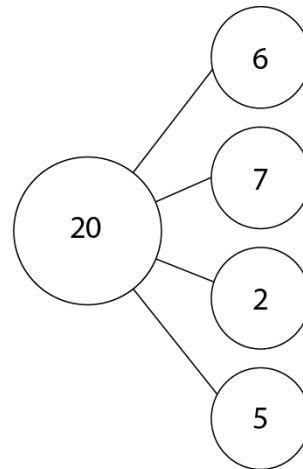
Use the stem sentences from step 2:2:

- 'The dividend is "20".'
- 'The divisor is "5" because there are five days.'
- 'The mean is $20 \div 5 = 4$.'

Draw children's attention to the second sentence, explaining that we divide by five as there are five days, even if the value for one of the days is zero. In the example opposite, Alicia is correct because she has included the zero value when counting the number of values.

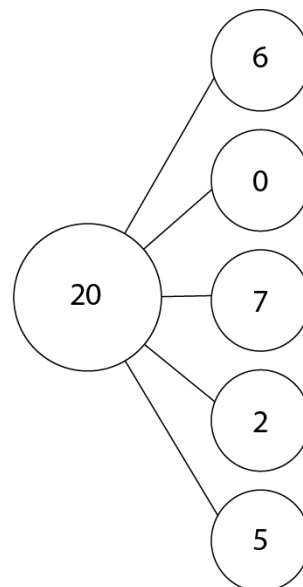
- 'This is Fatima's method and diagram:'

$$(6 + 7 + 2 + 5) \div 4 = 5$$



- 'This is Alicia's method and diagram:'

$$(6 + 0 + 7 + 2 + 5) \div 5 = 4$$



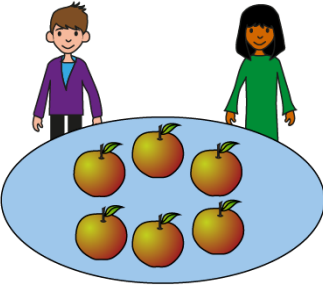
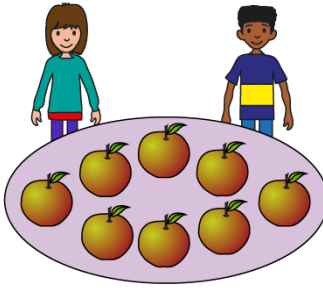
'Who is correct? Explain your answer.'

		<p>Dòng não jīn:</p> <p><i>'The following week there is a Bank Holiday and so the school is shut on Monday. What is the mean number of children per day for this week?'</i></p> <table border="1" data-bbox="794 380 1452 667"> <thead> <tr> <th>Day of the week</th> <th>Mon</th> <th>Tue</th> <th>Wed</th> <th>Thu</th> <th>Fri</th> </tr> </thead> <tbody> <tr> <td>Number of children</td> <td>0</td> <td>3</td> <td>7</td> <td>2</td> <td>4</td> </tr> </tbody> </table>	Day of the week	Mon	Tue	Wed	Thu	Fri	Number of children	0	3	7	2	4
Day of the week	Mon	Tue	Wed	Thu	Fri									
Number of children	0	3	7	2	4									
<p>2:8</p>	<p>To finish this teaching point, present problems where the mean is provided but some information is missing. Encourage children to use their previous knowledge to find the missing data.</p> <p>First encourage children to consider how they could work out the missing information. For the example opposite, you could ask questions such as:</p> <ul style="list-style-type: none"> • 'How many values are there?' (4) • 'What is the mean time?' (14) • 'How could we find the total time for all four runners?' (multiply the mean by the number of values) <p>Work towards the conclusion that we can add together the information we do have and subtract the sum from the total time. Children could also use the redistribution approach used in <i>Teaching point 1</i> to balance the columns.</p> <p>Next ask children: 'What other times would also generate a mean of fourteen?' Using counters or cubes, start with 14 in each column and explore how the columns can be redistributed while the mean remains 14.</p> <p>Include a dòng não jīn problem in your practice such as:</p> <p><i>'Four numbers have a mean of ten; a fifth number is added and the mean changes to eleven. What number was added?'</i></p>	<p><i>'The mean time for a 3 kilometre race is 14 minutes. What could Billy's time be?'</i></p> <table border="1" data-bbox="909 779 1337 902"> <thead> <tr> <th>Amir</th> <th>Billy</th> <th>Carla</th> <th>Dean</th> </tr> </thead> <tbody> <tr> <td>13</td> <td>?</td> <td>15</td> <td>14</td> </tr> </tbody> </table> <ul style="list-style-type: none"> • Step 1 – multiply the mean by the number of values in the set to find the total time for all runners: $14 \times 4 = 56$ • Step 2 – add together the values we have for three runners: $13 + 15 + 14 = 42$ • Step 3 – subtract this answer from the total time: $56 - 42 = 14$ • 'Billy's time is 14 minutes.' 	Amir	Billy	Carla	Dean	13	?	15	14				
Amir	Billy	Carla	Dean											
13	?	15	14											

Teaching point 3:

The mean can be used to compare data.

Steps in learning

	Guidance	Representations
3:1	<p>This teaching point explores using the mean to make comparisons between two sets of information.</p> <p>Show children a context using two unequal groups, such as the example opposite. There are two children on each table, but one table has six apples and one has eight apples. Ask children:</p> <ul style="list-style-type: none"> • 'Is this fair?' • 'Do all of the children get the same number of apples each?' • 'Which children get more apples, those on table A or those on table B?' <p>Explain that even though the apples can be shared equally on each table, they have not been shared equally between the two tables to begin with. Therefore, the children will not get the same number of apples. Ask children to work out the mean number of apples for table A and for table B.</p>	<p><i>'Mrs L has some apples that she is giving to her maths group as a reward. She places them on two tables.'</i></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>A</p>  </div> <div style="text-align: center;"> <p>B</p>  </div> </div> <ul style="list-style-type: none"> • 'Table A has six apples and two children.' • 'Table B has eight apples and two children.' • 'The two children on table A get three apples each.' • 'The two children on table B get four apples each.' • 'The mean number of apples for children on table A is three.' • 'The mean number of apples for children on table B is four.' <p>Dòng nǎo jīn:</p> <ul style="list-style-type: none"> • 'Class 1 has twenty-one sweets to share between three children.' • 'Class 2 has thirty sweets to share between six children.' <p><i>'Which class would you rather be in? Explain your answer.'</i></p>

3:2

Now explore another context where two sets of information are compared, such as the spelling test example opposite.

Ask children to consider different ways of assessing which group did better. Draw attention to the fact that there are a different number of children in each group, so we cannot make a direct comparison. You could ask questions such as:

- *'Agree or disagree? The blue group did the best because their scores sum to a larger number.'*
- *'Agree or disagree? The blue group did the best because someone in their group scored ten.'*

Discuss whether these are fair ways to assess which group did better, and ask children whether they can think of any other ways. Direct children to the conclusion that we can use the mean to find a single number that represents each set, and then compare the two means.

'The two tables below show the spelling test results for two groups of children. The maximum possible score in the test is 10 points. All of the results are whole numbers. Which group did better overall?'

Orange group:

Child	A	B	C	D	E
Score	9	9	5	8	9

Blue group:

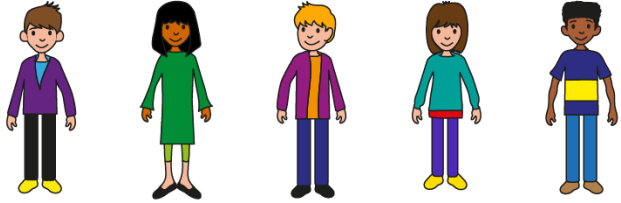
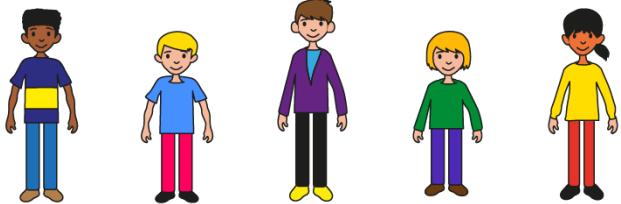
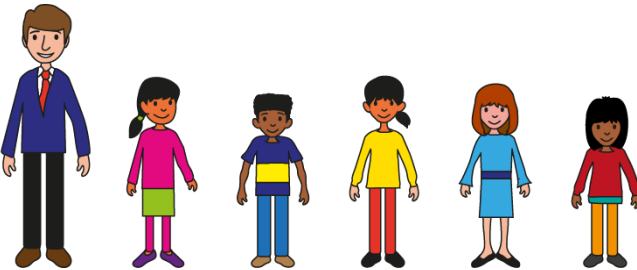
Child	A	B	C	D	E	F
Score	7	10	6	7	6	9

- *'The mean for the orange group is $(9 + 9 + 5 + 8 + 9) \div 5 = 8$.'*
- *'The mean for the blue group is $(7 + 10 + 6 + 7 + 6 + 9) \div 6 = 7.5$.'*
- *'The mean for the orange group is higher, so they did better overall.'*

Teaching point 4:

The mean is not always an appropriate representation of a set of data.

Steps in learning

	Guidance	Representations
4:1	<p>One of the key points in this segment is to consider whether calculating the mean is always an appropriate thing to do. Sometimes there can be one value that does not seem to fit and can therefore skew the mean. This is called an 'outlier'.</p> <p>To explore this idea provide children with several contexts, first with no outlier and then with an obvious outlier, such as the ones opposite.</p> <p>Look at the first picture, then ask:</p> <ul style="list-style-type: none"> • 'What is the mean age?' (12) • 'Do you think this is a good representation of the group overall?' (yes) • 'Are most people in the picture roughly that age?' (yes, they are all 12) <p>Repeat the questions for the second picture:</p> <ul style="list-style-type: none"> • 'What is the mean age?' (10) • 'Do you think this is a good representation of the group overall?' (yes) • 'Are most people in the picture roughly that age?' (yes, they are all close to 10) <p>Then show children the third picture, where there is an obvious outlier:</p> <ul style="list-style-type: none"> • 'What is the mean age?' (13) • 'Do you think this is a good representation of the group overall?' (no) • 'Are most people in the picture roughly that age?' (no, one is much older) <p>Come to the conclusion that the mean is not a useful representation when one</p>	<ul style="list-style-type: none"> • 'What is the mean age of the people in this picture?' age: 12 years 12 years 12 years 12 years 12 years  • 'What is the mean age of the people in this picture?' age: 11 years 9 years 11 years 8 years 11 years  • 'What is the mean age of the people in this picture?' age: 28 years 11 years 9 years 11 years 10 years 9 years 

	value in the set is very different to the others, and that this value is known as an outlier.													
4:2	<p>Now consider an example such as the one opposite where there are two extreme values that skew the data. Ask children what they notice about the prices. They should notice that</p> <ul style="list-style-type: none"> • one price is very low • one price is very high • three prices in the middle are similar. <p>Then ask children:</p> <ul style="list-style-type: none"> • <i>'What is the mean?'</i> • <i>'Is this a good representation of the typical price? Explain your answer.'</i> <p>With reference to outliers covered in step 4:1, explore the idea that there are two outliers and these might be affecting the usefulness of the mean.</p> <p>This is a scenario where we would typically use the median or the mode. We will not introduce these terms here, but mention to children that there are other types of average that can be useful when the mean is not suitable.</p> <p>Ensure children understand that it is not always appropriate to use the mean, and it is important to consider the data that is being used. The mean is a good measure to use when a set of data contains values that are relatively evenly spread with no exceptionally high or low values (outliers).</p> <p>In the example opposite, you could suggest that it might be better to ignore the highest and lowest prices and instead find the mean of the three middle values.</p>	<p><i>'On Sunny Avenue there is a row of terraced houses. House A is derelict and has been left empty for several years. House E is very modern with a large garden. What is the typical price of a house on Sunny Avenue?'</i></p> <table border="1" data-bbox="948 497 1299 864"> <thead> <tr> <th>Property</th> <th>Value in £</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>150,000</td> </tr> <tr> <td>B</td> <td>350,000</td> </tr> <tr> <td>C</td> <td>360,000</td> </tr> <tr> <td>D</td> <td>350,000</td> </tr> <tr> <td>E</td> <td>500,000</td> </tr> </tbody> </table>	Property	Value in £	A	150,000	B	350,000	C	360,000	D	350,000	E	500,000
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