MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.8 Composition of numbers: multiples of 10 up to 100

Teacher guide | Year 1

Teaching point 1:

One ten is equivalent to ten ones.

Teaching point 2:

Multiples of ten can be represented using their names or using numerals. We can count in multiples of ten.

Teaching point 3:

Knowledge of the 0–10 number line can be used to estimate the position of multiples of ten on a 0–100 number line.

Teaching point 4:

Adding ten to a multiple of ten gives the next multiple of ten; subtracting ten from a multiple of ten gives the previous multiple of ten.

Teaching point 5:

Known facts for the numbers *within* ten can be used to add and subtract in multiples of ten by unitising.

Overview of learning

In this segment, children will:

- develop understanding that 10 is one group of ten, 20 is two groups of ten, etc.
- use 'dual counting' to count in groups of ten, saying *'No tens, one ten, two tens, three tens…'* as well as *'Zero, ten, twenty, thirty…'*
- build on counting in multiples of ten, to add and subtract ten from a multiple of ten
- apply ordinal knowledge and number facts within ten, to understand the position of multiples of ten in the number system, and addition and subtraction of multiples of ten.

This segment introduces the important concept of unitising, where a 'unit' can be assigned any value. This concept is clearly seen in money, where one coin can have a value other than one. In this segment a unit is assigned the value of ten (one ten, two tens…). This work on unitising will provide the foundation for developing place-value concepts later, when the unit value may be one hundred, one thousand, or one tenth, and underpins much of the other maths presented in these materials. Multiplicative reasoning relies heavily on unitising (for example, working in a unit of five, or of 100 ml, or indeed anything else), and in some ways this whole segment is really a multiplicative segment, although here we build towards addition and subtraction of multiples of ten. Addition and subtraction of fractions also relies on unitising (for example, in $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$ we are working with a unit of 'one eighth').

Throughout this segment, connections will be made between number facts involving units of one and those involving units of ten. Connections will also be made between the relative positions of numbers on the 0–10 number line and the position of multiples of ten on the 0–100 number line.

Children will experience many representations of ten, which will become increasingly abstract. Careful thought has been given to the progression of representations used, to ensure that children develop a secure understanding. Initially, when representing ten, children will count out individual objects and group these into ten; the ten objects can then be separated again to reinforce the cardinality (quantity value) of ten. Children will then progress to using representations which cannot be split into ones, but where the cardinality of ten is obvious, for example, Dienes or ten frames. The final step is for children to work with representations of ten where the cardinality is not obvious, for example a ten-pence coin, or a box labelled as '10 items'.

It is important to note that, in English, the words for numbers above ten do not always make the structure of our number system explicit. Unlike in many other languages, the names which we give to two-digit multiples of ten do not all follow a regular pattern, which can lead to difficulties when children are in the early stages of counting. Teen and '–ty' numbers are often confused, and the names for some multiples of ten are more problematic than others. For example, 'eighty', which contains the word 'eight', is easier to understand and remember than 'thirty', which does not contain 'three'. Close attention should be paid to the way in which children say numbers, and frequent opportunities should be provided to practise counting, including dual counting (i.e. *'Zero, ten, twenty, thirty…'* and *'No tens, one ten, two tens, three tens…'*).

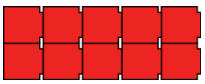
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks

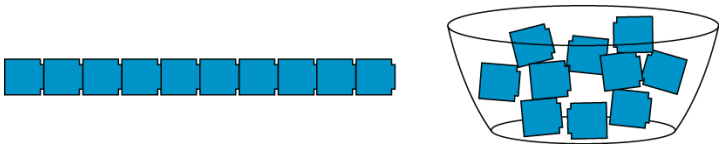
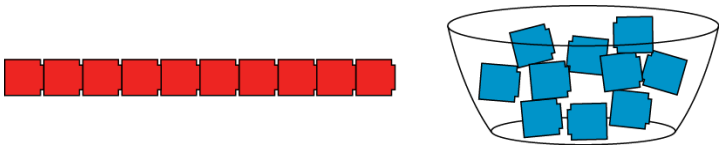
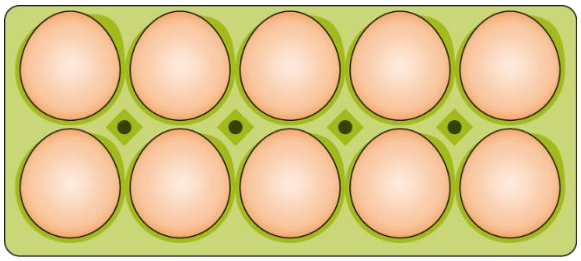
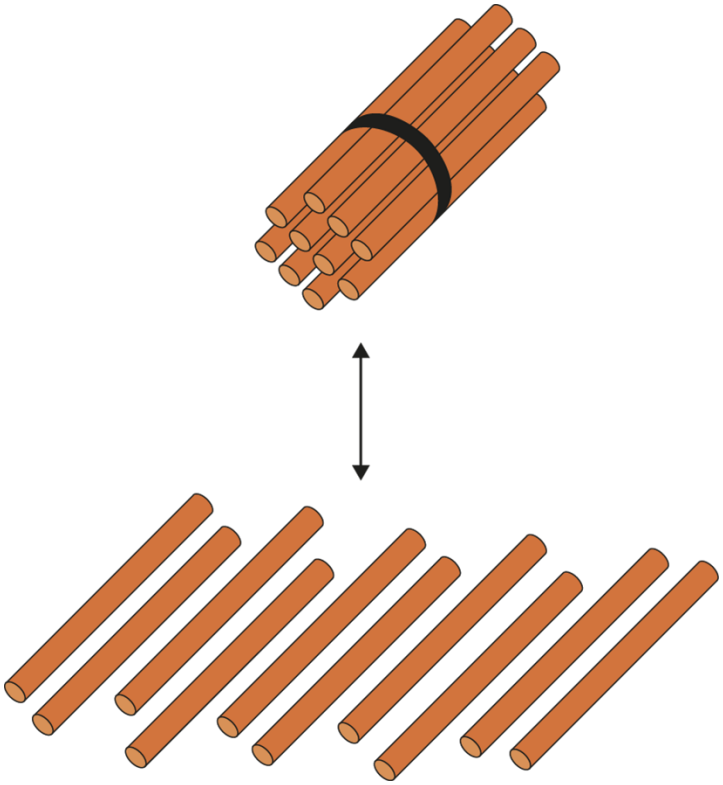
Teaching point 1:

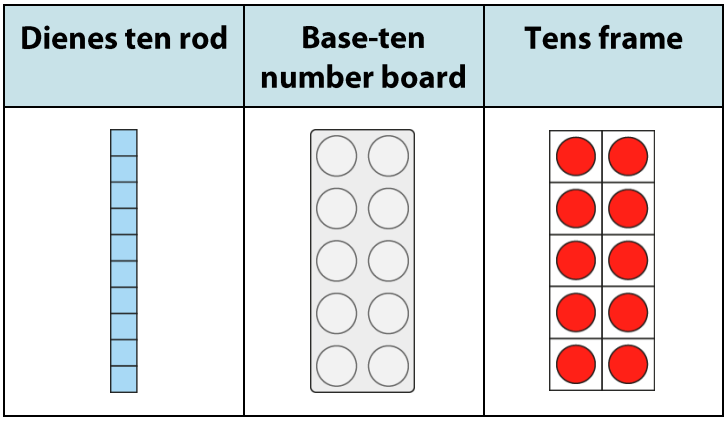
One ten is equivalent to ten ones.

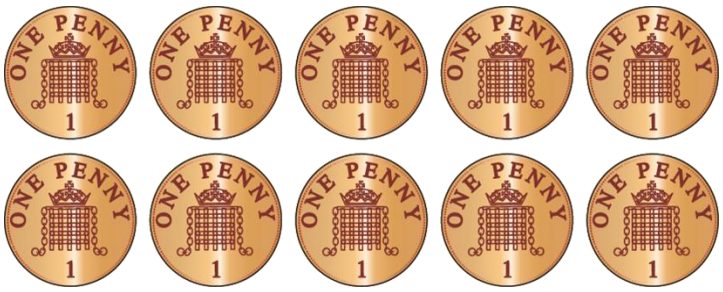
Steps in learning

	Guidance	Representations

1:1	The aim of this teaching point is to develop children's ability to group discrete (separate) objects into groups of ten and recognise each group as 'one ten'. This develops the concept of unitising, which can then later be applied to solve various problems using existing number-fact knowledge.
	Begin by working with multilink cubes, using the following progression:
	• Ask children to count out a set of ten cubes and join these to make a stick of ten. Ensure that all cubes in a stick are the same colour, so that the colour does not detract from the essential feature of ten.
	• To reinforce the equivalence between the stick of ten cubes and ten ones, ask children to break their stick of ten up into individual cubes and count them again.
	• Use variation to explore the essential feature of a group of ten, i.e. that it is composed of ten individual cubes; neither the orientation/arrangement of the cubes, nor their colour affect the 'ten-ness' of the group. Include the following:
	  • Introduce a stick of cubes of another colour, and a stick of cubes of mixed colours.
	  • Show ten cubes in an arrangement other than a stick.
	  • Compare ten cubes in a pot with a stick of ten cubes of the same colour.
	  • Compare ten cubes in a pot with a stick of ten cubes of a different colour; here you can check that
		Making a group of ten:
		'Is this a group of ten? How do we know?'
		 
		Non-essential features – orientation and colour:
		'Is this still a group of ten? How do we know?'
		 
		'Is this a group of ten? How do we know?'
		 
		'Is this a group of ten? How do we know?'
		 
		Non-essential features – arrangement and colour:
		'Is this a group of ten? How do we know?'
		 
		Look at the pictures below.
		• *'What's the same? What's different?'*
		• *'Would you be happy to swap your stick of cubes for this tub of cubes?'*
		• *'Do they have the same value? How do we know?'*

	children are happy to swap ten for ten, and not blue cubes for blue cubes. Use the following generalised statements to embed the idea of unitising: • ***'Ten ones are equal to one ten.'*** • ***'We have one group of ten.'*** • ***'We have one ten.'***	Scenario 1  Scenario 2 
1:2	Present pictorial contexts showing ten discrete objects represented as a single group/unit, such as: • ten fingers • packs of ten pencils/rubbers/ sharpeners • a vase of ten flowers • a bag of ten apples/oranges • a bundle of ten sticks. Reinforce the concept that we can prove that these are groups of ten because we can separate the objects and count them individually. When the objects are grouped into ten, ask the children if they can still see ten ones. Continue to use the unitising language of the generalised statements introduced in step *1:1*, for example ***'We have one group of ten.'*** To check understanding, present children with a concrete representation that does *not* show ten (such as a bundle of 11 sticks or a stick of nine multilink cubes) and ask whether they have a group of ten. Encourage children to explain or correct their answer, for example: *'This is not one group of ten, because when you separate the sticks there are not ten sticks.'*	 *'We have one group of ten eggs.'*  • *'We have one ten.'* • *'Ten ones are equal to one ten.'*

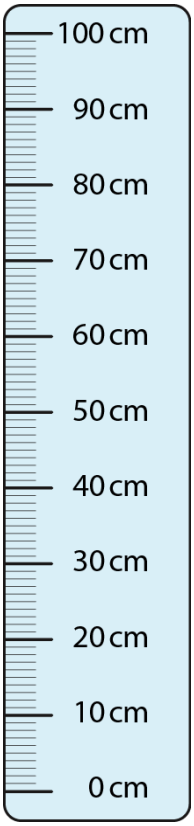
1:3	Reinforce the concept of equality by, for example, giving one child a pack of ten pencils and another child ten individual pencils. Ask: • *'Do they have the same number of pencils?'* • *'How do we know?'* Then use the same example to introduce the concept of efficiency by demonstrating giving ten pencils to a child, one at a time, and giving a pack of ten pencils.	
1:4	Once children have developed a good understanding of 'a group of ten' using resources where the group can be separated into individual objects, progress to looking at representations of ten that cannot be broken apart. These include: • Dienes ten rods • ten frames presented pictorially • base-ten number boards. Although these tens cannot be separated into ones, the cardinality (size) of ten is clear; the ten individual ones can be counted. Make sure that children can see the 'ten-ness' in each of these representations. Count the ones (*'one, two, three…'*, etc.) then summarise with the generalised statements as introduced in step *1:1*. Children may already be familiar with some of these generalised representations; they will be used later in this segment and in future segments. The purpose of this step is to give children the opportunity to explore the 'ten-ness' of these representations and shift their attention between ten ones and one ten.	

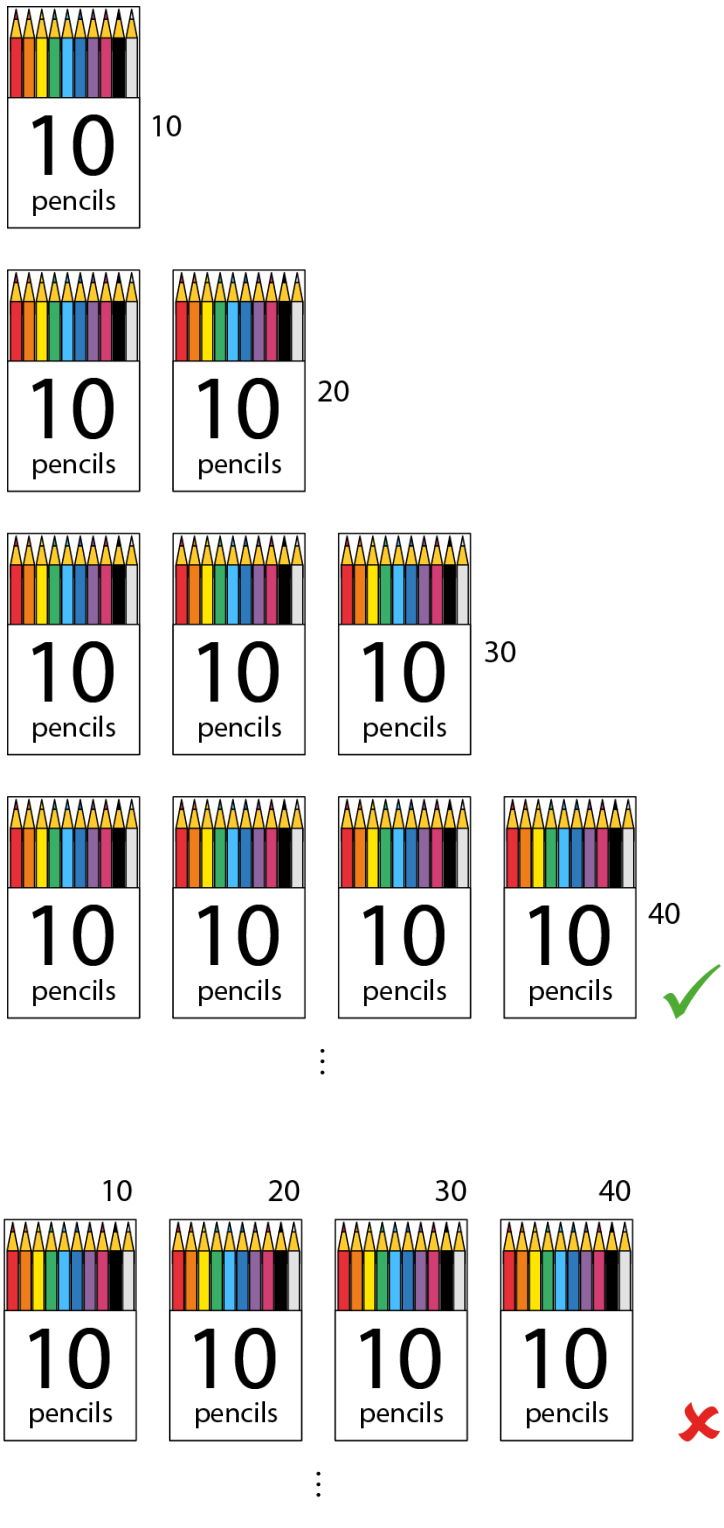
1:5	Now progress to exploring representations of ten which give no indication of cardinality other than that they show the word or numeral ('ten' or '10'). Examples include: • 'tens' place-value counters • ten-pence coins • £10 notes. Children can find the concept that a ten-pence coin has a value equivalent to ten one-pence coins very challenging. Provide plenty of opportunities for children to physically swap ten pennies for a ten-pence coin, so that children fully understand the meaning of the numeral 10 on the coin. Also investigate this unitising concept using place-value counters (tens counters and ones counters).	*'Would you be happy to swap these…'*  *'…for this?'* 

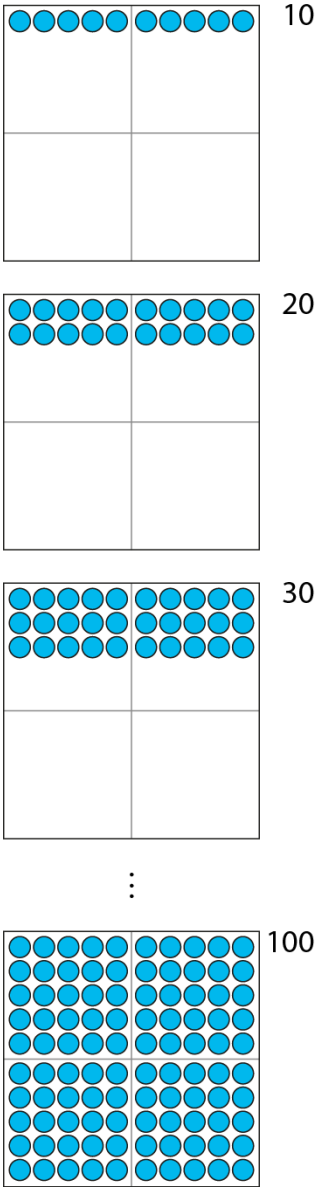
Teaching point 2:

Multiples of ten can be represented using their names or using numerals. We can count in multiples of ten.

Steps in learning

	Guidance	Representations
2:1	Begin this teaching point by linking the representations just used (step *1:5*), and other real-life contexts, to skip counting on and back in multiples of ten, between 0 and 100. At this point counting may still be quite procedural, with the class chanting while items are manipulated (or images of groups of ten items, or items with a value of ten, appear and disappear on the whiteboard). For now, ensure that children become familiar with using the pattern of numbers in a variety of situations – you will then build on this through the rest of the segment. So that children identify, for example, 'three groups of ten' with 'thirty' or 'three tens', count in two ways ('dual counting'): • *'Zero, ten, twenty, thirty…'* • *'No tens, one ten, two tens, three tens…'* Useful representations include: • objects: fingers, bags of ten marbles/apples, packs of ten pens, etc. • generalised representations: tens frames, Dienes, sticks of ten multilink cubes • money (here 'pence coins' has been contracted to 'pences' for easier recitation) and other measures (note that length measures are typically shown horizontally – ensure that you include variation by also presenting them vertically; this will provide a	Money: • *'No money, ten pence, twenty pence, thirty pence…'* • *'No ten-pences, one ten-pence, two ten-pences…'*  Measures: • *'Zero centimetres, ten centimetres, twenty centimetres, thirty centimetres…'* • *'No ten-centimetres, one ten-centimetres, two ten-centimetres…'* 

	useful foundation for when children begin to read scales and graphs) • scoring in tens: computer games, sports (archery or strikes in ten-pin bowling) and games in P.E.	
2:2	Now start to use numerals to represent multiples of ten. As children gain confidence you can begin to model use of the term 'multiple' in your teaching and explanations. Continue with skip counting, supported pictorially, with numerals representing the cardinal number (total number) as each new ten appears. As before, use dual counting, so that as children see, for example, three groups of ten and the numeral 30, they sometimes say *'thirty'* and sometimes say *'three tens'*.	Objects: 

		Generalised representation – hundred grid: 
2:3	Now that children have some familiarity with the numbers expressed as numerals, progress to a detailed examination of how multiples of ten are written in numerals. Children will be very familiar with the fact that '10' represents the number we call 'ten', but until now have not been taught why ten is represented by a 1 followed by a 0. Show the children a picture of one ten (such as a vase of ten flowers), labelled	

'10', alongside a picture two tens (two vases of ten flowers), labelled '20', and ask:

- *'What does the 1 represent in 10?'*
- *'What does the 2 represent in 20?'*

Draw out the fact that the 1 represents one ten and the 2 represents two tens. Then repeat with other multiples of ten, asking questions to draw out the pattern in the numeral representations:

- *'What do you notice about the way the numbers are written?'*
- *'How do we write five tens?'*
- *'How do we write six tens?'*
- *'What is the pattern?'*

Introduce the following stem sentence:

'This is the number ___. The ___ represents ___ tens.'

To truly understand the significance of the 0 in the ones place, children will need to meet numbers which *don't* have 0 in the ones place – this is covered in segment *1.9 Composition of numbers: 20–100*. Be aware that, in the meantime, children may find it difficult to conceptualise what the 'tens place' and 'ones place' really means, and what 'no ones' means. At this stage, focus on making sure that children are really confident in writing each multiple of ten, and that they don't, for example, write twenty as '02'. Use the following generalised statement as a class:
'All multiples of ten end with a zero.'

- *'What does the 1 represent?'*
- *'What does the 2 represent?'*

10 20

- *'The 1 represents one ten.'*
- *'The 2 represents two tens.'*

'This is the number ten. The 1 represents one ten.'

10s	1s
1	0

'What is the pattern?'

Digits	What it means
10	1 ten
20	2 tens
30	3 tens
40	4 tens
50	5 tens
60	6 tens
70	7 tens
80	8 tens
90	9 tens
100	10 tens

2:4

Once you have established the pattern, provide children with practice:

- Show a representation of a multiple of ten, and ask children to write the number. Note, when showing more than five groups for the children to enumerate, present them as shown in the top example opposite, to facilitate subitising; if presenting the items in a row ensure that you leave a small gap after the fifth item (otherwise children will have to count to deduce how many there are).
- Show a number written in digits, without saying it, and ask children to represent the quantity either practically or pictorially (the 'infinite cloner' setting on the interactive whiteboard is ideal for doing this). Provide the children with tens of the representation you are using, so that children do not have to prepare the tens themselves.

In both cases ask children to explain their reasoning, for example:

- when shown a representation of 70 – *'There are seven groups of ten so we write this seven-zero.'*
- when shown '40' - *'I know the number is a multiple of ten because it ends in zero; that means I can make it out of groups of ten. The four tells me there are four groups of ten.'*

'Each vase contains ten flowers. Write down the total number of flowers.'

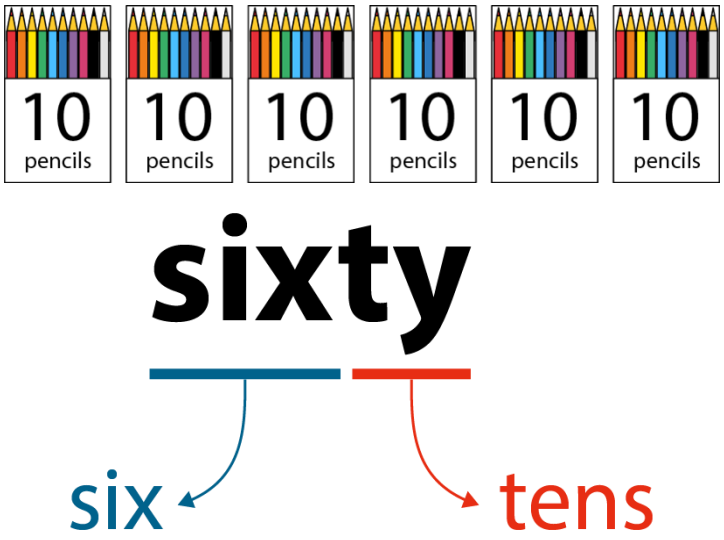
'Show me each number.'

10	50	30
		

10	60	40
		

2:5

Now look at the number names. Using a familiar representation that shows the cardinality of each ten, begin with sixty – the first 'regularly' named multiple. Discuss how the name 'sixty'

	is composed, and how it links to 'six tens'. Examine the name of each of the multiples of ten, exploring how they do or don't relate to the structure of the number: • The names 'sixty', 'seventy', 'eighty', and 'ninety' contain the number of tens, written as children will recognise, followed by '–ty'. • In 'twenty', 'thirty', 'forty' and 'fifty', the start of the name has some link to the number of tens, to a greater or lesser extent ('forty' of course *sounds* regular but is spelt differently). Link each multiple to the number of tens, using the stem sentence: ***'We have ___ tens. We call this ___.'*** For 'ten tens', show children '100' (written in numerals) and ask them how many tens it represents? Explain that it has a special name and is called 'one hundred'. Ensure that children say 'one hundred' rather than 'a hundred'.	Exploring 'sixty': *'We have six tens. We call this sixty.'* 
2:6	Summarise the link between names, digits and composition, as shown in the table opposite, and provide children with practice moving between the following representations: • the spoken number name • the written number name • the number written in numerals, for example, 20 • the number of tens (for example, 'two tens', or 'two groups of ten'). Use variation in terms of the concrete/pictorial representations used (for example, multilink, Dienes, bundles of straws, base-ten number boards, tens frames, fingers) Encourage children to: • practise counting in tens to check their answers	Linking number names, digits and composition: **Name** / **Digits** / **What it means** ten / 10 / 1 ten twenty / 20 / 2 tens thirty / 30 / 3 tens forty / 40 / 4 tens fifty / 50 / 5 tens sixty / 60 / 6 tens seventy / 70 / 7 tens eighty / 80 / 8 tens ninety / 90 / 9 tens one hundred / 100 / 10 tens

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talk in full sentences to explain their answers, for example: ‘I have five groups of ten cubes. I have fifty cubes.’

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Moving between representations:

‘Show me nine tens.’
‘In teams, show me forty fingers.’
‘Show me…’ 60 (written out as numerals)
‘Draw a picture to represent these numbers’:

<table>
<tr><th>twenty</th><th>forty</th><th>fifty</th></tr>
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‘Complete the table.’

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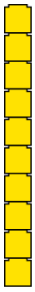
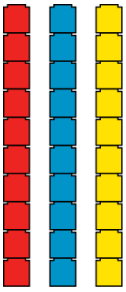
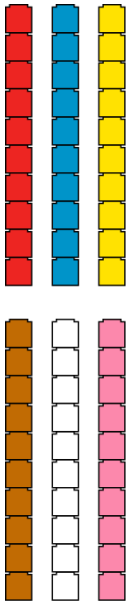
<tr><td>  </td><td>  </td><td>  </td></tr>
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<td>2:7</td>
<td>Now show pictures of multiples of ten, and ask children to represent each number with an expression in terms of repeated addition of tens. Then ask children to complete repeated addition equations. Children may find it difficult to recognise that ten tens are equal to 100, so make sure you include questions that enable this concept to be practised.</td>
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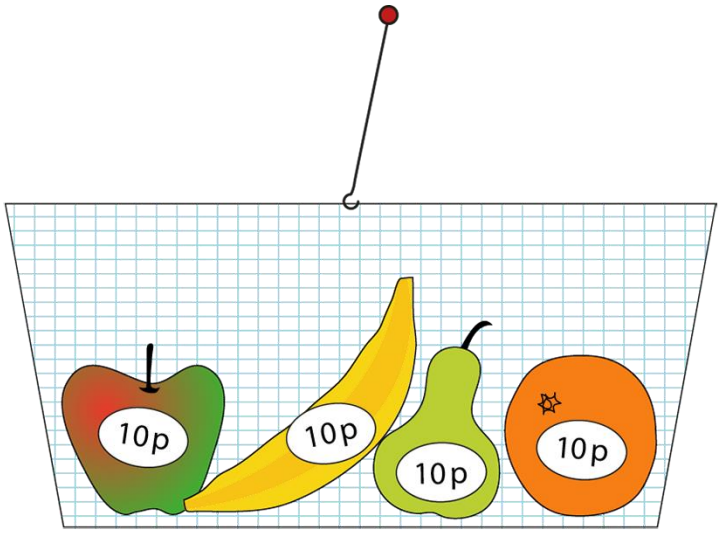
'Write an expression to represent this number.'

10 + 10 + 10 + 10 + 10 + 10 + 10

'Write an expression to represent the cost of the fruit.'



10 p + 10 p + 10 p + 10 p

'Complete the equations.'

10 + 10 + 10 = ☐

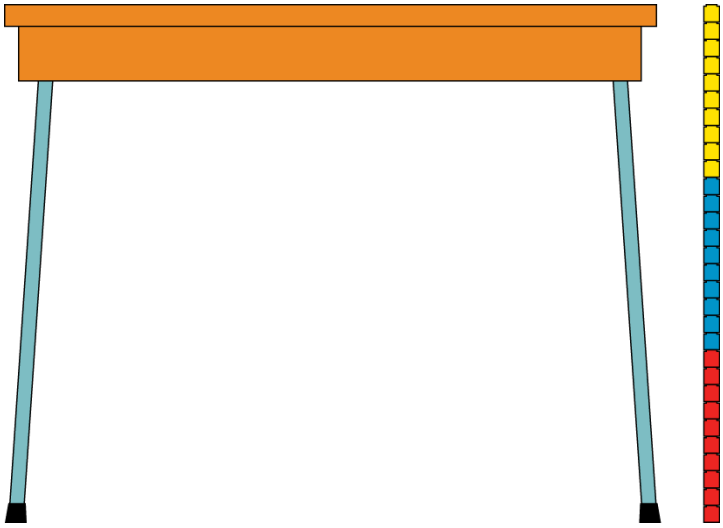
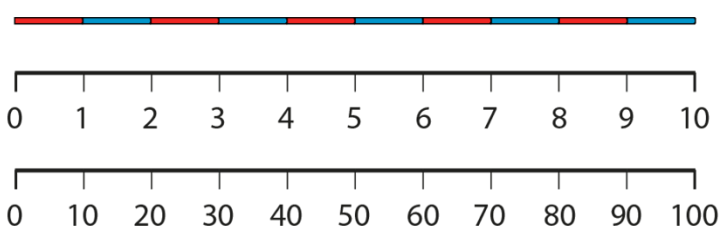
50 = 10 + 10 + _____

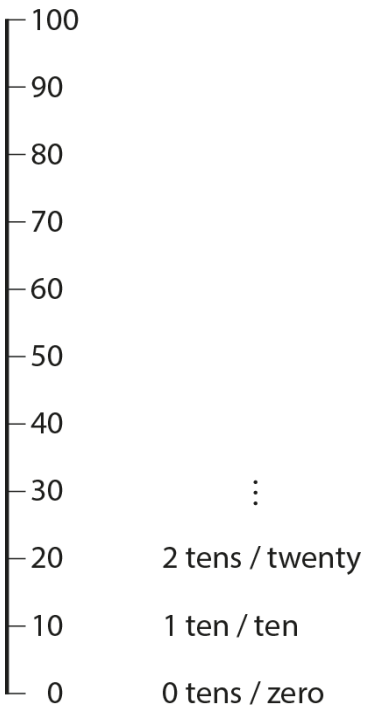
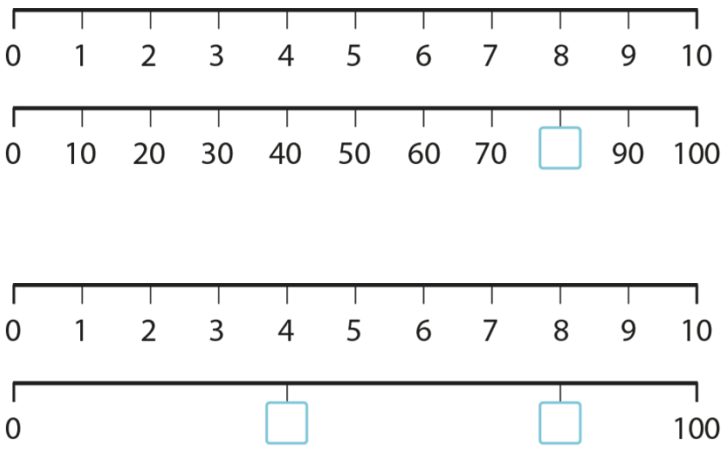
2:8	To finish this teaching point, use dòng nǎo jīn problems, such as: • *'Each piece of fruit costs ten pence. If I spent sixty pence, how many pieces of fruit did I buy?'* • *'How many children are needed to show seventy fingers?'* • *'Eggs are packed in boxes of ten. How many boxes do I need to buy so that I have twenty eggs altogether?'* These questions are, in fact, early examples of division (at this stage do not explicitly draw attention to this). The ability to solve these types of problem reveals that children are developing deep links between number and all four operations.	

Teaching point 3:

Knowledge of the 0–10 number line can be used to estimate the position of multiples of ten on a 0–100 number line.

Steps in learning

	Guidance	Representations
3:1	In order to help children to develop an understanding of the relative size of these numbers, begin with the familiar sticks of ten cubes, joined together: • Ask children to measure classroom objects using the sticks to get a feel for the relative size of the numbers. • Practise dual counting again. • Show a tower representing a multiple of ten and ask *'What's the next multiple of ten?'* and *'What's the previous multiple of ten?'* After some practice with the sticks, ask children to complete missing number sequences for the multiples of ten.	• *'About how many towers tall is the table?'* • *'How many tens is this? How many cubes is this?'*  • *'What's the next multiple of ten?'* • *'What's the previous multiple of ten?'*  *'Fill in the missing numbers.'* 0 | | 20 | 30 | | 50 | | 70
3:2	Children will already be familiar with the 0–10 number line (see, for example, segment *1.4 Composition of numbers: 6–10*). Now introduce the 0–100 number line alongside a set of ten multilink tens sticks, and the 0–10 number line. Ask children to compare the two number lines, identifying the similarities and differences. Draw attention to the link between the unit size of one and the unit size of ten. It is important to show both horizontal and vertical number lines	Introducing the 0–100 number line: 

	to develop children's conceptual understanding (conceptual variation), and to help them make a link to measures in various contexts. The Gattegno chart is a particularly useful representation here, since it draws parallels between the numbers 1–9 and the numbers 10– 90. Continue to practise dual counting – now at the same time as tapping the number line or the Gattegno chart.	Dual counting with the number line:  Gattegno chart: 1000 2000 3000 4000 5000 6000 7000 8000 9000 100 200 300 400 500 600 700 800 900 10 20 30 40 50 60 70 80 90 1 2 3 4 5 6 7 8 9
3:3	In segment *1.4 Composition of numbers: 6–10*, children practised estimating and placing numbers on a 0–10 number line. Now, using the 0–10 number line as a scaffold as shown opposite, ask children to: • identify missing numbers on the 0–100 number line • place given numbers on to the 0–100 number line.	*'Fill in the missing numbers.'* 

		'Mark the numbers twenty and sixty on the number line.' 
3:4	Now present similar problems, but remove the scaffold of the 0–10 number line. When children are identifying or placing numbers on the number line, ensure they are considering the positions relative to *both* the 0 and the 100 (for more information see segment *1.4 Composition of numbers: 6–10, Teaching point 2*); look out for children who are counting up in tens to place the larger numbers and model how to work relative to both ends of the number line. Teach children to first mark on the midpoint (50), before estimating the position of other numbers. After each estimation, reveal the full set of numbers on the number line to support checking. There is no need for children to be completely accurate with their placements; rather they should be making reasonable judgements that demonstrate they are developing proportional thinking.	*'Estimate where each of the numbers lie on the number line.'*  *'Which multiple of ten is this?'* 
3:5	To provide further challenge, use a dòng nǎo jīn problem: ask children to estimate the position of numbers on open-ended number lines with different multiples of ten marked as reference points.	*'Where does seventy lie on this number line?'*  *'Where does forty lie on this number line?'* 

3:6

Finally, look at how multiples of ten up to 100 can be compared using either understanding of the cardinality (size) of the number, or ordinality (position of the number in the number system).

Provide opportunities for children to compare multiples of ten using concrete resources, then build towards more abstract representations, including the number line, and missing number/symbol problems.

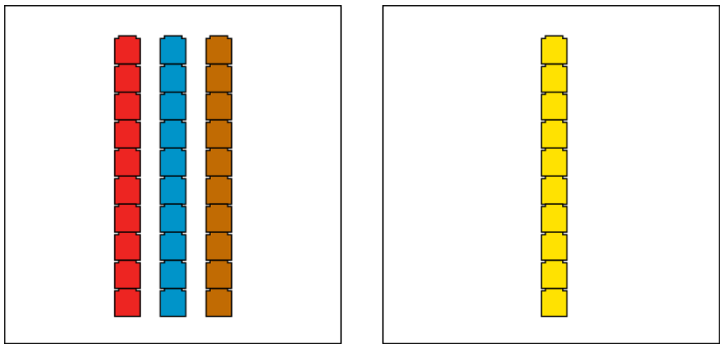
Use comparative language ('greater than', 'less than', and 'equal to'), before moving to use of the symbols >, < and =.

The missing symbol problems opposite use procedural variation to develop children's understanding. Initially, use examples that compare numbers written in the same way (i.e. using number names, number of tens, or numerals for both of the numbers being compared) before moving to examples where children need to work with more than one representation. Also include examples that will draw out potential misconceptions, for example, when comparing 40 with 7 tens, children may just look at the number of digits and say that 40 is larger than 7 tens.

You can provide number lines or the Gattegno chart to support children as they work through problems.

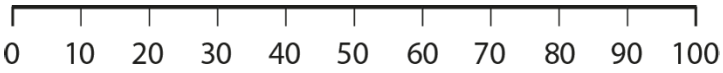
Comparing using cardinality:

- *'Three tens are greater than one ten.'*
- *'Thirty is greater than ten.'*



Comparing using ordinality:

'Teddy says that forty is less than sixty. Is he right? Convince me.'



Missing symbol/number problems:

- *'Fill in the missing symbols.'* (<, >, =)

6 tens ◯ 8 tens

40 ◯ 2 tens

90 ◯ 80

40 ◯ 7 tens

10 + 10 ◯ 10 + 10 + 10 + 10

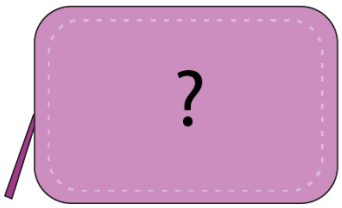
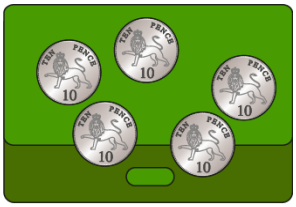
40 ◯ 10 + 10 + 10 + 10

7 tens ◯ 30

100 ◯ 10 tens

- *'Fill in the missing number. Explain why you made your choice.'*

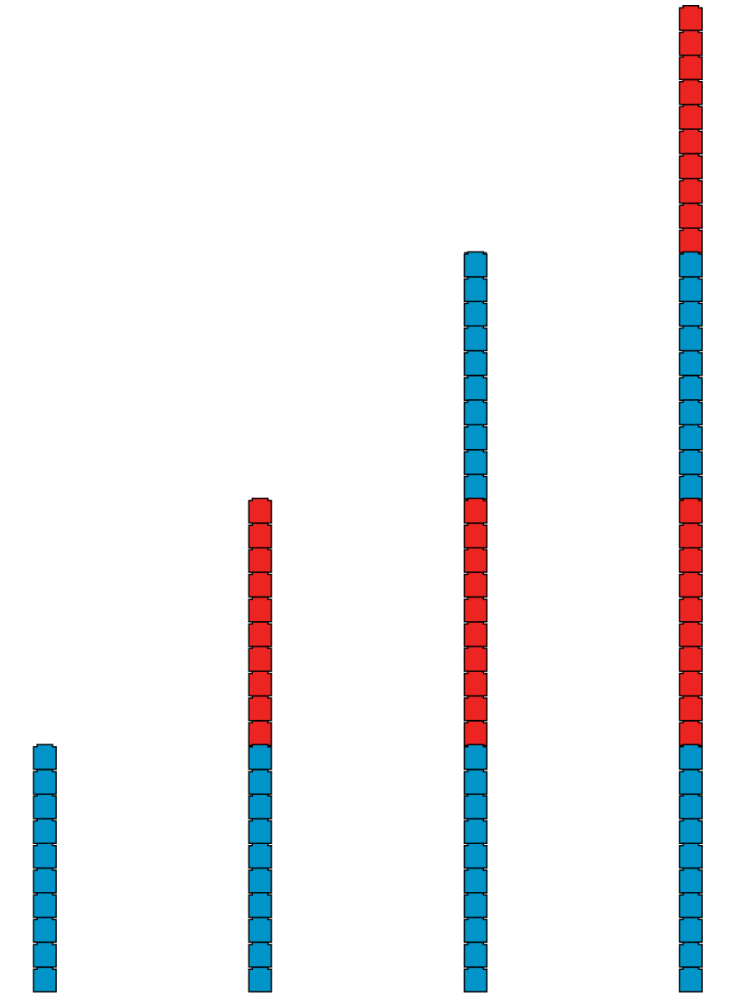
3 tens > ☐

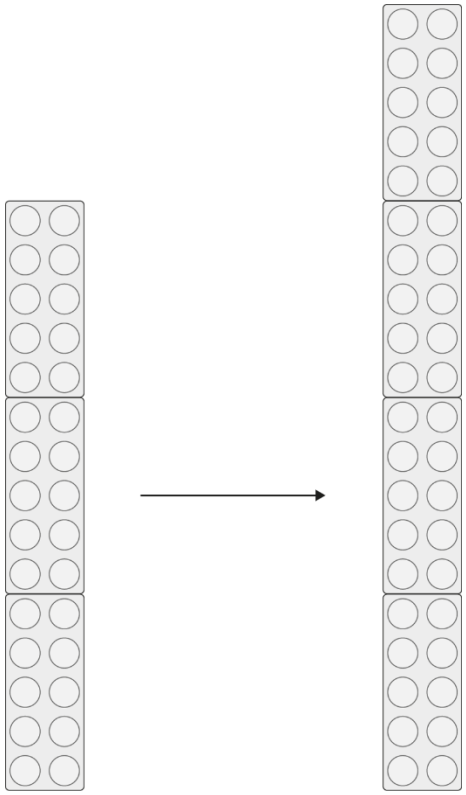
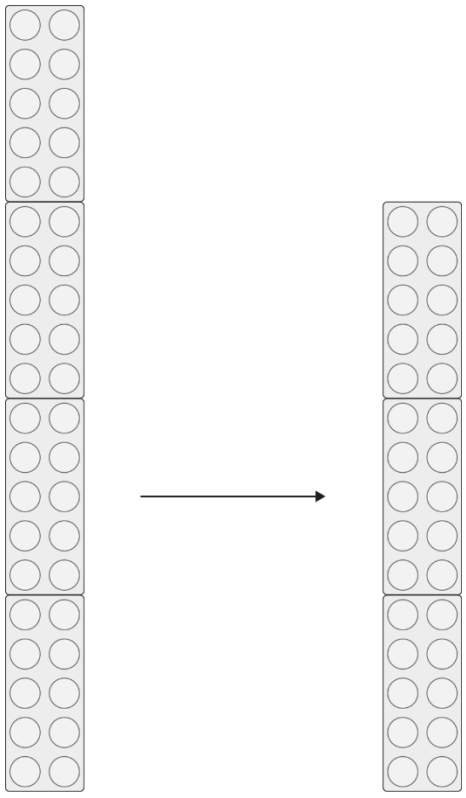
3:7	Use a dòng nǎo jīn question to promote and assess depth of understanding, such as: *'I'm thinking of a number. It is greater than twenty and less than six tens. What could it be?'* Then ask children to create similar problems of their own.	*'Sarah and Deb have some ten-pence coins. Sarah has fewer coins than Deb. If Deb has fifty pence, how much money might Sarah have?'* Sarah's money  Deb's money 

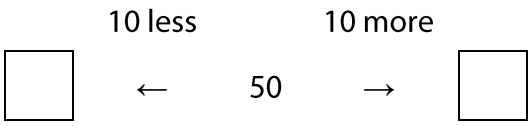
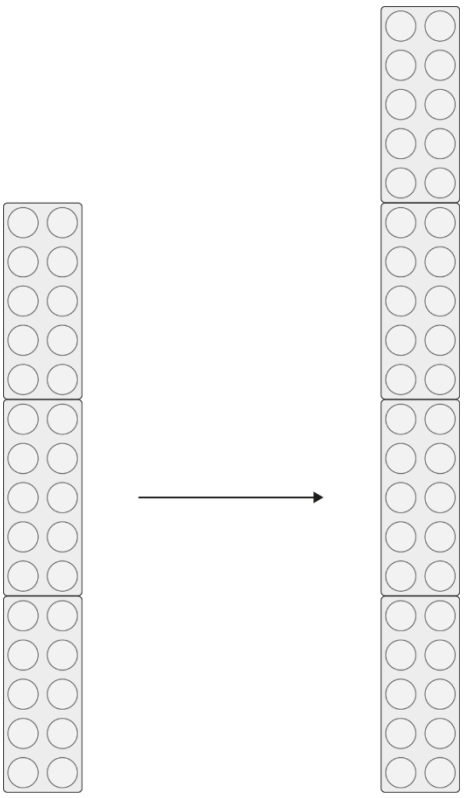
Teaching point 4:

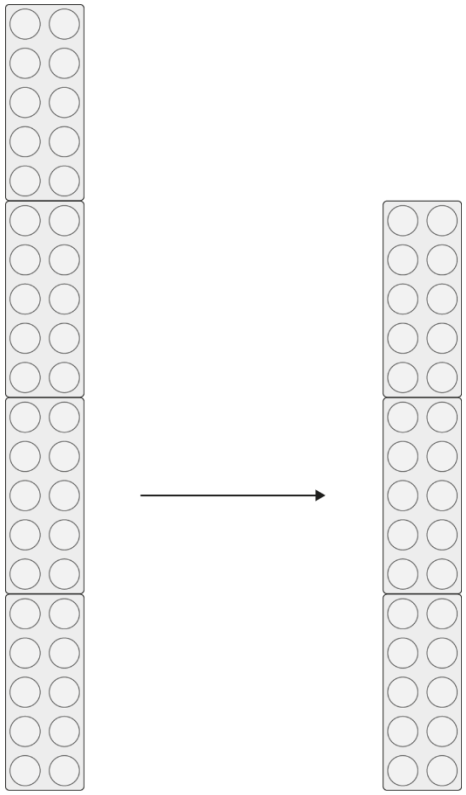
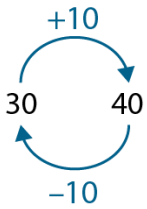
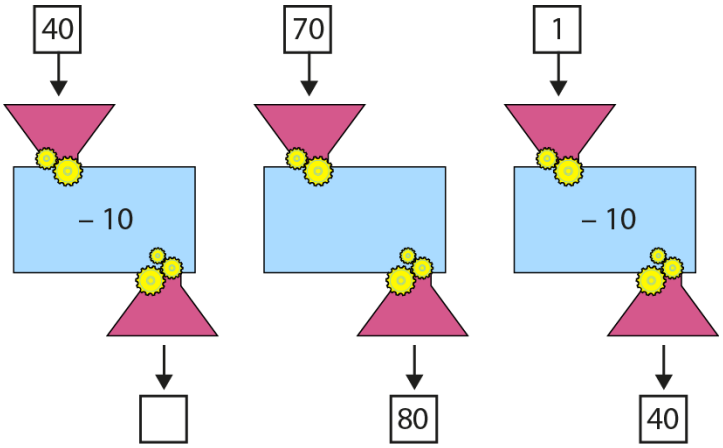
Adding ten to a multiple of ten gives the next multiple of ten; subtracting ten from a multiple of ten gives the previous multiple of ten.

Steps in learning

	Guidance	Representations
4:1	Begin by considering 'ten more' and 'ten less' using the towers representation from step *3:1*. Place towers side-by-side in increasing multiples of ten and ask children to look for, and describe, patterns in the sequence of numbers. It is important for children to recognise that the numbers are increasing and decreasing by the same amount each time. Move forwards then backwards through the sequence. Beginning with multiples of ten language, ask children questions to draw attention to the difference of ten between consecutive multiples of ten, for example: • *'This is one ten. How many more do we need to make two tens?'* • *'This is four tens. How many fewer to make three tens?'* Then progress to using number names: • *'This is ten. How many more do we need to make twenty?'* • *'This is forty. How many fewer to make thirty?'* Then muddle up the sequence of towers and challenge children to rearrange them in either increasing or decreasing multiples of ten. Draw attention to *'ten more, ten more, ten more…'* as you move through the sequence of increasing multiplies of ten. Similarly, draw attention to *'ten less, ten less, ten less…'* as you move through the sequence of decreasing multiplies of ten.	 1 ten ten 10 2 tens twenty 20 3 tens thirty 30 4 tens forty 40

4:2	When children are confident moving through the sequence of multiples of ten, choose *any* multiple of ten and ask how many more/fewer is needed to make the next/previous multiple of ten. Vary the representations (for example, bundles of ten straws, tens frames, ten-pence coins, boxes of eggs). Repeatedly add and then remove an additional ten, using the stem sentences: • ***'This is ___. Ten more than ___ is ___.*** ***___ is ten more than ___.'*** • ***'This is ___. Ten less than ___ is ___.*** ***___ is ten less than ___.'***	 • *'This is thirty. Ten more than thirty is forty.'* • *'Forty is ten more than thirty.'*  • *'This is forty. Ten less than forty is thirty.'* • *'Thirty is ten less than forty.'*

4:3	Once children have had plenty of experience using resources where the tens can be manipulated, progress to use of a more abstract representation such as the number line or Gattegno chart. With the chosen representation as a scaffold, ask children to identify ten more and ten less than a given multiple of ten.	
4:4	In time children need to be able to move away from the scaffold of the manipulatives and the number line/Gattegno chart and be able to give ten less or ten more than any multiple of ten within 100. Use missing number problems for practice.	
4:5	In preparation for writing equations, progress to using the vocabulary of 'plus' and 'minus' in the stem sentences. Ensure children understand that the term 'plus' links to ten more and the term 'minus' links to ten less. Initially, use concrete or pictorial resources to support these links.	 *'Thirty plus ten is forty.'*

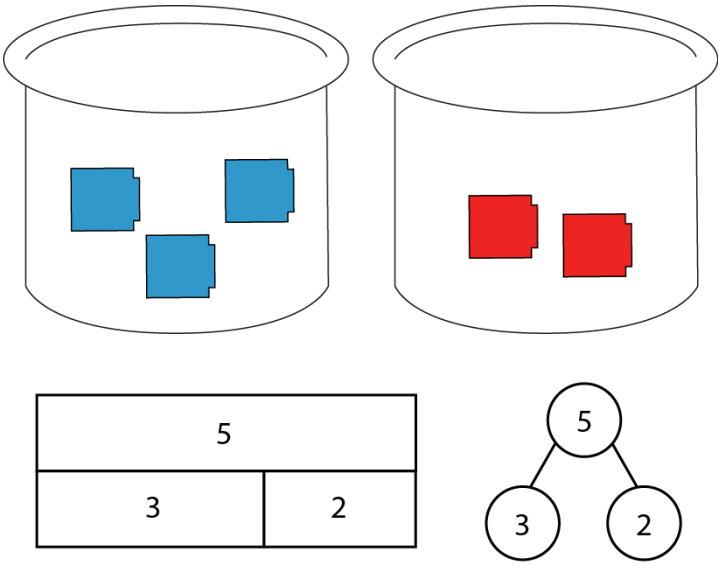
		 *'Forty minus ten is thirty.'*
4:6	Demonstrate how the connection between 'plus ten' and 'minus ten' can be represented using diagrams like the one opposite (top), or a function machine (bottom). Provide diagrams or function machines with missing numbers/operations, varying the unknown.	Relationship between '+ 10' and '– 10':  Missing number/operation problems: *'Fill in the empty boxes.'* 

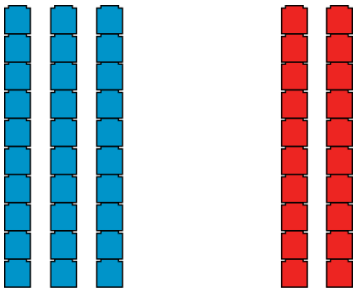
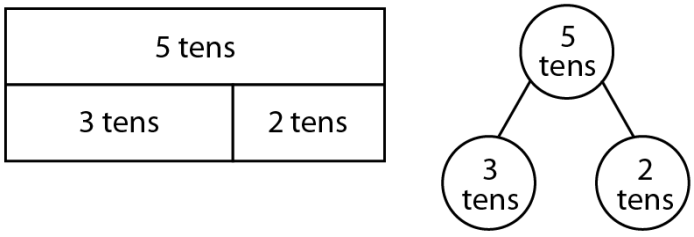
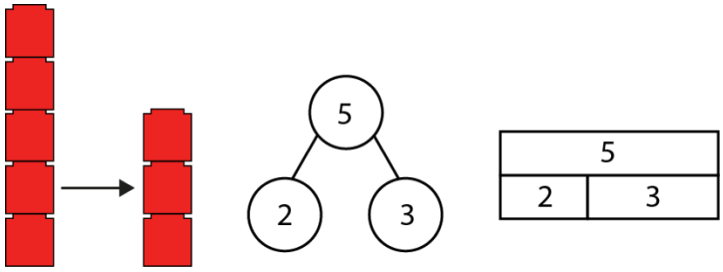
4:7	Finally, provide varied practice for the addition and subtraction of ten.	*Fill in the missing numbers.* 10 + 10 = ☐ 100 – 10 = ☐ 20 + 10 = ☐ 90 – 10 = ☐ 30 + 10 = ☐ 80 – 10 = ☐ …etc. …etc. ☐ + 10 = 20 ☐ – 10 = 10 ☐ + 10 = 30 ☐ – 10 = 30 ☐ + 10 = 50 ☐ – 10 = 50 70 = 60 + ☐ 70 = ☐ – 10 70 = ☐ + 10 90 = ☐ – 10

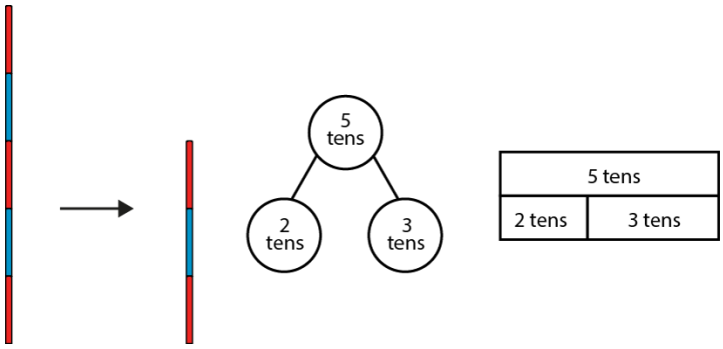
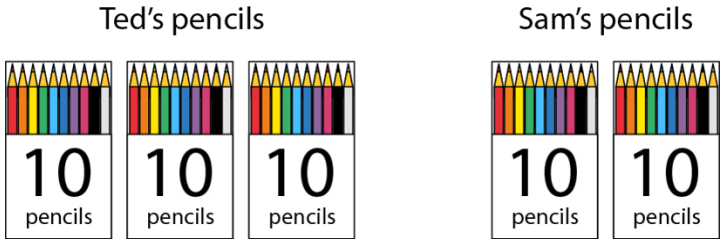
Teaching point 5:

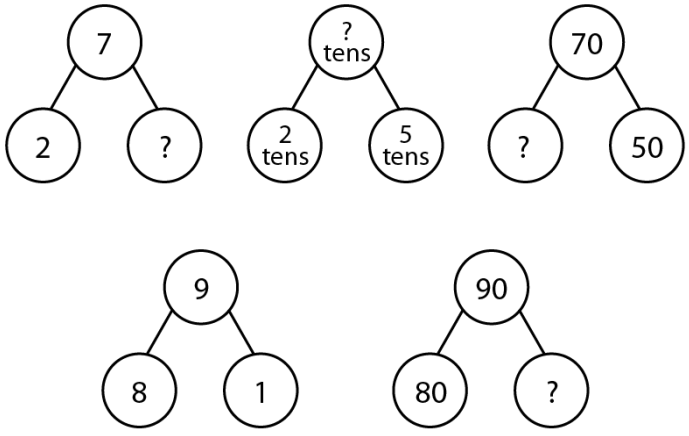
Known facts for the numbers *within* ten can be used to add and subtract in multiples of ten by unitising.

Steps in learning

	Guidance	Representations
5:1	In steps *5:1–5:5*, we make use of children's knowledge of addition and subtraction facts to ten, and their ability to unitise, to add and subtract *groups* of ten, for example, 5 tens + 3 tens = 8 tens. Step *5:6* then progresses to use of number names and numerals to represent the multiples of ten. Throughout the entire teaching point, use a range of familiar representations of ten, as introduced in the previous teaching points. Initially, pose a familiar single-digit aggregation problem, for example 'three blue cubes plus two red cubes'. Ask children to represent the problem using concrete resources, part–part–whole models (cherry and bar-model representations) and equations. From their work in previous segments, children should be very secure in doing this. Reinforce the language associated with the calculation, as in previous segments, by ensuring that children talk in full sentences and describe what each number represents.	*'I have three blue cubes and two red cubes. How many cubes do I have altogether?'*  3 + 2 = 5 • *'Three blue cubes plus two red cubes equals five cubes altogether.'* • *'The 3 represents the three blue cubes.'* • *'The 2 represents the two red cubes.'* • *'The 5 represents the total number of cubes.'*
5:2	Move to presenting a similar problem, this time involving tens. Use the same number of tens as you used ones in the previous step. Ask children to describe what they see, and compare this problem with the previous one. Draw attention to the fact that in the previous step there were three *ones* and now there are three *tens*; the size of the unit has	

	changed. Similarly, for the other addend, before there were two *ones*, and now there are two *tens*. Represent this calculation on a part–part–whole diagram and as an equation, expressing the quantities as 'three tens' and 'two tens' for now and explicitly linking this calculation to the previous one: • *'We know that three plus two is equal to five.'* 3 + 2 = 5 • *'So three tens plus two tens is equal to five tens.'* 3 tens + 2 tens = 5 tens Through previous segments, children will have had many opportunities to talk about problems requiring the addition and subtraction of the same nouns, for example, '*Three cars plus two cars is equal to five cars.*' Adding tens by unitising applies the same principle; the noun is now 'tens'.	*'I have three tens and two tens. How many tens do I have altogether?'*  *'What's the same and what's different between this and the last example?'*  3 tens + 2 tens = 5 tens *'Three tens plus two tens is equal to five tens.'*
5:3	Provide varied practice using different representations of ten. Throughout, ensure children continue to use single-digit number facts applied to units of ten, using the following stem sentences: • ***'I know that ___ plus ___ is equal to ___.'*** • ***'So, ___ tens plus ___ tens is equal to ___ tens.'***	 • *'I know that two plus five is equal to seven.'* • *'So, two tens plus five tens is equal to seven tens.'*
5:4	Now explore subtraction of multiples of ten in the same way, representing the calculation with concrete resources, part–part–whole models and equations: • Begin with a single-digit subtraction calculation; a reduction structure using multilink cubes works well here. • Then present the analogous problem in units of ten.	Single-digit calculation:  5 – 2 = 3

	• Provide practice with a range of representations. Continue to use stem sentences to link the calculation in units of ten with the corresponding single-digit calculation: • ***'I know that ___ minus ___ is equal to ___.'*** • ***'So, ___ tens minus ___ tens is equal to ___ tens.'***	Tens calculation:  5 tens – 2 tens = 3 tens • *'I know that five minus two is equal to three.'* • *'So, five tens minus two tens is equal to three tens.'*
5:5	Due to their previous experience and knowledge of numbers to ten, children should be able to move quickly to working with missing number problems, without concrete/pictorial resources for scaffolding. As previously, vary the position of the empty boxes and the equals sign.	*'Fill in the missing numbers.'* 8 tens + 2 tens = ☐ tens 6 tens – ☐ tens = 2 tens 5 tens = ☐ tens + 2 tens
5:6	Now progress to using the names and numerals for the multiples of ten. Present a concrete or pictorial real-world context, such as that shown opposite. Maintaining the link to calculations within ten, and using the concepts of dual-counting learnt earlier, discuss the different ways that the problem can be represented, using the following progression: • *'Three boxes plus two boxes is equal to five boxes.'* 3 boxes + 2 boxes = 5 boxes 3 + 2 = 5 • *'Three tens plus two tens is equal to five tens.'* 3 tens + 2 tens = 5 tens • *'Thirty plus twenty is equal to fifty.'* 30 + 20 = 50	*'How many pencils do Ted and Sam have altogether?'*  3 + 2 = 5 3 tens + 2 tens = 5 tens 30 + 20 = 50 *'Ted and Sam have fifty pencils altogether.'*

	Ensure that children can explain what each number in the equation represents, for example: • *"3 tens' represents the number of pencils that Ted has. '2 tens' represents the number of pencils that Sam has. '5 tens' represents the total number of pencils.'* • *'The 30 represents Ted's pencils. The 20 represents Sam's pencils. The 50 represents the total number of pencils.'* Also use a range of familiar generalised representations to describe the context and provide further practice, including: • tens frames • base-ten number boards • Dienes • ten value place-value counters • part–part–whole diagrams (cherry or bar model).	
5:7	Provide practice completing missing number problems that draw attention to the connection between numbers to ten and multiples of ten. At this stage, children should be able to work without concrete representations, drawing solely on their fluency with single-digit numbers.	Part–part–whole diagrams:  *'Fill in the missing numbers.'* 9 = 8 + 1 90 = ☐ + 10 9 = 2 + 7 90 = ☐ + 70 9 = 3 + 6 90 = ☐ + 60

5:8

Now provide a range of practice, without the single-digit scaffolding.

Include:

- real-world contexts, using a mixture of aggregation, partitioning, augmentation and reduction structures, for example:
  - *'Ted has thirty pence and Sam has forty pence. How much do they have altogether?'* (aggregation)
  - *'The teacher buys sixty pencils. If twenty of them are red, and the rest are blue, how many blue pencils are there?'* (partitioning)
  - *'Sam has forty pence and her mum gives her another fifty pence. How much does she have now?'* (augmentation)
  - *'I have eighty centimetres of ribbon and use sixty centimetres of it to wrap a present. How much do I have left?'* (reduction)
- part–part–whole models with missing numbers (varying the position of the missing number)
- equations with missing numbers (varying the position of the missing numbers, the order or addends, and the position of the equals sign).

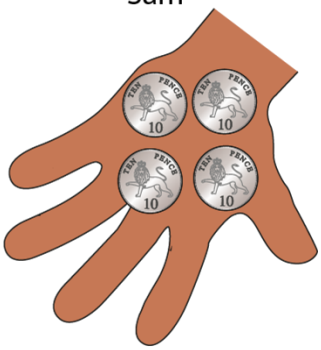
Throughout, when writing an equation to represent a contextual problem, make sure children describe the meaning of each number in the equation. Also ensure that children give their answers to word problems using full sentences.

Real-world context:

'Ted has thirty pence and Sam has forty pence. How much do they have altogether?'

Ted

Sam



3 tens + 4 tens = 7 tens

30 p + 40 p = 70 p

Varied fluency practice:

20 + 40 = ☐ 80 – 40 = ☐

30 + 40 = ☐ 50 – 40 = ☐

10 + 90 = ☐ 90 – 10 = ☐

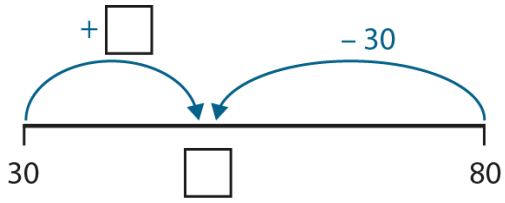
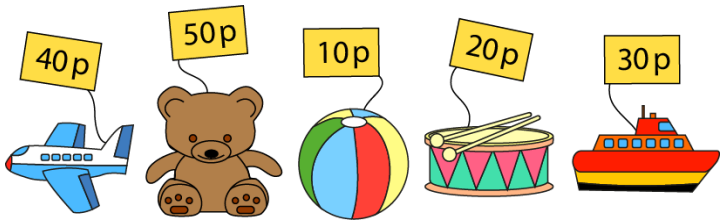
20 + 70 = ☐ 70 – 30 = ☐

20 + 40 = ☐

50 = ☐ – 10

☐ = 70 – 30

☐ + 30 = 90

		Dòng nǎo jīn: 
5:9	To complete this segment, you can bring together a range of concepts covered, using the context of a simple shop, in which each item costs a multiple of ten up to 50 pence. Limiting the cost of the items in this way will ensure that children work within 100 pence. (Note: it is not necessary at this stage for children to equate 100 pence with £1.) Ask questions that require children to compare, add, and subtract multiples of ten, such as those shown opposite. Note: buying three toys could take the children to a total cost greater than £1.	 • *'Which is the most expensive toy?'* • *'Which toys cost more than the drum?'* • *'Put the toys in order from cheapest to most expensive?'* • *'Buy two toys and calculate the total cost.'* • *'Which two toys would cost the most money?'* • *'I bought two toys which cost ninety pence in total. What did I buy?'* • *'I had eighty pence and now have sixty pence. What could I have bought? Can you find more than one answer?'* • *'I bought two of the same toy and spent sixty pence. What did I buy?'*