MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

# Mastery Professional Development

*Multiplication and Division*

## 2.11 Times tables: 11 and 12

Teacher guide | Year 4

### Teaching point 1:

The distributive law can be used to build up the 11 times table by partitioning 11 into 10 and 1. Adjacent multiples of 11 have a difference of 11.

### Teaching point 2:

The distributive law can be used to build up the 12 times table by partitioning 12 into 10 and 2. Adjacent multiples of 12 have a difference of 12.

### Teaching point 3:

Products in the 12 times table are double the products in the six times table; products in the six times table are half of the products in the 12 times table.

### Teaching point 4:

Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by 11 or 12.

## Overview of learning

In this segment children will:

- build on segment *2.10 Connecting multiplication and division, and the distributive law*, applying the distributive law when 11 is a factor by partitioning 11 into 10 and 1, working systematically to build up the 11 times table
- practise skip counting in multiples of 11 and reciting the 11 times table
- explore patterns in the 11 times table
- use the 11 times table to solve:
  - multiplication problems about groups of 11
  - multiplication problems about 11 equal groups
  - both quotitive and partitive division problems with a divisor of 11
- work through the same sequence of steps for the 12 times table
- explore links between the 6 and 12 times tables, applying knowledge of doubling and halving
- learn and apply divisibility rules for 11 and 12.

Teachers are encouraged to continue building up the class multiplication chart as each times table is covered (first introduced in *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*):

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **1** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| **2** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| **3** | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| **4** | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| **5** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| **6** | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| **7** | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| **8** | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| **9** | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| **10** | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| **11** | 0 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| **12** | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

Key: 'new' facts
previously learnt facts
relevant previously learnt facts (commutativity)


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The chart should be used to help children to see that there are very few truly 'new' facts to be learnt. However, in order for children to become fluent with these times tables and to gain a deeper understanding of the patterns within them, the 11 and 12 times tables are still built up 'from scratch' in the segment. Regular practice will be needed both in reciting the times tables (for example, '*One eleven is eleven, two elevens are twenty-two…*') and with isolated multiplication facts (for example, *'I know that seven times eleven is equal to seventy-seven.*').

*Teaching points 1 and 2* differ from other times-table teaching points in previous segments; instead of building up the 11 and 12 times tables by skip counting in multiples of 11 or 12, these times tables are built up by systematically applying the distributive law, relating the 'new' facts to known facts in the ten and one/two times tables. Skip counting and chanting is then used to develop fluency with the resulting times tables. Applications and practice then follow a similar format to that in previous segments. Exploration of the connection between the 12 times table and the 6 times table also follows a similar progression to that used for similar learning in previous segments.

*An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast:* www.ncetm.org.uk/primarympdpodcast*. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.*


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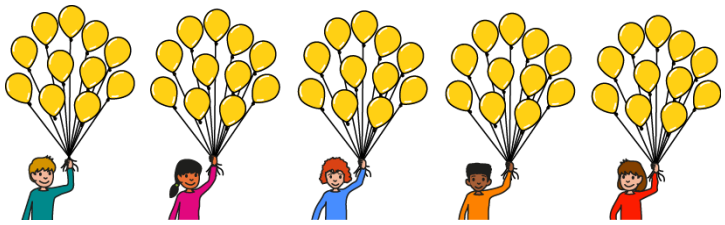
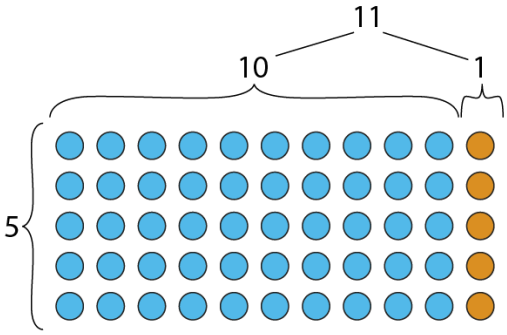
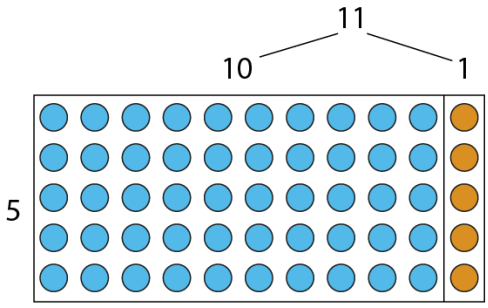
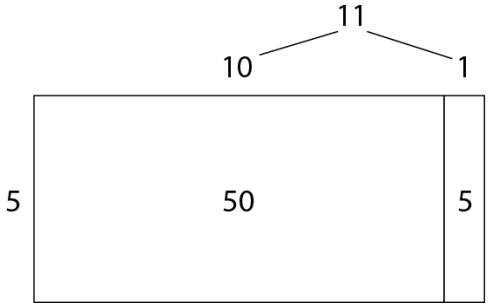
## Teaching point 1:

The distributive law can be used to build up the 11 times table by partitioning 11 into 10 and 1. Adjacent multiples of 11 have a difference of 11.

Steps in learning

| | Guidance | Representations |
|---|---|---|
| **1:1** | Children have already learnt how the distributive law can be used to derive multiplication facts where one of the factors is a teen number (segment *2.10 Connecting multiplication and division, and the distributive law, Teaching point 3*). They also learnt to apply the distributive law irrespective of whether the factor being partitioned represents the group size or the number of groups.<br>Since children are used to building up each times table based on group size (for example, building up the three times table based on groups of three), begin by presenting an example of groups of eleven, such as the five groups of eleven balloons shown opposite. Then ask children how we can work out the total number of balloons. Support them to apply the distributive law by partitioning eleven into ten and one, using the array/grid representation as in segment *2.10*. |  <br>$5 \times 11 =$ ☐ $11 \times 5 =$ ☐<br>  <br>  <br>  |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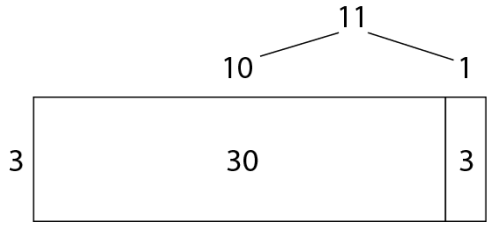
| | | |
|---|---|---|
| | | $5 \times 11 = 5 \times 10 + 5 \times 1$<br>$= 50 + 5$<br>$= 55$<br>So…<br>$5 \times 11 =$ 55 $\quad$ $11 \times 5 =$ 55<br>• *'Five groups of eleven is equal to fifty-five.'*<br>• *'Eleven, five times is equal to fifty-five.'* |
| **1:2** | Work through some different examples of groups of eleven, keeping the context the same as in step *1:1*.<br>Gradually remove the array/grid scaffold, until children are confident working with equations only. Vary the order of the factors, using a mixture of *11 × single-digit* and *single-digit × 11* examples. | Example 1 – less scaffolding, factor order varied:<br>  <br><br>$11 \times 3 = 10 \times 3 + 1 \times 3$<br>$= 30 + 3$<br>$= 33$<br><br>Example 2 – no scaffolding:<br>$7 \times 11 = 7 \times 10 + 7 \times 1$<br>$= 70 + 7$<br>$= 77$ |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| | | |
|---|---|---|
| **1:3** | Provide children with practice applying the distributive law when 11 is a factor. | Missing-number problems:<br>*'Fill in the missing numbers.'*<br>$9 \times 11 = 9 \times 10 + 9 \times 1 = \square + \square = \square$<br>$8 \times 11 = 8 \times \square + 8 \times 1 = \square + \square = \square$<br>$\square \times 11 = 3 \times 10 + 3 \times \square = \square + \square = \square$<br>$11 \times 2 = 10 \times 2 + 1 \times 2 = \square + \square = \square$<br>$11 \times 4 = \square \times 4 + 1 \times 4 = \square + \square = \square$<br>$11 \times 6 = 10 \times \square + 1 \times \square = \square + \square = \square$<br>True/false problems:<br>*'Use a tick or a cross to show whether each equation is correct or not.'*<br>$7 \times 11 = 7 \times 1 + 7 \times 10$<br>$5 \times 10 - 1 \times 5 = 5 \times 11$ |


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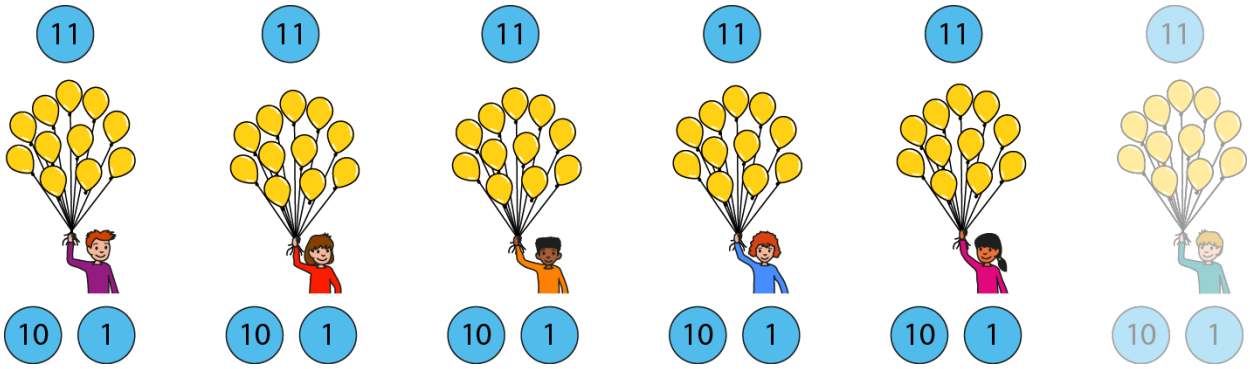
**1:4**

Now work systematically to build up the eleven times table using the distributive law and a ratio chart (below right). For each fact, write out the pair of resulting multiplication equations (below left).

At each stage:

- encourage children to describe what each equation represents, for example:
  - *'There are six groups of eleven balloons.'*
  - *'There are sixty-six balloons altogether.'*
  - *'The product of six and eleven is sixty-six.'*
- then add another eleven balloons, and work with children to complete the next column of the table using the distributive law.




| | |
|---|---|
| $0 \times 11 = 0$ | $11 \times 0 = 0$ |
| $1 \times 11 = 11$ | $11 \times 1 = 11$ |
| $2 \times 11 = 22$ | $11 \times 2 = 22$ |
| $3 \times 11 = 33$ | $11 \times 3 = 33$ |
| $4 \times 11 = 44$ | $11 \times 4 = 44$ |
| $5 \times 11 = 55$ | $11 \times 5 = 55$ |
| **$6 \times 11 = 66$** | **$11 \times 6 = 66$** |

- *'Six groups of eleven is equal to sixty-six.'*
- *'Six times eleven is equal to sixty-six.'*
- *'Eleven, six times is equal to sixty-six.'*
- *'Eleven times six is equal to sixty-six.'*

| Number of bunches of balloons | × 10 | × 1 | Total number of balloons (× 11) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 10 | 1 | 11 |
| 2 | 20 | 2 | 22 |
| 3 | 30 | 3 | 33 |
| 4 | 40 | 4 | 44 |
| 5 | 50 | 5 | 55 |
| **6** | **60** | **6** | **66** |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**1:5**

Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the chart/equations for support, for example:

- *'If there are nine bunches of balloons, how many balloons are there altogether?'*
- *'How many bunches are there if there are seventy-seven balloons?'*
- *'If the product is one hundred and ten, what are the factors?'*
- *'Why are eight times eleven and eleven times eight both equal to eighty-eight?'*

Complete ratio chart and eleven times table:

| | |
|---|---|
| $0 \times 11 = 0$ | $11 \times 0 = 0$ |
| $1 \times 11 = 11$ | $11 \times 1 = 11$ |
| $2 \times 11 = 22$ | $11 \times 2 = 22$ |
| $3 \times 11 = 33$ | $11 \times 3 = 33$ |
| $4 \times 11 = 44$ | $11 \times 4 = 44$ |
| $5 \times 11 = 55$ | $11 \times 5 = 55$ |
| $6 \times 11 = 66$ | $11 \times 6 = 66$ |
| $7 \times 11 = 77$ | $11 \times 7 = 77$ |
| $8 \times 11 = 88$ | $11 \times 8 = 88$ |
| $9 \times 11 = 99$ | $11 \times 9 = 99$ |
| $10 \times 11 = 110$ | $11 \times 10 = 110$ |
| $11 \times 11 = 121$ | $11 \times 11 = 121$ |
| $12 \times 11 = 132$ | $11 \times 12 = 132$ |

| Number of bunches of balloons | × 10 | × 1 | Total number of balloons (× 11) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 10 | 1 | 11 |
| 2 | 20 | 2 | 22 |
| 3 | 30 | 3 | 33 |
| 4 | 40 | 4 | 44 |
| 5 | 50 | 5 | 55 |
| 6 | 60 | 6 | 66 |
| 7 | 70 | 7 | 77 |
| 8 | 80 | 8 | 88 |
| 9 | 90 | 9 | 99 |
| 10 | 100 | 10 | 110 |
| 11 | 110 | 11 | 121 |
| 12 | 120 | 12 | 132 |


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**1:6**

Now practise chanting the eleven times table, with the written times table for support, using a variety of representations, including:

- stacked number lines (as shown below)
- the Gattegno chart, tapping out the multiples of 11 as they are counted to reinforce the patterns (tap '10' then '1' as you count '11', tap '20' then '2' as you count '22' and so on)
- concrete representations
- pictorial representations.

Use the following language:

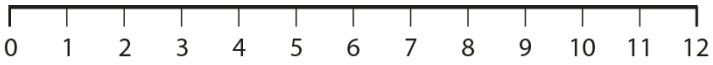
- *'One group of eleven is equal to eleven.'*
  *'Two groups of eleven is equal to twenty-two…'*
- *'One times eleven is equal to eleven.'*
  *'Two times eleven is equal to twenty-two…'*
  then shortening to
  *'One eleven is eleven, two elevens are twenty-two…'*

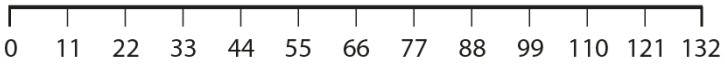
and

- *'Eleven, one time is equal to eleven…'*
  *'Eleven, two times is equal to twenty-two…'*
- *'Eleven times one is equal to eleven…'*
  *'Eleven times two is equal to twenty-two…'*

Regular practice should be undertaken, including outside the main maths lesson, until children are fluent. Also practise skip counting, both forwards and backwards in elevens, between zero and one hundred and thirty-two.

Number line:




Gattegno chart:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1000 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 | 8000 | 9000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |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**1:7**

Children are likely to already have spotted patterns in the 11 times table. Now discuss those patterns including the following:

- The products alternate between odd and even.
- Each two-digit multiple of 11 has equal digits (i.e. the tens and ones digits are the same); however, this pattern does not continue into the three-digit multiples.
- Working down the list, the product increases by eleven each time.

Focus in on the fact that adjacent multiples of 11 have a difference of 11, and that this knowledge can be used to find the next or previous multiple of 11 from a given multiple. Use the same representations as in earlier segments to illustrate this (ratio chart, number line, mixed-operation equations and arrays).

Remind children that:

- 11 can be added to a number by adding 10 then 1 (or 1 then 10)
- 11 can be subtracted from a number by subtracting 10 then 1 (or 1 then 10)

Show how this strategy can be used to help us recite the 11 times table/count in multiples of 11, and to find the next/previous multiple of 11 from a given multiple.

Finding adjacent multiples – ratio chart and number line:

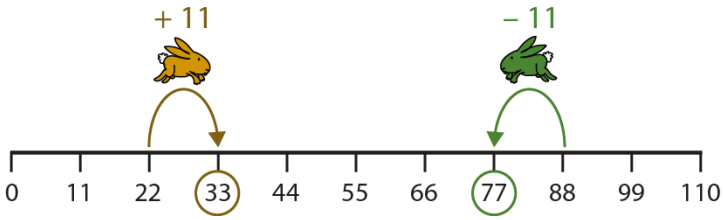
| | × 11 |
|---|---|
| **0** | 0 |
| **1** | 11 |
| **2** | 22 |
| **3** | |
| **4** | 44 |
| **5** | 55 |
| **6** | 66 |
| **7** | |
| **8** | 88 |
| **9** | 99 |
| **10** | 110 |
| **11** | 121 |
| **12** | 132 |

↓ + 11

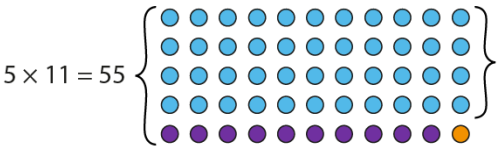
↑ −11

$3 \times 11 = 2 \times 11 + 11$
$= 22 + 10 + 1$

$7 \times 11 = 8 \times 11 - 11$
$= 88 - 10 - 1$




Finding adjacent multiples – array:




$4 \times 11 = 55 - 11$
$= 55 - 10 - 1$
$= 45 - 1$
$= 44$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| | | |
|---|---|---|
| **1:8** | The new facts for the 11 times table (*11* × *11* and *12* × *11*), along with *10* × *11*, can be challenging for children to memorise since they are three-digit multiples of 11 that depart from the 'same-digit pattern' (…88, 99, **110, 121, 132**). Work with children to generate strategies to help recall these facts, including:<br>• using the 'adjacent-multiples rule', for example:<br>$10 \times 11 = 9 \times 11 + 11$<br>$= 99 + 11$<br>$= 99 + 1 + 10$<br>$= 100 + 10$<br>$= 110$<br>• using the distributive law, partitioning 11 into 10 and 1 (as practised earlier in this teaching point). | |
| **1:9** | Provide children with varied practice:<br>• in the context of groups of 11<br>• in the context of 11 equal groups<br>• based on their knowledge that adjacent multiples of 11 have a difference of 11.<br>Include:<br>• missing-number sequences and problems, including division calculations with a divisor of 11<br>• missing-symbol problems (<, > or =)<br>• true/false style questions<br>• word problems, including measures contexts, for example:<br>  • *'There are eight football teams in a tournament. Each team has eleven players. How many players are there in the tournament?'* (multiplication; groups of 11)<br>  • *'How many players will there be if one of the teams drops out of the tournament?'*<br>  • *'There are eleven packs of biscuits. Each pack contains twelve biscuits. How many biscuits are there altogether?'* (multiplication; 11 equal groups)<br>  • *'There are sixty-six chairs for a concert. They are arranged in rows of eleven. How many rows are there?'* (quotitive division)<br>  • *'If £132 is shared equally between eleven people, how much money will each person get?'* (partitive division) | |


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- *'The Sunnyside football team has eleven players. At half time, they share some fruit. There are fourteen oranges, five apples and three bananas. If the players get the same number of pieces of fruit, how many do they each get?'*
  (addition, and partitive division)
- *'I have six trays with eleven cakes on each. Sarah takes half of the trays. How many cakes will I have left?'*
  (halving, and multiplication; groups of 11)
- *'Write a story to go with this equation.'*
  $6 \times 11 + 4$

At this stage, children can use the multiplication chart for reference. Plenty of practice will be needed over an extended period until children are fluent in the multiplication facts, particularly those discussed in step *1:8*, for which they may be tempted to over-apply the 'same digit' pattern.

Sequences and multiplication:

Missing-number sequences/problems:
*'Fill in the missing numbers.'*

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 11 | 22 | 33 | 44 | | | | | | | | |
| 132 | 121 | 110 | | | | | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 11 × | 1 | = | | 0 | | ×11 = | | |
| | 3 | | | 2 | | | | |
| | 5 | | | 4 | | | | |
| | 7 | | | 6 | | | | |
| | 9 | | | 8 | | | | |
| | 11 | | | 10 | | | | |
| | | | | 12 | | | | |

Dòng nǎo jīn:

- *'What do you notice about the pattern of odd and even numbers in the products of the eleven times table? How can you explain this?'*
- *'What is the relationship between these calculations?'*
  $2 \times 11 = ?$ $4 \times 11 = ?$ $8 \times 11 = ?$


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Multiplication and division:

Missing-number problems:
*'Fill in the missing numbers.'*

| | × 11 |
|---|---|
| 0 | 0 |
| 1 | 11 |
| 2 | |
| 3 | 33 |
| | 44 |
| 5 | |
| 6 | |
| | 77 |
| | 88 |
| | 99 |
| 10 | |
| | 121 |
| 12 | |

$8 \times \square = 88$

$11 \times \square = 77$

$99 = 11 \times \square$

$88 \div 11 = \square$

$77 \div 11 = \square$

$\square \div 11 = 9$

$66 \div 11 = \square$

$5 = \square \div 11$

$132 \div 11 = \square$

Dòng nǎo jīn:

*'True or false?'*

★ ÷ 11 = ♦ because ♦ × 11 = ★



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Problems on adjacent multiples/the distributive law:

*'Fill in the missing symbols (<, > or =).'*

$9 \times 11 \bigcirc 8 \times 11$

$9 \times 11 \bigcirc 8 \times 11 + 11$

$9 \times 11 \bigcirc 9 \times 11 + 11$

$9 \times 11 \bigcirc 10 \times 11 - 11$

$5 \times 11 \bigcirc 3 \times 11 + 2 \times 11$

$11 \times 4 - 11 \times 2 \bigcirc 11 \times 3$

$11 \times 11 - 1 \times 11 \bigcirc 11 \times 6 + 4 \times 11$

Dòng nǎo jīn:

*'Write an equation like this of your own.'*

Dòng nǎo jīn:

*'Fill in the missing numbers.'*

$20 \times 11 = 220$

so

$21 \times 11 = \square$

$11 \times 18 = \square \times 18 + 1 \times 18$

$= 180 + \square$

$= \square$



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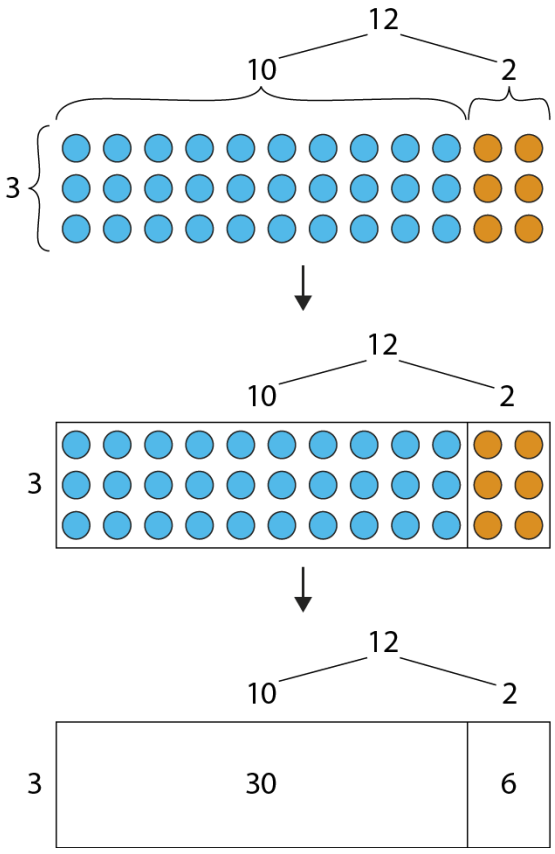
## Teaching point 2:

The distributive law can be used to build up the 12 times table by partitioning 12 into 10 and 2. Adjacent multiples of 12 have a difference of 12.

Steps in learning

| | Guidance | Representations |
|---|---|---|
| **2:1** | This teaching point follows the same progression as *Teaching point 1*. Guidance is summarised here, and representations are shown for the 12 times table. For more detail, please see the corresponding steps in *Teaching point 1*.<br><br>Begin by presenting an example of groups of 12, such as the three groups of 12 cans shown opposite. Ask children how many cans there are altogether. Support them to apply the distributive law by partitioning 12 into 10 and 2, using the array/grid representation as in step *1:1*. |  <br>$3 \times 12 =$ ☐ $12 \times 3 =$ ☐<br>  <br>$3 \times 12 = 3 \times 10 + 3 \times 2$<br>$= 30 + 6$<br>$= 36$ |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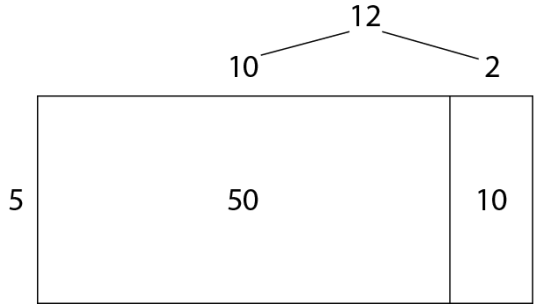
| | | |
|---|---|---|
| | | So…<br>$3 \times 12 =$ 36 $12 \times 3 =$ 36<br>• *'Three groups of twelve is equal to thirty-six.'*<br>• *'Twelve, three times is equal to thirty-six.'* |
| **2:2** | Work through some different examples of groups of 12, gradually removing the array/grid scaffold, until children are confident working with equations only. | Example 1 – less scaffolding, factor order varied:<br>  <br><br>$12 \times 5 = 10 \times 5 + 2 \times 5$<br>$= 50 + 10$<br>$= 60$<br>Example 2 – no scaffolding:<br>$8 \times 12 = 8 \times 10 + 8 \times 2$<br>$= 80 + 16$<br>$= 96$ |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| | | |
|---|---|---|
| **2:3** | Provide children with practice applying the distributive law when 12 is a factor. | Missing-number problems:<br>*'Fill in the missing numbers.'*<br>$7 \times 12 = 7 \times 10 + 7 \times 2 = \square + \square = \square$<br>$12 \times 12 = 12 \times 10 + 12 \times 2 = \square + \square = \square$<br>$\square \times 12 = 6 \times 10 + 6 \times \square = \square + \square = \square$<br>$12 \times 4 = 10 \times 4 + 2 \times 4 = \square + \square = \square$<br>$12 \times 9 = 10 \times 9 + 2 \times 9 = \square + \square = \square$<br>$12 \times \square = 10 \times 10 + 2 \times \square = \square + \square = \square$<br>True/false problems:<br>*'Use a tick or a cross to show whether each equation is correct or not.'*<br>$8 \times 12 = 8 \times 10 + 8 \times 2$<br>$10 \times 7 - 2 \times 7 = 12 \times 7$<br>$12 \times 11 = 11 \times 10 + 2 \times 11$ |


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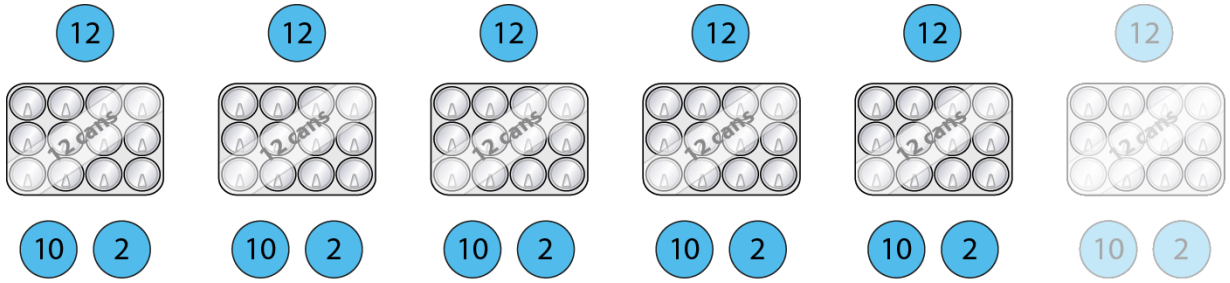
**2:4** Now work systematically to build up the 12 times table, in the same way as described in step *1:4*.




| | |
|---|---|
| $0 \times 12 = 0$ | $12 \times 0 = 0$ |
| $1 \times 12 = 12$ | $12 \times 1 = 12$ |
| $2 \times 12 = 24$ | $12 \times 2 = 24$ |
| $3 \times 12 = 36$ | $12 \times 3 = 36$ |
| $4 \times 12 = 48$ | $12 \times 4 = 48$ |
| $5 \times 12 = 60$ | $12 \times 5 = 60$ |
| **$6 \times 12 = 72$** | **$12 \times 6 = 72$** |

| Number of packs of cans | × 10 | × 2 | Total number of cans (× 12) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 10 | 2 | 12 |
| 2 | 20 | 4 | 24 |
| 3 | 30 | 6 | 36 |
| 4 | 40 | 8 | 48 |
| 5 | 50 | 10 | 60 |
| **6** | **60** | **12** | **72** |

- *'Six groups of twelve is equal to seventy-two.'*
- *'Six times twelve is equal to seventy-two.'*
- *'Twelve, six times is equal to seventy-two.'*
- *'Twelve times six is equal to seventy-two.'*

**2:5** Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the chart/equations for support, for example:

- *'If there are nine packs of cans, how many cans are there altogether?'*
- *'How many packs are there if there are ninety-six cans?'*
- *'If the product is one hundred and thirty-two, what are the factors?'*
- *'Why are seven times twelve and twelve times seven both equal to eighty-four?'*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Complete ratio chart and three times table:

| | |
|---|---|
| 0 × 12 = 0 | 12 × 0 = 0 |
| 1 × 12 = 12 | 12 × 1 = 12 |
| 2 × 12 = 24 | 12 × 2 = 24 |
| 3 × 12 = 36 | 12 × 3 = 36 |
| 4 × 12 = 48 | 12 × 4 = 48 |
| 5 × 12 = 60 | 12 × 5 = 60 |
| 6 × 12 = 72 | 12 × 6 = 72 |
| 7 × 12 = 84 | 12 × 7 = 84 |
| 8 × 12 = 96 | 12 × 8 = 96 |
| 9 × 12 = 108 | 12 × 9 = 108 |
| 10 × 12 = 120 | 12 × 10 = 120 |
| 11 × 12 = 132 | 12 × 11 = 132 |
| 12 × 12 = 144 | 12 × 12 = 144 |

| Number of packs of cans | × 10 | × 2 | Total number of cans (× 12) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 10 | 2 | 12 |
| 2 | 20 | 4 | 24 |
| 3 | 30 | 6 | 36 |
| 4 | 40 | 8 | 48 |
| 5 | 50 | 10 | 60 |
| 6 | 60 | 12 | 72 |
| 7 | 70 | 14 | 84 |
| 8 | 80 | 16 | 96 |
| 9 | 90 | 18 | 108 |
| 10 | 100 | 20 | 120 |
| 11 | 110 | 22 | 132 |
| 12 | 120 | 24 | 144 |

**2:6**

Now practise chanting the 12 times table, with the written times table for support, using a variety of representations, as described in step *1:6*.

Use the following language:

- *'One group of twelve is equal to twelve. Two groups of twelve is equal to twenty-four…'*
- *'One times twelve is equal to twelve. Two times twelve is equal to twenty-four…'*

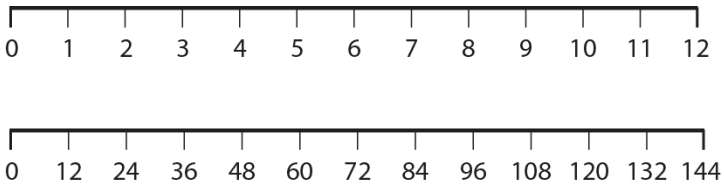
then shortening to

- *'One twelve is twelve, two twelves are twenty-four…'*

and

- *'Twelve, one time is equal to twelve.' 'Twelve, two times is equal to twenty-*

Number line:




Gattegno chart:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1000 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 | 8000 | 9000 |
| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |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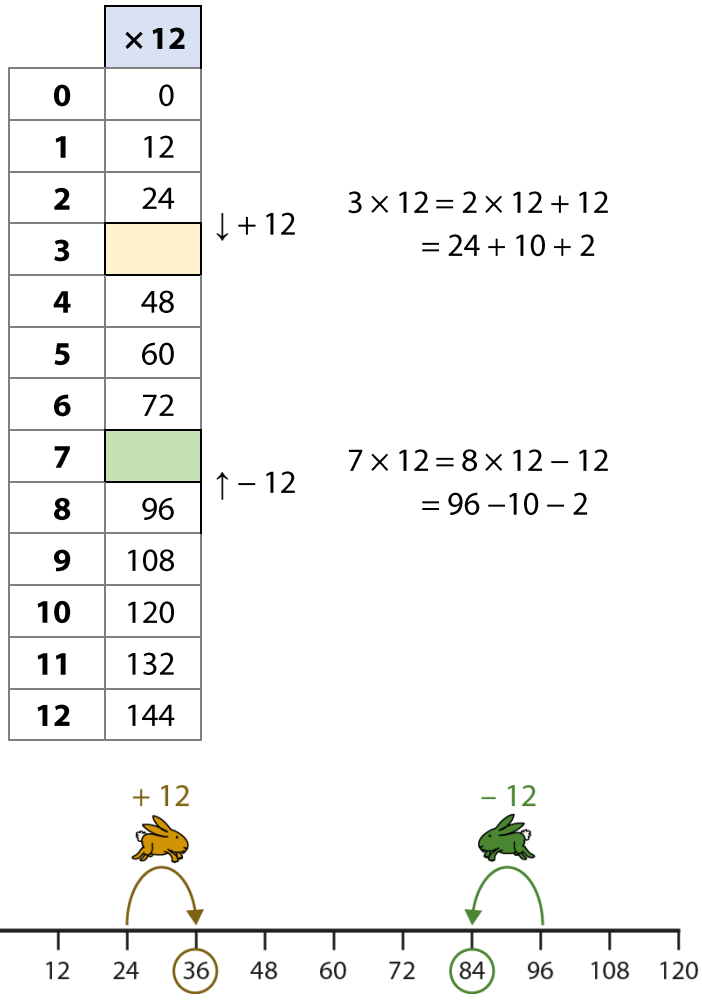
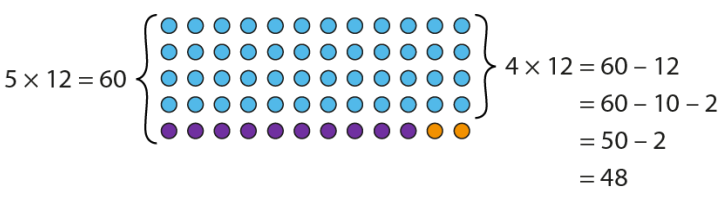
| | | |
|---|---|---|
| | *four…'*<br>• *'Twelve times one is equal to twelve.' 'Twelve times two is equal to twenty-four…'*<br>Regular practice should be undertaken, including outside the main maths lesson, until children are fluent.<br>Also practise skip counting, both forwards and backwards in 12s, between zero and 144. | |
| **2:7** | Now ask children what patterns they can see in the twelve times table, prompting for the following:<br>• The products are all even.<br>• The ones digit follows a pattern (0, 2, 4, 6, 8, 0, 2, 4…) (some children may notice that the tens digit follows a pattern too: 1, 1, 1, 1, 2, 1, 1, 1, 1, 2…).<br>• Working down the list, the product increases by twelve each time.<br>Focus in on the fact that adjacent multiples of 12 have a difference of 12.<br>Remind children that:<br>• 12 can be added to a number by adding 10 then 2 (or 2 then 10)<br>• 12 can be subtracted from a number by subtracting 10 then 2 (or 2 then 10).<br>Show how this strategy can be used to help us recite the 12 times table/count in multiples of 12, and to find the next/previous multiple of 12 from a given multiple. | Finding adjacent multiples – ratio chart and number line:<br>  <br><br>Finding adjacent multiples – array:<br>  <br> |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| | |
|---|---|
| **2:8** | Provide children with varied practice, such as that described in step *1:9*:<br>• in the context of groups of 12<br>• in the context of 12 equal groups<br>• based on their knowledge that adjacent multiples of 12 have a difference of 12.<br>At this stage, children can use the multiplication chart for reference. Plenty of practice will be needed over an extended period until children are fluent in the multiplication facts.<br>Example word problems:<br>• *'Roses are sold in bunches of twelve; this is called 'a dozen' roses. If a shop has seven bunches, how many roses are there altogether?'*<br>(multiplication; groups of 12)<br>• *'How many roses will the shop have left if they sell one bunch?'*<br>• *'There are twelve months in a year. If Sally saves £5 each month, how much money will she have after a year?'*<br>(multiplication; 12 equal groups)<br>• *'There are eighty-four cupcakes, arranged in rows of twelve. How many rows are there?'*<br>(quotitive division)<br>• *'I share forty-eight litres of water equally between twelve buckets. How much water is in each bucket?'*<br>(partitive division)<br>• *'If I have seven packs of twelve cards and another two cards, how many cards have I got altogether?'*<br>(multiplication – groups of 12; addition)<br>• '*Dog food comes in packs of twelve cans. Yesterday, Niamh bought six packs of dog food; her dog has already eaten three cans of food. How many cans are left?'*<br>(multiplication – groups of 12; subtraction)<br>• *'Write a story to go with this equation.'*<br>$7 \times 12 + 2$ |


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Sequences and multiplication:

Missing-number sequences/problems:
*'Fill in the missing numbers.'*

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 12 | 24 | 36 | 48 | | | | | | | | |
| 144 | 132 | 120 | | | | | | | | | | |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 × | 1 | 3 | 5 | 7 | 9 | 11 | = | | | | | |
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | × 12 = | | | | |

Dòng nǎo jīn:

*'Why are all the products in the twelve times table even? Can there ever be an odd multiple of twelve? How do you know?'*


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Multiplication and division:

Missing-number problems:
*'Fill in the missing numbers.'*

| | × 12 |
|---|---|
| 0 | 0 |
| 1 | 12 |
| 2 | |
| 3 | 36 |
| | 48 |
| 5 | |
| 6 | |
| | 84 |
| | 96 |
| | 108 |
| 10 | |
| | 132 |
| 12 | |

$8 \times \square = 96$ $\quad 12 \times \square = 84$ $\quad 108 = 12 \times \square$

$96 \div 12 = \square$ $\quad 84 \div 12 = \square$ $\quad \square \div 12 = 9$

$72 \div 12 = \square$ $\quad 5 = \square \div 12$ $\quad 144 \div 12 = \square$

Dòng nǎo jīn:

*'True or false?'*

★ ÷ 12 = ♦ because ★ × 12 = ♦


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Problems on adjacent multiples/the distributive law:

*'Fill in the missing symbols (<, > or =).'*

$6 \times 12 \bigcirc 5 \times 12$

$6 \times 12 \bigcirc 5 \times 12 + 12$

$6 \times 12 \bigcirc 6 \times 12 + 12$

$6 \times 12 \bigcirc 7 \times 12 - 12$

Dòng nǎo jīn:

- *'Work systematically to write all the possible answers to this calculation. '*

  $6 \times 12 > \square \times 12 + \square \times 12$

- *'How is this calculation different?'*

  $6 \times 12 \geq \square \times 12 + \square \times 12$

  *'Will you get the same number of possible answers? Explain your thinking.'*
  (Teacher note: take a moment to discuss what the ≥ symbol means.)

Matching contexts to equations:

*'Draw a line to match each story with the expression that represents it.'*

| | |
|---|---|
| *'Stickers are sold in packs of twelve. Sam had three packs but has lost two stickers. How many stickers does he have now?'* | $12 \times 8$ |
| *'Stamps are sold in books of six or twelve. Janina buys three books of twelve stamps and a book of six stamps. How many stamps does she have?'* | $12 \times 3 - 2$ |
| *'Each month, Iniko gets £5 pocket money and £3 for delivering a newsletter. How much money does Iniko get in a year (twelve months)?'* | $12 \times 3 + 6$ |


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## Teaching point 3:

Products in the 12 times table are double the products in the six times table; products in the six times table are half of the products in the 12 times table.

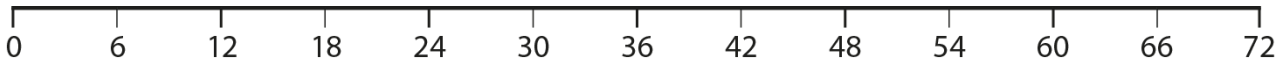
### Steps in learning

**3:1** This teaching point explores the relationship between the twelve times table and the six times table. Children have already made similar comparisons for other times tables, so the guidance here is brief. Use similar representations to segment *2.8 Times tables: 3, 6 and 9, and the relationship between them*, *Teaching point 3* (for more detail, refer back to segment *2.8*):

1. Double-skip count in sixes and twelves, supported by a number line and Gattegno chart (half the class should count the multiples of six, and on every other 'beat' the other half of the class should count only the multiples of twelve). Summarise the results in a table, as shown below, and discuss the pattern.
2. Use a familiar pictorial representation, alongside the bar model, to explore the relationship between a quantity grouped into sixes or twelves; generalise: ***'For every one group of twelve, there are two groups of six.'***
   Use this as the basis to compare pairs of equations with the same product, e.g.:
   $8 \times 6 = 48$
   $4 \times 12 = 48$
3. Compare pairs of equations where one factor is the same and the other is either six or twelve, e.g.:
   $5 \times 6 = 30$
   $5 \times 12 = 60$

Note: segment *2.10 Connecting multiplication and division, and the distributive law* (step *3:2*), briefly explored how the distributive law can be applied when there is a factor of twelve, by partitioning twelve into two sixes (e.g. $7 \times 12 = 7 \times 6 + 7 \times 6$).

**1.** Double skip counting




| **Number** | **0** | 1 | 2 | 3 | 4 | 5 | **6** | 7 | 8 | 9 | 10 | 11 | **12** | 13 | 14 | 15 | 16 | 17 | **18** | 19 | 20 | 21 | 22 | 23 | **24** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Counting in 6s** | ✓ | | | | | | ✓ | | | | | | ✓ | | | | | | ✓ | | | | | | ✓ |
| **Counting in 12s** | ✓ | | | | | | | | | | | | ✓ | | | | | | | | | | | | ✓ |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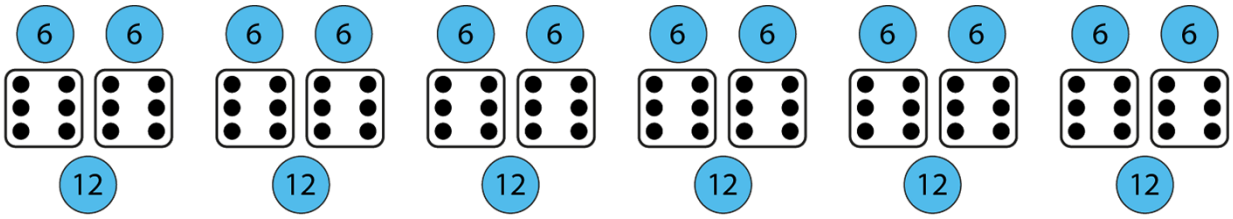
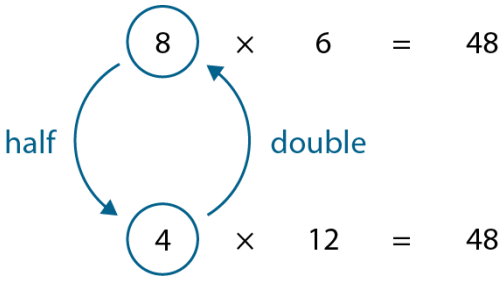
**2.** Grouping into sixes vs. grouping into twelves (same product)

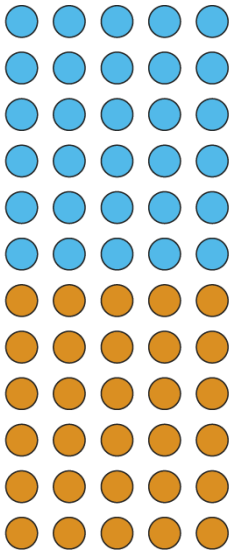


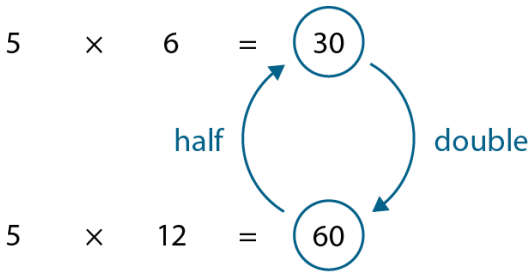

| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | | 12 | | 12 | | 12 | | 12 | | 12 | |

- *'Twelve is double six. Six is half of twelve.'*
- *'Four times twelve is equal to forty-eight, so double-four times six is equal to forty-eight.'*
- *'Eight times six is equal to forty-eight, so half-of-eight times twelve is equal to forty-eight.'*




**3.** Multiplying by six vs. multiplying by twelve (related products)






- *'Six is half of twelve, so five sixes is half of five-twelves.'*
- *'Twelve is double six, so five twelves is double five-sixes.'*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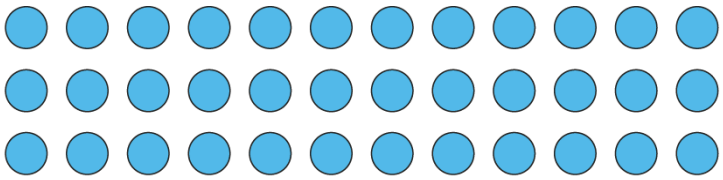
**3:2**

Provide children with practice based on the relationship between the six and twelve times tables, as shown opposite and on the next page.

Example word problems:

- *'Stefan has ten six-litre buckets of water. Ebony has five twelve-litre buckets of water. Ebony says she's got more water than Stefan. Is she right? Why/why not?'*
- *'There are sixty marbles.'*
  - *'If the marbles are shared between six children, how many marbles does each child get?'*
  - *'If the marbles are shared between twelve children, how many marbles does each child get?'*
    (partitive division)
- *'Ninety-six cupcakes are made in a bakery.'*
  - *'How many packs of twelve can be made?'*
  - *'How many packs of six can be made?'*
    (quotitive division)

Array problem:



- *'How many groups of six are there?'*

$\square \times 6 = \square$

- *'How many groups of twelve are there?'*

$\square \times 12 = \square$

Missing-number problems – same product:

*'Fill in the missing numbers.'*

$2 \times 6 = 1 \times 12$

$4 \times 6 = 2 \times 12$

$6 \times 6 = \square \times 12$

$\square \times 6 = 4 \times 12$

$\square \times 6 = 400 \times 12$

$20 \times 6 = \square \times 12$

$40 \times 6 = \square \times 12$

$60 \times 6 = \square \times 12$

$\square \times 6 = 40 \times 12$

$\square \times 6 = 90 \times 12$

$24 \div 6 = 4$

$36 \div 6 = \square$

$48 \div 6 = \square$

$24 \div 12 = \square$

$36 \div 12 = \square$

$48 \div 12 = \square$

Missing-number problems – related products:

*'Fill in the missing numbers.'*

$2 \times 6 = \square$

$2 \times 12 = \square$

$6 \times 5 = \square$

$12 \times 5 = \square$

$6 \times 7 = \square$

$12 \times 7 = \square$



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*'Fill in the missing numbers.'*

| | × 6 | × 12 |
|---|---|---|
| **0** | 0 | 0 |
| **1** | 6 | 12 |
| **2** | | 24 |
| **3** | 18 | |
| **4** | 24 | 48 |
| **5** | | 60 |
| **6** | 36 | 72 |
| **7** | 42 | |
| **8** | | 96 |
| **9** | 54 | 108 |
| **10** | | 120 |
| **11** | 66 | 132 |
| **12** | 72 | |

Dòng nǎo jīn:

- *'Explain why every second multiple of six is a multiple of twelve. Draw a picture to help you explain.'*
- *'Explain why the product of "3" and "12" is double the product of "3" and "6".'*
- *'Decide whether each of these sentences is always true, sometimes true or never true. Tick the correct box for each sentence.'*

| | Always | Sometimes | Never |
|---|---|---|---|
| Multiples of 6 are also multiples of 12. | | | |
| Multiples of 12 are also multiples of 6. | | | |
| Products in the 12 times table are half the value of products in the 6 times table. | | | |


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## Teaching point 4:

Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by 11 or 12.

Steps in learning

| | Guidance | Representations |
|---|---|---|
| **4:1** | Knowing if a dividend is divisible exactly by a divisor (to give a whole number), without having to do a full calculation, is a useful skill. Children have already learnt the divisibility rules for divisors of two, three, four, five, six, eight, nine and ten; these are summarised and applied in segment *2.9 Times tables: 7 and patterns within/across times tables.*<br><br>In this teaching point, children explore and apply the divisibility rules for divisors of 11 and 12. Note that, since children are working within the context of integers, throughout this teaching point the statement 'can be divided by' implies 'gives a whole number when it is divided by'.<br><br>Children have already noted (in step *1:7*) that each two-digit multiple of 11 has equal digits (i.e. the tens and ones digits are the same); revisit the pattern now, and explore how it can be used as a divisibility check for dividends up to and including 99: ***'A two-digit number is divisible by eleven if the digits are the same.'***<br><br>This pattern does not continue into the three-digit multiples, so the divisibility rule taught at this stage is somewhat limited and corresponds only to known multiplication facts in the 11 times table. As discussed in step *1:8*, ensure that children do not over apply the 'same-digit pattern' beyond two-digit numbers. | *'A two-digit number is divisible by eleven if the digits are the same.'*<br><br>\| | × 11 \|<br>\|---\|---\|<br>\| 0 \| 0 \|<br>\| 1 \| 11 \|<br>\| 2 \| 22 \|<br>\| 3 \| 33 \|<br>\| 4 \| 44 \|<br>\| 5 \| 55 \|<br>\| 6 \| 66 \|<br>\| 7 \| 77 \|<br>\| 8 \| 88 \|<br>\| 9 \| 99 \|<br>\| 10 \| 110 \|<br>\| 11 \| 121 \|<br>\| 12 \| 132 \|<br><br>*'Circle the numbers that are divisible by eleven.'*<br><br>3 11 99<br>33 21 89<br>63 31 79<br>93 41 69 |


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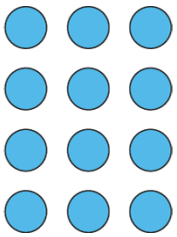
**4:2**

Now move on to the divisibility rule for 12. Since this rule is based on the connection between 3, 4 and 12, and relies on the divisibility rules for divisors of 3 and 4, review the following points:

- Remind children, using an array and bar models, that twelve is equal to three groups of four or four groups of three.
- Remind children of the divisibility rule for three, and practise applying it: '***For a number to be divisible by three, the sum of the digits of the number must be divisible by three.'***
- Remind children of the divisibility rules for four, and practise applying them:
  - ***'If halving a number gives an even value, then the number is divisible by four.'***
  - ***'For numbers with more than two digits: if the final two digits are divisible by four then the number is divisible by four.'***

Then introduce, and practise applying, the divisibility rule for twelve: ***'For a number to be divisible by twelve, the number must be divisible by <u>both</u> three <u>and</u> four.'***

Connection between three, four and twelve:



$12 = 4 \times 3$

$12 = 3 \times 4$

| 12 | | | |
|---|---|---|---|
| 3 | 3 | 3 | 3 |

| 12 | | |
|---|---|---|
| 4 | 4 | 4 |

Applying the divisibility rules for 3, 4 and 12:

*'Put a tick in the correct boxes to show which numbers are divisible by three, four and twelve.'*

| | Number is divisible by… | | |
|---|---|---|---|
| **Number to test** | **3** | **4** | **12** |
| **12** | | | |
| **15** | | | |
| **16** | | | |
| **18** | | | |
| **20** | | | |
| **21** | | | |
| **24** | | | |
| **30** | | | |
| **96** | | | |
| **109** | | | |
| **144** | | | |
| **240** | | | |
| **304** | | | |
| **1200** | | | |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| | | |
|---|---|---|
| **4:3** | Now provide some general practice, across all divisibility rules learnt so far. You can now extend to problems with four-digit dividends.<br>Dòng nǎo jīn: *'It is not possible for a number to be divisible by both five and twelve. True or false? Explain.'* | *'For each example below, circle the numbers that are possible.'*<br>• A cake factory needs to put an equal number of cakes in every box. If the factory makes 1164 cakes, how many could go in each box?<br>3 4 5 6 12<br>(quotitive division)<br>• Staff at a factory must sit down to eat at the same time. If there are 55 staff, and an equal number of people need to sit at each table, how many tables could there be?<br>2 3 5<br>8 10 11<br>(partitive division) |


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