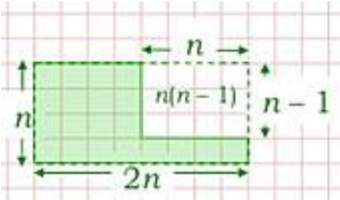
Secondary Magazine 141

National Centre for Excellence in the Teaching of Mathematics

Welcome to the second edition of our new-look Secondary Magazine. This time we offer some ideas and resources for teaching (or revising) quadratic sequences for the new GCSE (9-1). And we report on the latest publication from the Further Mathematics Support Programme, in their continued campaign to encourage more girls to study Maths and Further Maths at AS/A level. For those embroiled in GCSE revision, or about to be, there are plenty of useful links in the 'Some other things...' section below.

Don't forget that all previous issues are available in the Archive.



New at GCSE: Quadratic sequences

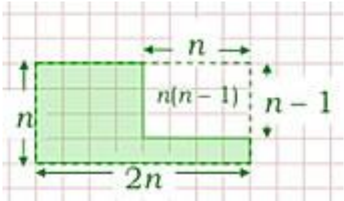
In this article we look at the importance of approaching the nth term both through geometric patterns and through algebraic calculations using differences. We suggest that understanding of one supports the other. And there are some great patterns to try out with your classes, both in Y10 and for revision…

What Makes More Girls Want to Study Maths and Further Maths at AS/A Level?

What is the gender balance of your A/AS level Maths/Further Maths classes? A recent study has reported on five case studies of sixth forms with above average numbers of girls choosing Maths and Further Maths. We report here on the common features found in those schools/colleges, and suggest where schools might find further advice, ideas and resources for improving the gender balance in their Maths and Further Maths classes.

And some other things to draw to your attention:

- Ofqual's blog reports on a recent meeting with teachers about the new 9-1 GCSE, in which they explain about grade boundary setting, ensuring fairness between cohorts and across both tiers (with slides from the meeting and a link to sign up for their 9-1 newsletter).
  And here, they offer some insight into how a handful of schools are making those difficult decisions about tier of entry.
- As 'that' time of year comes steaming towards us, Jo Morgan, maths teacher and blogger (@mathsjem, known for resourceaholic.com), has put together her recommendations for the best online revision resources for the new GCSE 9-1 curriculum.
- Amir Arezoo (@WorkEdgeChaos), maths teacher and vice-principal, provokes a debate in the Twittersphere, when he asks for a best method – does such a thing exist, or should we encourage multiple ways to tackle a question? His account of the exchange is here, and is just one example of many such pedagogical discussions happening (mostly through Twitter) in the online maths education community.
- Cambridge Assessment are undertaking research to understand how current reforms to AS/A levels are affecting schools and colleges. They have put out a call for Heads of Department to answer this 15-minute survey (with a chance of winning a £100 Amazon voucher).
- Hidden Figures is the story of three African-American women mathematicians who served a vital role in NASA space programme in the 1960s. Confounding both racial and gender stereotypes of the time, the film is uplifting and encouraging: a must-see for secondary maths teachers and their students alike. Watch the trailer.



New at GCSE: Quadratic sequences

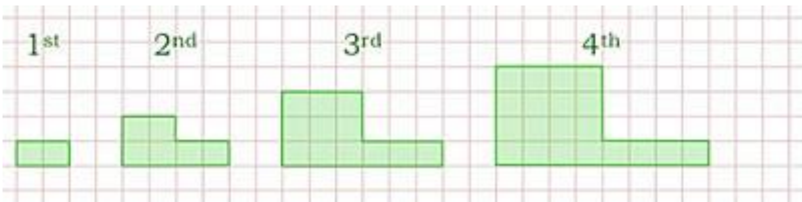
An item of content newly introduced into GCSE (though it has come and gone over the years) is finding the *n*th terms of quadratic sequences. Expressing general rules that relate the *n*th terms of pattern sequences to their values is a way to harness pupils' natural pattern-seeking to develop their understanding of, and fluency with, algebra. (See: Key Ideas in Teaching Mathematics, Anne Watson, Keith Jones, Dave Pratt, 2013, Oxford University Press, Page 31).

Here are some examples of tasks that provide opportunities for pupils to see visual patterns in various different ways, and thereby arrive at equivalent quadratic expressions for the *n*th term. Alternatively pupils could count the number of squares in each image to obtain a numerical sequence, and then use an algebraic method to derive an expression for the nth term. Usually the numerical-sequence method is more laborious and is often used without pupils understanding why it works. Therefore pupils should appreciate that, with sequences of images it is often easiest to use the geometric structure of the images - to see how the images 'grow' to form the sequence – so that when they arrive at an expression for the *n*th term they know precisely why it must be correct.

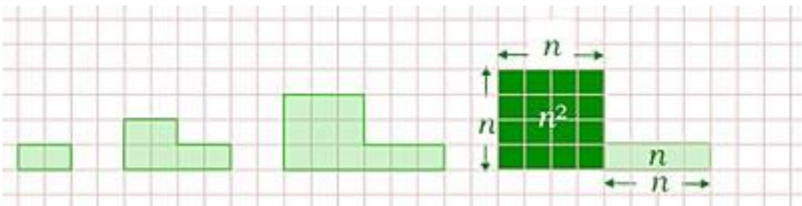
In all the following examples the task is to

find an expression for the number of squares in the nth image

Example 1

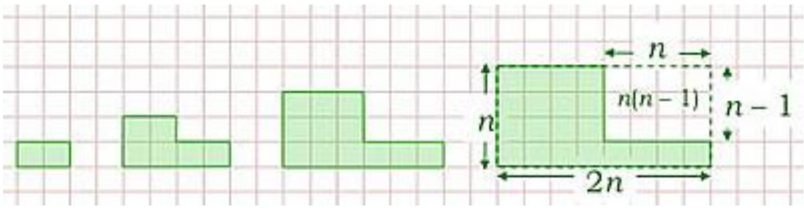


Pupils may see each image as a square with side-length *n* together with a rectangle of height 1, and length *n*:



The number of squares in the *n*th image = $n^2 + n$.

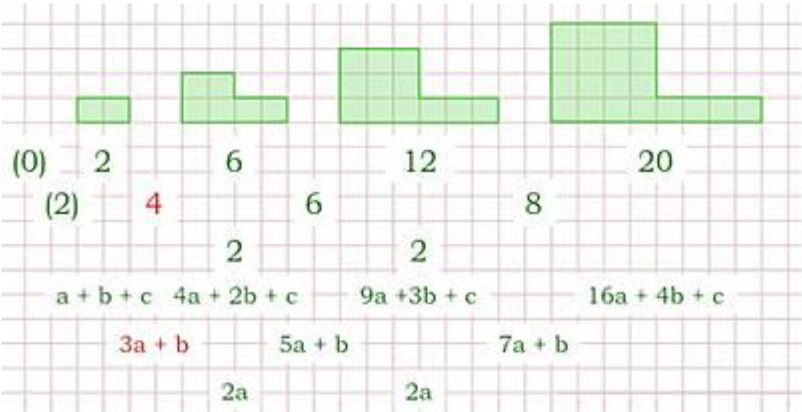
A different way to see each image is as a rectangle with side-lengths *n* and 2*n* with a smaller rectangle of side-lengths *n* and *n* – 1 removed from it:



The number of squares in the nth image $= n \times 2n - n(n-1)$
$= 2n^2 - n^2 + n$
$= n^2 + n$

The numerical approach entails counting the number of squares in each of the first few images and then working with the numerical sequence so produced. Pupils first have to understand that the general term of a quadratic sequence is of the form $an^2 + bn + c$. They can then use reasoning to deduce the values of a, b and c for the numerical sequence they have formed, as follows.

First write the sequence of number of squares in each image, and underneath write the first differences, and below that write the second differences. Then write the same first few terms algebraically by substituting 1, then 2, then 3, and so on, in turn for n in $an^2 + bn + c$. Also write the first and second differences algebraically.



By comparing the second differences expressed algebraically (2a) with the numerical second differences pupils will see that, for this sequence, 2a = 2, so a = 1.

By equating any of the first differences expressed algebraically with the corresponding numerical first difference pupils can now find the value of b. For example, 3a + b = 4. Since a = 1, 3 + b = 4, so b = 1.

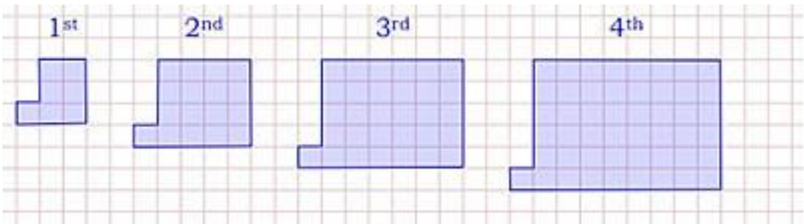
To find the value of c pupils have to understand that when $n = 0$, the term expressed algebraically is just c, since an^2 and bn are both 0. By extending the sequence of first differences backwards (in this example to get 2, shown in brackets above) they can extend the quadratic sequence itself back to arrive (in this example) at 0 as the term when $n = 0$.

Therefore the nth term, $an^2 + bn + c$, is $n^2 + n$.

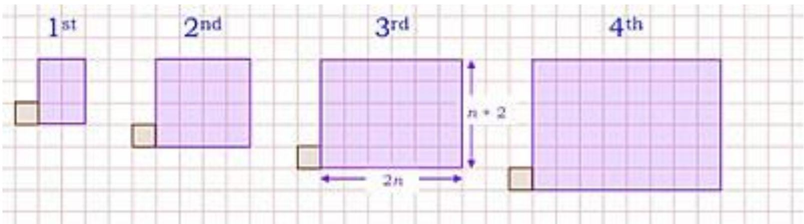
Pupils can compare the relatively complex third method to the previous direct ways of seeing the nth term using the structure of the images.

It is helpful to remind pupils that usually the images can be cut up into a combination of rectangles and squares, or every image can be seen as a 'surrounding' rectangle from which one or more rectangles have been removed. They then just have to decide how the side-lengths of the squares and rectangles composing, or removed from, an image are related to the position of the image in the sequence. Here is another example in which the images can be cut up into squares and rectangles in various ways.

Example 2



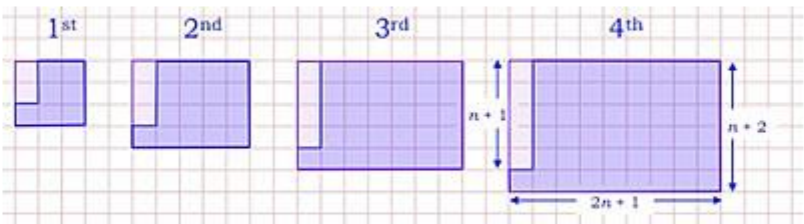
Pupils could imagine cutting each image into a rectangle of side-lengths $n + 2$ and $2n$ and a unit square, as shown below:



The number of squares in the nth image $= 2n(n + 2) + 1$

$= 2n^2 + 4n + 1$

Or pupils may see a rectangle with side-lengths $2n + 1$ and $n + 2$ from which a rectangle of unit width and height $n + 1$ has been removed, as shown below:

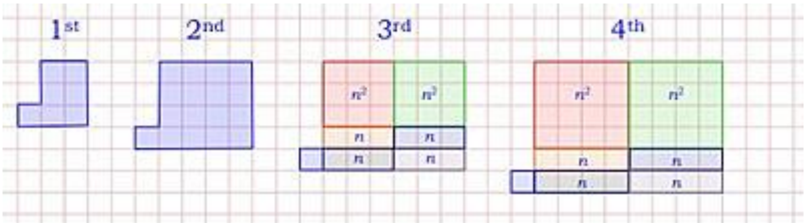


The number of squares in the nth image $= (2n + 1)(n + 2) - (n + 1)$

$= 2n^2 + 5n + 2 - n - 1$

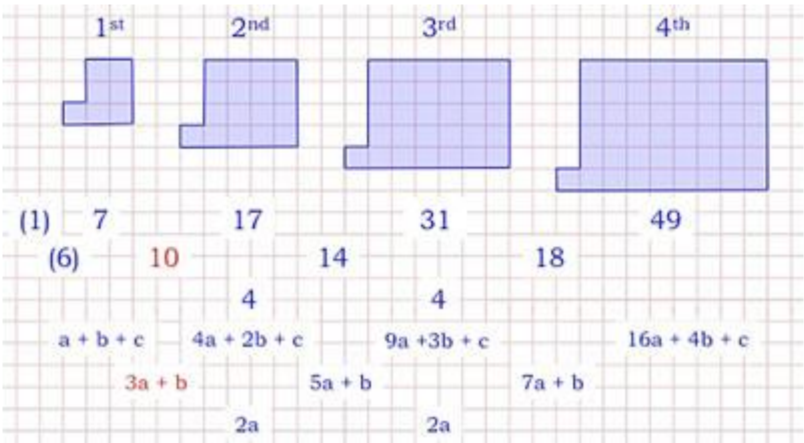
$= 2n^2 + 4n + 1$

Or they may see each image as two n-by-n squares of area n^2 plus four n-by-1 rectangles of area n plus a unit square, possibly as shown below:



The number of squares in the *n*th image $= n^2 + n^2 + n + n + n + n + 1$
$= 2n^2 + 4n + 1$

Pupils can now compare the relatively simple way in which they found the expression for the nth term by seeing a structure in the images with the more laborious procedure of counting the squares in the first few images and working algebraically on the resulting numerical sequence, as shown below:



By equating the second differences $2a = 4$
$a = 2$
By equating the first differences $3a + b = 10$
Since $a = 2$, $6 + b = 10$
$b = 4$

By extending the sequence back to $n = 0$, using the pattern of first differences,

when $n = 0$, $an^2 + bn + c = c = 1$

Therefore the *n*th term $an^2 + bn + c = 2n^2 + 4n + 1$.

Pupils may enjoy devising their own examples for others to solve. In the meantime here are some more.

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What Makes More Girls Want to Study Maths and Further Maths at AS/A Level?

Our Twitter #mathscpdchat discussion on 25 April, 7 - 8 pm, will focus on successful strategies to encourage more girls to study Maths and Further Maths at AS/A Level

There are strong reasons for wanting to increase the numbers of girls opting for Maths/Further Maths (FM) at AS/A level. As well as the clear social justice implications (for future career and salary parity), girls tend to be more successful at AS/A level, with consistently lower proportions getting E and U grades, and higher proportions getting A grades (though A*s are awarded more often to boys). Justice Secretary Liz Truss said in a speech recently that her "main career advice" for girls was to study maths for whatever career they wanted to pursue.

To discover the secrets to attracting more female candidates, four schools and one FE college were studied by a team from UCL IoE (University College London, Institute of Education) on behalf of the FMSP (Further Maths Support Programme). The full report can be read here.

The schools and college studied were chosen because of their above average proportions of girls choosing to take Maths and FM at AS/A level. None had employed specific initiatives to raise girls' participation rates, but researchers were keen to see if they could find common factors that were increasing girls' participation rates.

It is worth noting, first of all, that gender participation is measured as the proportion of girls (or boys) choosing maths out of the entire cohort of students taking one or more A level. The national figure for girls is 20% for Maths and 2.5% for Further Maths (boys are twice as likely as girls to take Maths and three times as likely to take FM). Alternative measures that compare the number of girls taking maths with the number of boys, have drawbacks such as appearing successful if the number of boys opting for maths is reduced (a scenario which improves nothing!).

Researchers observed lessons, collected participation data and conducted interviews with teachers and Y11 and Y12 girls, in order to understand higher than average female participation rates. No obvious blueprint was evident from the study, but the following themes emerged as common to more than one of the institutions:

- A strong **culture of maths** in the institution – this was seen as encouraging all students to aspire to study maths, particularly in the schools, where such expectations were evident from as early as Y8. Some of the institutions were as successful in recruiting above average numbers of boys, meaning that gender imbalance in classes has not necessarily been eradicated.
- A pervasive **expectation** that those capable of B grade and above in Maths would continue to study the subject, and be successful at AS/A level.
- In some institutions,**Y11 top sets studied A level topics** as standard, in class time. In some cases this was part of a L2 Further Maths qualification (though girls reported the qualification as less important than the opportunity to 'try out' harder maths). There was a belief that this 'demystified' A level maths and challenged the reputation that it was 'too hard'.
- **Careers input** was overwhelmingly positive about the wide applicability of maths and emphasised that studying the subject would keep options open. In some cases this positivity was reinforced by classroom/corridor displays.

- Girls reported it significant to their enjoyment when teachers took the time to explain **how abstract mathematics topics can be applied** in practical situations. It was significant to their decision-making when teachers talked to them personally about their futures.
- Two institutions offered **AS level Further Maths in Y13** for those students that only discovered a particular love or capability for Maths in Y12, or who realised the need for FM for their university choice. There was evidence that this was an option welcomed by girls in particular.
- All institutions had evidence of **stable Further Maths** provision, with a commitment to keep the course timetabled through years with uneconomically small classes.
- Strong **female role models** were important – all institutions had at least two female maths teachers who taught Y11 top sets and A-level classes, and were cited as influential by staff and students.
- Various **in-class practices** were reported as being encouraging to girls – in particular, teachers directly asking for contributions from less vocal members of the class and valuing a variety of ways of working. Girls also reported that they valued their teachers being available for individual help (in and out of lessons), and feeling that their teachers knew them and encouraged them personally.
- The schools all offered a Mathematics A-level option that included **statistics** in Y12 and they promoted this as beneficial because of its social-science applications. Girls are more likely than boys to consider careers supported by studying statistics, so this school–level offer is understood to match their needs and promote participation.

Evidence suggests that girls tend to be less confident about their mathematical abilities. For example, fewer girls with GCSE grade B or C will opt for AS/A level maths than boys with those grades. They seem to be more deterred by the reputation that A level maths is a 'hard' subject and more likely to take a 'safer' route. The institutions studied seemed to use a variety strategies to build confidence by giving 'tasters' of AS level maths, by expectations of success, and to avoid shutting down options for mathematical study too early.

The report expressed concern that some of the forthcoming changes to AS/A levels, to performance measures, and to school funding might mitigate against some of the factors found to improve the uptake of Maths and FM amongst girls. They recommend that policy change should be actively monitored for unintended impact on girls' participation in all forms of Level 3 mathematics.

We'd love to hear of any successful strategies that you have used in your school/college to increase the numbers of girls studying AS/A level Maths and Further Maths. Please let us know.

And for those schools wishing to pursue strategies to increase girls' participation, the FMSP has produced this briefing document (with recommended strategies on the final page), and these resources for teachers (including an enrichment session, 'promoting maths' presentation, and guidance for parents). There is also an information for students page. All the information available from the FMSP is summarised in their recent Focus of the Month.