Maths HUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.19 Calculation: ×/÷ decimal fractions by whole numbers

Teacher guide | Year 5

Teaching point 1:

Decimal fractions (with a whole number of tenths or hundredths) can be multiplied by a whole number by using known multiplication facts and unitising.

Teaching point 2:

Multiplying by 0.1 is equivalent to dividing by 10; multiplying by 0.01 is equivalent to dividing by 100. Understanding of place value can be used to divide a number by 10/100: when a number is divided by 10, the digits move one place to the right; when a number is divided by 100, the digits move two places to the right.

Teaching point 3:

To multiply a single-digit number by a decimal fraction with up to two decimal places, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the product by dividing by 10 or 100.

Teaching point 4:

If the multiplier is less than one, the product is less than the multiplicand; if the multiplier is greater than one, the product is greater than the multiplicand.

Teaching point 5:

To divide any decimal fraction with up to two decimal places by a single-digit number, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the quotient by dividing by 10 or 100.

Overview of learning

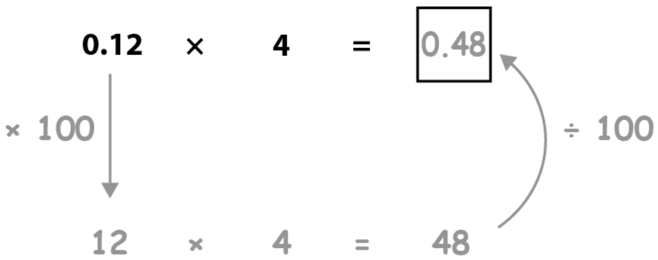
In this segment children will:

- use known multiplication facts and unitising to multiply whole numbers of tenths (between 0.1 and 0.9) or hundredths (between 0.01 and 0.09) by a whole number, for example:

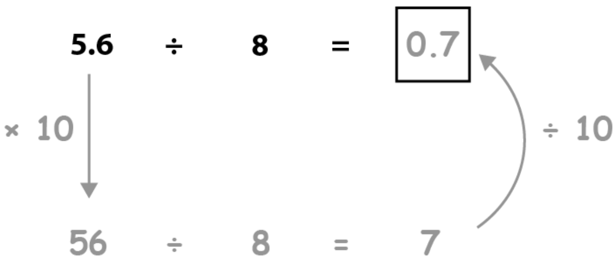
6×4	$= 24$	6×4	$= 24$
6×4 ones	$= 24$ ones	6×4 ones	$= 24$ ones
6×4 tenths	$= 24$ tenths	6×4 hundredths	$= 24$ hundredths
6×0.4	$= 2.4$	6×0.04	$= 0.24$

- learn the equivalence of:
  - multiplying by 0.1 and dividing by 10
  - multiplying by 0.01 and dividing by 100

  and use their understanding of place value to carry out these calculations
- recognise that multiplying a positive number can result in a smaller value than we started with (challenging the misconception that multiplication always 'makes things bigger'/increases the value)
- efficiently multiply decimal fractions by whole numbers, using integer multiplication and adjusting, for example:



- efficiently multiply decimal fractions by whole numbers, using short multiplication with the decimal point already in place
- efficiently divide decimal fractions by whole numbers, using integer division and adjusting, for example:



- efficiently divide decimal fractions by whole numbers, using short division with the decimal point already in place.

The purpose of this segment is to provide children with strategies for multiplying and dividing decimal fractions by whole numbers, based on their existing knowledge of multiplication facts and strategies for whole numbers. Essentially, with just their times-table facts and the short multiplication/division methods they have learnt so far, children are able to solve a range of related problems involving decimal fractions.

In *Teaching point 1*, times-table facts are combined with unitising, building on a comparison of skip counting in whole-number multiples (e.g. *'four, eight, twelve…'*) and skip counting in multiples of tenths (e.g. *'four tenths, eight tenths, twelve tenths…'* / *'zero-point-four, zero-point-eight, one-point-two…'*) and hundredths (e.g. *'four hundredths, eight hundredths, twelve hundredths…'* / *'zero-point-zero-four, zero-point-zero-eight, zero-point one-two…'*). Arrays of place-value counters are used to support understanding of the connection to known times-table facts. This strategy enables children to multiply whole numbers of tenths (between 0.1 and 0.9) or hundredths (between 0.01 and 0.09) by any whole number from 0 to 12.

Teaching point 2 builds on what children have learnt about multiplying and dividing whole numbers by 10 or 100 (segment *2.13 Calculation: multiplying and dividing by 10 or 100*), using place-value charts to understand strategies for multiplying by 0.1/dividing by 10 and for multiplying by 0.01/dividing by 100.

In our decimal number system, the value of a digit depends on its place (position) in the number, with each place having a value ten times that of the place to its right. Place value is based on unitising, treating a group of things as one 'unit'; children have already gained experience of unitising throughout *Spine 1: Number, addition and subtraction*, *Spine 2: Multiplication and division*, and *Spine 3: Fractions*, and explored this in the context of place-value in *Spine 1*, segments *1.23* and *1.24*, where they developed their understanding of tenths, hundredths and thousandths. In place value, the 'units' are powers of ten (…1,000, 100, 10, 1, 0.1, 0.01, 0.001…). The decimal point is the symbol used in numerical representations to separate the integer part of a number (to the left of the decimal point) from its fractional part (to the right of the decimal point). When writing integers, there is no need to write a decimal point, because the fractional part is zero (and the value of the places can be determined with reference to the right-most digit – in other words, the ones place). When the fractional part of a number is non-zero, the decimal point allows us to determine the value of each digit, based on its placement relative to the decimal point, i.e. the decimal point acts as a reference point from which we can determine the place-value of the surrounding digits.

When, for example, a dividend is divided by ten (e.g. $15 \div 10 = 15 \times 0.1 = 1.5$), we can interpret the effect on the dividend in two possible ways. We can say that the digits have moved one place to the right, or we can say that the decimal point has moved one place to the left. Both actions serve the same purpose – they enable a reference point for the value of each digit: if the digits are moved one place to the right, they each become one-tenth times their original value; if the decimal point is moved one place to the left, each digit becomes one-tenth times its original value. Rather than saying '*to divide by ten, move the decimal point one place to the left*', you could say '*to divide by ten, place the decimal point so that each digit is one-tenth the size*' or *'to divide by ten, move each digit one place to the right'*. In this, and other segments, we refer to movement of the digits, not to movement of the decimal point.

In *Teaching points 3* and *5*, children use their understanding of multiplying and dividing by 10 or 100 (and of place value) to transform multiplication calculations involving one decimal fraction to calculations involving only whole numbers; they perform the resulting whole-number calculations using known strategies and then adjust the product/quotient to solve the original calculation. These teaching points also encourage the important habit of using estimation to check that answers are reasonable, because errors can easily occur when using 'adjusting' strategies. *Teaching point 5* has connections with *Spine 3: Fractions*, segment *3.9* (year 6) in which children learn to divide fractions by whole numbers.

Up until this segment, children only had experience of multiplying whole-number multiplicands by whole-number multipliers, and in these cases (except when the multiplier is zero or one) the product is always larger than the multiplicand. Through *Teaching points 1–3* children will gain experience multiplying whole-number multiplicands by decimal fractions that are either less than or greater than one. *Teaching point 4* allows children to explore and formalise the fact that when the multiplier is less than one, the product is less than the multiplicand and, conversely, that if the multiplier is greater than one (including decimal fractions greater than one), the product is greater than the multiplicand. *Teaching point 4* has connections with *Spine 3: Fractions*, segment *3.9* (year 6), in which children will extend their understanding to the multiplication of any fraction multiplicand by a proper-fraction multiplier, resulting in a product that is less than the multiplicand.

Throughout this segment, teachers should avoid using the term 'factor' to refer to a multiplicand/ multiplier that is a decimal fraction; factors are positive integers (see segment *2.21 Factors, multiples, prime numbers and composite numbers*).

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast*. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.*

Teaching point 1:

Decimal fractions (with a whole number of tenths or hundredths) can be multiplied by a whole number by using known multiplication facts and unitising.

Steps in learning

1:1 In *Spine 1: Number, Addition and Subtraction*, segments *1.23* and *1.24*, numbers with tenths or with hundredths, respectively, were decomposed multiplicatively, supported by unitising language, for example:

- $0.3 = 3 \times 0.1$
- $1.8 = 18 \times 0.1$
- $0.03 = 3 \times 0.01$
- $0.43 = 43 \times 0.01$

In this teaching point, children will use known multiplication facts and unitising to explore calculations of the type:

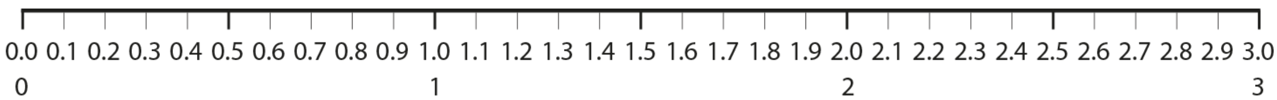
- $9 \times 0.4 = 3.6$
  (steps *1:1–1:6*; at this stage, keep to multiplication of whole numbers 0–12 by decimal fractions with whole numbers of tenths 0.1–0.9 inclusive)
- $9 \times 0.04 = 0.36$
  (steps *1:7–1:12*; at this stage, keep to multiplication of whole numbers 0–12 by decimal fractions with whole numbers of hundredths 0.01–0.09 inclusive)

Begin by reviewing the fact that there are ten tenths in one whole, and then skip counting in tenths up to and beyond one, using a number line and Gattegno chart. Count in two ways:

- *'Zero tenths, one tenth, two tenths, three tenths…'*
- *'Zero, zero-point-one, zero-point-two, zero-point-three…'*

(For more guidance, see *Spine 1*, segment *1.23, Teaching point 3)*.

Number line:



Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

1:2

Now practise skip counting in multiples of tenths, building up 'decimal times tables'.

First skip count in multiples of a whole number, using a number line and an appropriate context, for example: *'Ben is designing a building. Each floor is 4 m tall. How tall could the building be?'*

Then introduce a context that involves the related multiple of tenths, for example: *'Susan is stacking crates. Each crate is 0.4 m tall. How tall could a stack be?'*

Skip count in multiples of four tenths. As in step *1:1*, count in two ways:

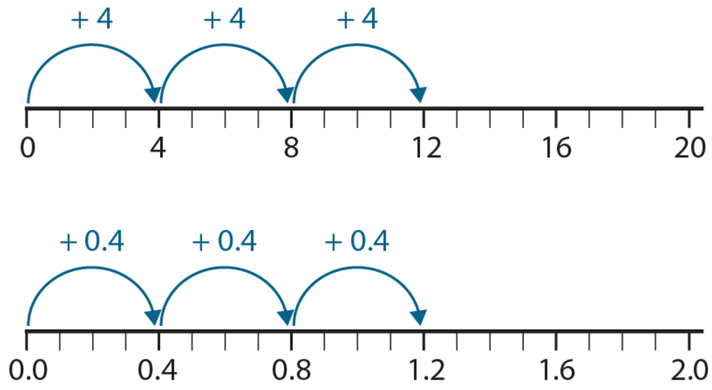
- *'Zero tenths, four tenths, eight tenths, twelve tenths…'*
- *'Zero, zero-point-four, zero-point-eight, one-point-two…'*

You could use a number line marked in tenths, and highlight or circle the multiples of 0.4 as you count. Then count again, recording the '0.4 times table' initially using repeated addition equations (e.g. *0.4 + 0.4 + 0.4 = 1.2*) then, as this becomes cumbersome, prompt children to recognise that multiplication equations can be used.

A common mistake is for children to write *0.4 × 3 = 0.12* and say *'zero-point-twelve'*. To challenge this error, return to the repeated addition structure, adding together the decimal numbers using known facts and unitising, and emphasising that there are ten tenths in one whole:

- *'Eight tenths plus four tenths is equal to twelve tenths.'*
- *'There are ten tenths in one whole, so twelve tenths is equal to one-and-two-tenths.'*

Skip counting in multiples of 4 and of 0.4 – number line:



	0 × 0.4 = 0.0	0.4 × 0 = 0.0
0.4	1 × 0.4 = 0.4	0.4 × 1 = 0.4
0.4 + 0.4 = 0.8	2 × 0.4 = 0.8	0.4 × 2 = 0.8
0.4 + 0.4 + 0.4 = 1.2	3 × 0.4 = 1.2	0.4 × 3 = 1.2

1:3

Once you have built up the 0.4 times table as far as *12×0.4* (and *0.4×12*), compare it to the four times table, and recite the facts in pairs, using unitising language to show the connection:

- *'Three times four ones is equal to twelve ones.'*
- *'Three times four tenths is equal to twelve tenths.'*

Note that, throughout this teaching point, multiplication facts within a decimal times table have been represented to the same precision; for example *$0.4 \times 5 = 2.0$* rather than *$0.4 \times 5 = 2$*. You can discuss with children when it is appropriate to omit zero place-value holders (i.e. here it would be acceptable to write *$0.4 \times 5 = 2$*).

Complete '0.4 times table':

$0 \times 0.4 = 0.0$	$0.4 \times 0 = 0.0$
$1 \times 0.4 = 0.4$	$0.4 \times 1 = 0.4$
$2 \times 0.4 = 0.8$	$0.4 \times 2 = 0.8$
$3 \times 0.4 = 1.2$	$0.4 \times 3 = 1.2$
$4 \times 0.4 = 1.6$	$0.4 \times 4 = 1.6$
$5 \times 0.4 = 2.0$	$0.4 \times 5 = 2.0$
$6 \times 0.4 = 2.4$	$0.4 \times 6 = 2.4$
$7 \times 0.4 = 2.8$	$0.4 \times 7 = 2.8$
$8 \times 0.4 = 3.2$	$0.4 \times 8 = 3.2$
$9 \times 0.4 = 3.6$	$0.4 \times 9 = 3.6$
$10 \times 0.4 = 4.0$	$0.4 \times 10 = 4.0$
$11 \times 0.4 = 4.4$	$0.4 \times 11 = 4.4$
$12 \times 0.4 = 4.8$	$0.4 \times 12 = 4.8$

Comparing the 4 and 0.4 times tables:

$0 \times 4 = 0$	$0 \times 0.4 = 0.0$
$1 \times 4 = 4$	$1 \times 0.4 = 0.4$
$2 \times 4 = 8$	$2 \times 0.4 = 0.8$
$3 \times 4 = 12$	$3 \times 0.4 = 1.2$
$4 \times 4 = 16$	$4 \times 0.4 = 1.6$
$5 \times 4 = 20$	$5 \times 0.4 = 2.0$
$6 \times 4 = 24$	$6 \times 0.4 = 2.4$
$7 \times 4 = 28$	$7 \times 0.4 = 2.8$
$8 \times 4 = 32$	$8 \times 0.4 = 3.2$
$9 \times 4 = 36$	$9 \times 0.4 = 3.6$
$10 \times 4 = 40$	$10 \times 0.4 = 4.0$
$11 \times 4 = 44$	$11 \times 0.4 = 4.4$
$12 \times 4 = 48$	$12 \times 0.4 = 4.8$

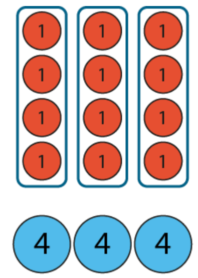
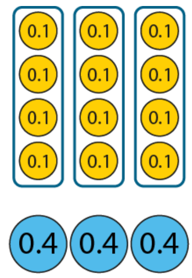
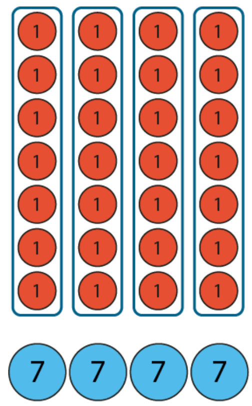
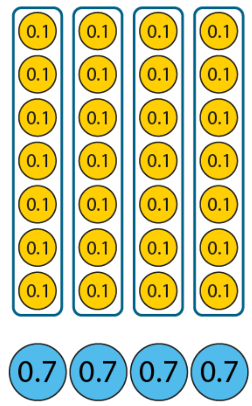
Example comparison:

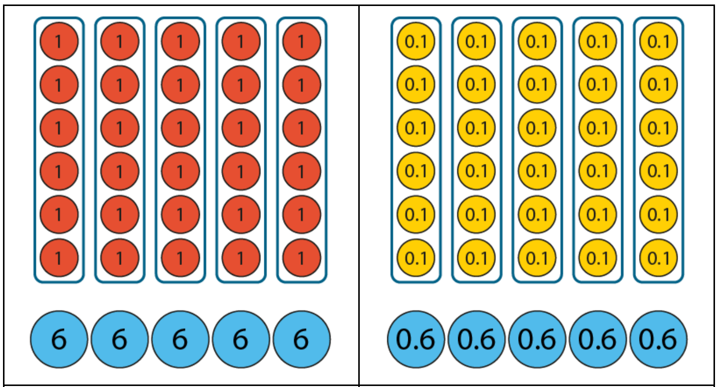
$6 \times 4 = 24$

6×4 ones $= 24$ ones

6×4 tenths $= 24$ tenths

$6 \times 0.4 = 2.4$

1:4	Take a moment to explore how pairs of related facts from the 4 and 0.4 times tables could be represented with manipulatives or pictures, sharing suggestions as a class. Then focus on array representations, drawing attention to how the same layout can be used to represent two related facts by changing the place value of the counters. Similarly, compare the facts represented with 4-value counters and with 0.4-value counters. Use the following stem sentence: ***'___ times ___ ones is equal to ___ ones, so ___ times ___ tenths is equal to ___ tenths.'***	  $3 \times 4 = 12$ 3×4 ones $= 12$ ones $3 \times 0.4 = 1.2$ 3×4 tenths $= 12$ tenths • *'Three times four ones is equal to twelve ones, so three times four tenths is equal to twelve tenths.'*
1:5	Repeat steps *1:2–1:4* with other decimal times tables. Continue to use the stem sentence from step *1:4* to draw attention to unitising in ones vs unitising in tenths. Vary the order in which the factors are presented (both *whole number × decimal fraction* and *decimal fraction × whole number* calculations).	Example 1:   $4 \times 7 = 28$ 4×7 ones $= 28$ ones $4 \times 0.7 = 2.8$ 4×7 tenths $= 28$ tenths • *'Four times seven ones is equal to twenty-eight ones, so four times seven tenths is equal to twenty-eight tenths.'*

		Example 2:  $6 \times 5 = 30$ 6 ones × 5 = 30 ones $0.6 \times 5 = 3.0$ 6 tenths × 5 = 30 tenths • *'Six ones times five is equal to thirty ones, so six tenths times five is equal to thirty tenths.'*

1:6	At this point, provide children with practice, including:
	• missing-number/symbol problems
	• contextual problems, including both grouping and scaling structures, as shown opposite and here:
	  ◦ *'It takes 0.3 kg of flour to make one cake. How much flour is needed to make seven cakes?'*
	  ◦ *'A dressmaker has a blue ribbon that is five metres long. She has a red ribbon that is three-tenths times that length. How long is the red ribbon?'*
		Missing-number problems:
		'Fill in the missing numbers.'
		$4 \times 8 = \square$ $\square \times 6 = 18$
		$4 \times 0.8 = \square$ $\square \times 0.6 = 18$
		$5 \times \square = 45$ $\square \times 9 = 45$
		$5 \times \square = 4.5$ $\square \times 9 = 4.5$
		$11 \times 0.4 = \square$ $0 \times 0.9 = \square$
		$0.5 \times 7 = \square$ $0.4 \times 10 = \square$

Missing-symbol problems:

'Fill in the missing symbols (<, > or =)'

5×0.4 ◯ 6×0.4

8×0.9 ◯ 9×0.9

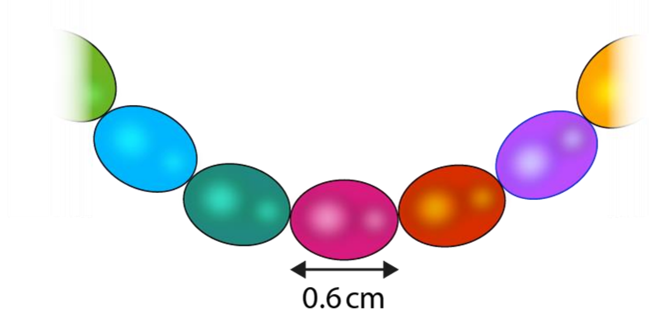
7×0.2 ◯ 5×0.2

6×0.8 ◯ 0.8×6

$4 \times 0.3 + 1 \times 0.3$ ◯ 5×0.3

Contextual problem:

'Dinesh is making a necklace of beads. Each bead is 0.6 cm long.'



'How long could the necklace be? Tick the possible lengths of the necklace.'

3.6 cm

4.6 cm

6.0 cm

3.2 cm

48 cm

Dòng nǎo jīn

'Jack writes:'

I know that $7 \times 7 = 49$

So $0.7 \times 7 = 0.49$

'Is he correct? Explain why/why not.'

1:7

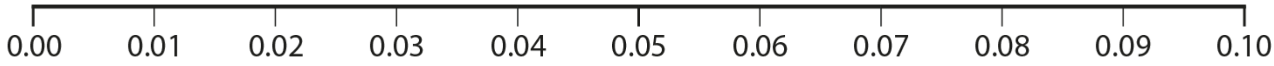
Now follow a similar progression for hundredths instead of tenths. Guidance here is kept brief; for more detail see steps *1:1–1:6* and adjust accordingly.

Review the fact that there are 100 hundredths in one whole, and ten hundredths in one-tenth. Revisit skip counting in hundredths up to and beyond one-tenth, using a number line and Gattegno chart. As in step *1:1*, count in two ways:

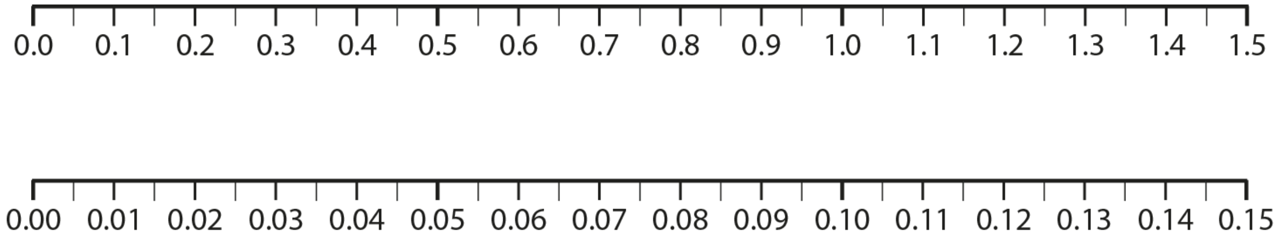
- *'Zero hundredths, one hundredth, two hundredths, three hundredths…'*
- *'Zero, zero-point-zero-one, zero-point-zero-two, zero-point-zero-three…'*

(For further guidance, see *Spine 1: Number, Addition and Subtraction* segment *1.24, Teaching point 3)*.

Number line – ten hundredths in one tenth:



Stacked number lines:



Gattegno chart:

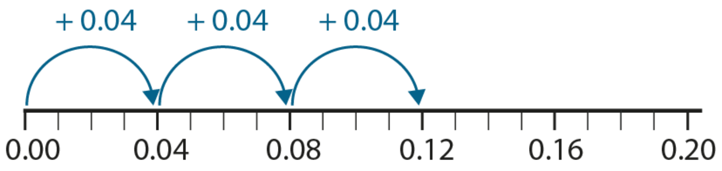
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

1:8

Now practise skip counting in multiples of hundredths (for example, in multiples of 0.04), building up 'decimal times tables', continuing to count in two ways, similar to step *1:2*.

As before, count again while recording the '0.04 times table'; initially use repeated addition equations (e.g. $0.04 + 0.04 + 0.04 = 0.12$) then, prompt children to recognise that multiplication equations can be used.

Skip counting in multiples of 0.04 – number line:



	$0 \times 0.04 = 0.00$	$0.04 \times 0 = 0.00$
0.04	$1 \times 0.04 = 0.04$	$0.04 \times 1 = 0.04$
$0.04 + 0.04 = 0.08$	$2 \times 0.04 = 0.08$	$0.04 \times 2 = 0.08$
$0.04 + 0.04 + 0.04 = 0.12$	$3 \times 0.04 = 0.12$	$0.04 \times 3 = 0.12$

1:9

Once you have built up the 0.04 times table as far as 12×0.04 (and 0.04×12), compare it to the four times table, and recite the facts in pairs, using unitising language to show the connection:

- *'Three times four ones is equal to twelve ones.'*
- *'Three times four hundredths is equal to twelve hundredths.'*

As noted in step *1:3*, multiplication facts within a decimal times table have been represented to the same precision; for example $5 \times 0.04 = 0.20$. Again, discuss when it is appropriate to omit zero place-value holders (i.e. we can write $5 \times 0.04 = 0.2$).

Comparing the 4 and 0.04 times tables:

$0 \times 4 = 0$	$0 \times 0.04 = 0.00$
$1 \times 4 = 4$	$1 \times 0.04 = 0.04$
$2 \times 4 = 8$	$2 \times 0.04 = 0.08$
$3 \times 4 = 12$	$3 \times 0.04 = 0.12$
$4 \times 4 = 16$	$4 \times 0.04 = 0.16$
$5 \times 4 = 20$	$5 \times 0.04 = 0.20$
$6 \times 4 = 24$	$6 \times 0.04 = 0.24$
$7 \times 4 = 28$	$7 \times 0.04 = 0.28$
$8 \times 4 = 32$	$8 \times 0.04 = 0.32$
$9 \times 4 = 36$	$9 \times 0.04 = 0.36$
$10 \times 4 = 40$	$10 \times 0.04 = 0.40$
$11 \times 4 = 44$	$11 \times 0.04 = 0.44$
$12 \times 4 = 48$	$12 \times 0.04 = 0.48$

Example comparison:

$6 \times 4 = 24$

6×4 ones $= 24$ ones

6×4 hundredths $= 24$ hundredths

$6 \times 0.04 = 0.24$

1:10 To avoid errors in the position of digits, when multiplying by a number with hundredths it can be useful to think of multiplying by tenths first, before considering multiplying by hundredths. Take a moment to explore how triplets of related facts from the 4, 0.4 and 0.04 times tables can be represented as arrays.

Use the following stem sentence to directly connect unitising in ones and hundredths:
'___ times ___ ones is equal to ___ ones, so ___ times ___ hundredths is equal to ___ hundredths.'

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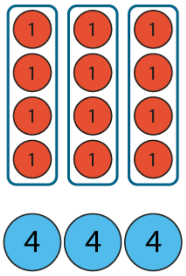
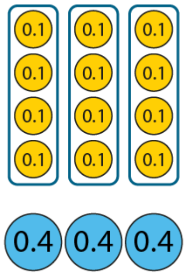
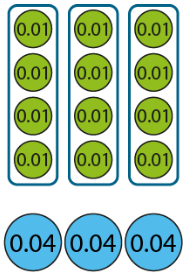
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$3 \times 4 = 12$</td><td>$3 \times 0.4 = 1.2$</td><td>$3 \times 0.04 = 0.12$</td></tr>
<tr><td>3 × 4 ones = 12 ones</td><td>3 × 4 tenths = 12 tenths</td><td>3 × 4 hundredths = 12 hundredths</td></tr>
</table>

- *'Three times four ones is equal to twelve ones, so three times four tenths is equal to twelve tenths.'*
- *'Three times four ones is equal to twelve ones, so three times four hundredths is equal to twelve hundredths.'*

1:11 Repeat steps *1:8–1:10* with other decimal times tables. Continue to use the stem sentence from step *1:10* to draw attention to unitising in ones vs unitising in hundredths. Also, vary the order in which the factors are presented (both *whole number* × *decimal fraction* and *decimal fraction* × *whole number* calculations).

1:12 As with multiplication by tenths (step *1:6*), provide children with practice, including:

- missing-number/symbol problems
- contextual problems, including both grouping and scaling structures, as shown opposite and here:
  - *'It takes 0.01 kg of baking powder to make one cake. How much baking powder is needed to make seven cakes?'*
  - *'A blue kite is flying 0.04 km above the ground. A hot air balloon is flying at nine times this height. How far above the ground is the hot air balloon?'*

Missing-number problems:

'Fill in the missing numbers.'

$4 \times 8 = \square$ $\square \times 6 = 18$

$4 \times 0.08 = \square$ $\square \times 0.6 = 0.18$

$5 \times \square = 45$ $\square \times 9 = 45$

$5 \times \square = 0.45$ $\square \times 9 = 0.45$

$12 \times 0.04 = \square$ $0 \times 0.07 = \square$

$0.05 \times 7 = \square$ $0.06 \times 10 = \square$

Missing-symbol problems:

'Fill in the missing symbols (<, > or =)'

9×0.04 ◯ 6×0.04

8×0.09 ◯ 9×0.09

7×0.02 ◯ 5×0.02

6×0.08 ◯ 0.08×6

$2 \times 0.07 + 1 \times 0.07$ ◯ $0.07 \times 5 - 0.07 \times 2$

Dòng nǎo jīn:

'Decide whether each statement is always true, sometimes true or never true. Explain your answers.'

- Ten groups of any number of tenths is always a whole number.
- Ten groups of any number of hundredths is always a whole number.

1:13 Converting between units of measure is a useful context for considering related multiplication facts with tenths and hundredths. Explore situations for which we can convert between units of measure in order to move from values with tenths or hundredths to integer values, or vice versa, as shown opposite. You can use money contexts as an opportunity to discuss the role of zeros and the convention when recording in pounds (see also *Spine 1: Number, Addition and Subtraction*, segment *1.25*).

- *'I am building a tower out of blocks. Each block is 0.8 cm tall. How tall could the tower be?'*

  1 cm = 10 mm

  so

  0.1 cm = 1mm

Number of blocks in the tower	Height of the tower in centimetres	Height of tower in millimetres
1	$1 \times 0.8 = 0.8$	$1 \times 8 = 8$
2	$2 \times 0.8 = 1.6$	$2 \times 8 = 16$
3	$3 \times 0.8 = 2.4$	$3 \times 8 = 24$

	When solving problems involving units of measure, encourage children to solve the same problem in different units of measure to describe equivalent situations and check that answers are reasonable; for example, with the kite and balloon example in the previous step: 9×0.04 km = 0.36 km and 0.04 km = 40 m 9×40 m = 360 m	• *'I put 60 p in my money box each week. How much money could there be in my money box?'* £1 = 100 p so £ 0.10 = 10 p

Number of weeks I've been saving	Amount of money in money box in pence	Amount of money in money box in pounds
5	$5 \times 60 = 300$	$5 \times 0.60 = 3.00$
6	$6 \times 60 = 360$	$6 \times 0.60 = 3.60$
7	$7 \times 60 = 420$	$7 \times 0.60 = 4.20$

• *'I fill a jug with water using 0.3 litre cups. How much water could be in the jug?'*

1 litre = 1,000 ml

so

0.1 litres = 100 ml

Number of cups of water in the jug	Volume of water in the jug in litres	Volume of water in the jug in millilitres
2	$2 \times 0.3 = 0.6$	$2 \times 300 = 600$
3	$3 \times 0.3 = 0.9$	$3 \times 300 = 900$
4	$4 \times 0.3 = 1.2$	$4 \times 300 = 1{,}200$

Teaching point 2:

Multiplying by 0.1 is equivalent to dividing by 10; multiplying by 0.01 is equivalent to dividing by 100. Understanding of place value can be used to divide a number by 10/100: when a number is divided by 10, the digits move one place to the right; when a number is divided by 100, the digits move two places to the right.

Steps in learning

2:1

This teaching point builds on what children learnt in:

- *2.13 Calculation: multiplying and dividing by 10 or 100*
- *2.17 Structures: using measures and comparison to understand scaling*
- *Spine 3: Fractions*, segment *3.6*.

Briefly recap the following points:

- To divide a multiple of ten by ten, we move the digits one place to the right; this is the same as removing the zero from the ones place, e.g. $90 \div 10 = 9$ (segment *2.13*).
- Multiplying a whole number by a unit fraction is the same as dividing that number by the denominator of the fraction, e.g. $15\text{ cm} \times \frac{1}{3} = 15\text{ cm} \div 3 = 5\text{ cm}$ (segments *2.17* and *3.6*).

Present a context with a multiplicand of 1 and a multiplier of 0.1; for example: '*William has a one-metre length of string. Mary's string is one-tenth times the length of William's string. How long is Mary's string?*'

Write the corresponding multiplication equation, using what children learnt in *Teaching point 1* to multiply by 0.1. Continue, systematically increasing the multiplicand (the length of William's string), until the patterns are revealed:

- $1 \times 0.1 = 0.1$
- $2 \times 0.1 = 0.2$
- $3 \times 0.1 = 0.3$
- …

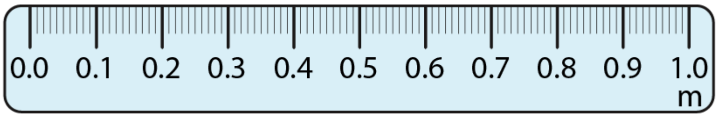
Use the following stem sentence to connect the equations to the context: ***'One-tenth of ___ metre(s) is ___ metre(s).'***

Then continue into two-digit multiplicands, including a multiplicand of 10.

- $10 \times 0.1 = 1$
- $11 \times 0.1 = 1.1$
- $12 \times 0.1 = 1.2$
- …
- $20 \times 0.1 = 2$

You can discuss with children when it is appropriate to omit zero place-value holders (i.e. here we have written $10 \times 0.1 = 1$, rather than $10 \times 0.1 = 1.0$).

- *'William has a* *<u>one</u>-metre length of string. Mary's string is one-tenth times the length of William's string. How long is Mary's string?'*



William's string: 1 m

Mary's string: 1 m × 0.1 = 0.1 m

  - *'One-tenth of one metre is zero-point-one metres.'*
  - *'One-tenth of one metre is one-tenth of a metre.'*

- *'William has a* *<u>two</u>-metre length of string. Mary's string is one-tenth times the length of William's string. How long is Mary's string?'*

William's string: 2 m

Mary's string: 2 m × 0.1 = 0.2 m

  - *'One-tenth of two metres is zero-point-two metres.'*
  - *'One-tenth of two metres is two-tenths of a metre.'*

⋮

- *'William has a* *<u>twelve</u>-metre length of string. Mary's string is one-tenth times the length of William's string. How long is Mary's string?'*

William's string: 12 m

Mary's string: 12 m × 0.1 = 1.2 m

  - *'One-tenth of twelve metres is one-point-two metres.'*
  - *'One-tenth of twelve metres is twelve-tenths of a metre.'*

⋮

2:2

Now connect to division. Ask children how else we can find one-tenth of a number, prompting for division by ten. Present the problem from step *2:1*, with a multiplicand of 10 and then of 20 (*10 × 0.1 = 10 ÷ 10 = 1* and *20 × 0.1 = 20 ÷ 10 = 2*), revisiting how we can use the place-value chart, moving the digits to the right, to divide by ten (as in segment *2.13 Calculation: multiplying and dividing by 10 or 100, Teaching point 2*). Use unitising language, as in segment *2.13*, to describe the 'before' and 'after' values; for example, *'We had two tens. We now have two ones.'*

Then apply the method to:

- cases with two-digit multiplicands that are not multiples of ten (i.e. two-digit dividends that are not multiples of ten, e.g. *12 × 0.1 = 12 ÷ 10 = 1.2*); draw attention to the fact that we now have a non-zero digit in the tenths column, and that we need to show this after the decimal point (when there was a '0' in the tenths place, we could just omit it)

- cases with a single-digit multiplicand (i.e. single-digit dividend, e.g. *$4 \times 0.1 = 4 \div 10 = 0.4$*); draw attention to the fact that we now need to include a zero place-value holder before the decimal point (according to convention).

Prompt children to notice that, in each case, we are moving the digits one place to the right, and generalise: ***'When a number is divided by ten, the digits move one place to the right.'*** Because children know that multiplying by 0.1 equivalent to dividing by ten, you can also generalise: ***'When a number is multiplied by zero-point-one/one-tenth, the digits move one place to the right.'***

Gradually remove the scaffolding of the place-value chart until children can complete calculations without them.

Dividing a multiple of ten by ten:

× 0.1 × 0.1 × 0.1

Divide '20' by ten. ↓ ÷ 10

1,000s	100s	10s	1s
		2	0
			2

0

÷ 10 ÷ 10 ÷ 10

$20 \times 0.1 = 2$ $20 \div 10 = 2$

Dividing a two-digit number (that is not a multiple of ten) by ten:

× 0.1 × 0.1 × 0.1 × 0.1

Divide '12' by ten. ↓ ÷ 10

1,000s	100s	10s	1s	0.1s
		1	2	
			1	2

÷ 10 ÷ 10 ÷ 10 ÷ 10

$12 \times 0.1 = 1.2$ $12 \div 10 = 1.2$

Dividing a single-digit number by ten:

× 0.1 × 0.1 × 0.1 × 0.1

Divide '4' by ten. ↓ ÷ 10

1,000s	100s	10s	1s	0.1s
			4	
			0	4

÷ 10 ÷ 10 ÷ 10 ÷ 10

$4 \times 0.1 = 0.4$ $4 \div 10 = 0.4$

2:3 Extend to cases with a multiplicand/dividend with a whole number of tenths, from 0.1–0.9 (e.g. *0.9 × 0.1 = 0.9 ÷ 10 = 0.09*). Draw attention to the fact that we now need to include a zero place-value holder before the decimal point (according to convention).

× 0.1 × 0.1 × 0.1 × 0.1 × 0.1

Divide '0.9' by ten. ↓ ÷ 10

1,000s	100s	10s	1s	0.1s	0.01s
			0	9	
			0	0	9

÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10

$0.9 \times 0.1 = 0.09$ $0.9 \div 10 = 0.09$

2:4 Now repeat steps *2:1–2:3* for multiplying by 0.01 and dividing by 100:

- Apply the contextual string problem from step *2:1*, now with a scale factor of 0.01/one-hundredth; use what children learnt in *Teaching point 1* to multiply by 0.01.
- Connect to division by 100, using the place-value chart, and generalising:
  - ***'When a number is divided by one hundred, the digits move two places to the right.'***
  - ***'When a number is multiplied by zero-point-zero-one/one-hundredth, the digits move two places to the right.'***

Gradually remove the scaffolding of the place-value chart until children can complete calculations without them.

Multiplying by 0.01 – string context:

'Mary's string is one-hundredth the length of William's string. How long is Mary's string?'

Length of William's string	2 m	12 m	200 m
Length of Mary's string	2 m × 0.01 = 0.02 m	12 m × 0.01 = 0.12 m	200 m × 0.01 = 2 m
	• *'One-hundredth of two metres is zero-point-zero-two metres.'* • *'One-hundredth of two metres is two-hundredths of a metre.'*	• *'One-hundredth of twelve metres is zero-point-one-two metres.'* • *'One-hundredth of twelve metres is twelve-hundredths of a metre.'*	• *'One-hundredth of two hundred metres is two metres.'*

Connecting to division by 100:

- Example 1 – dividend is a multiple of 100

× 0.01

Divide 200 by 100. ↓ ÷ 100

1,000s	100s	10s	1s		
	2	0	0		
			2	0	0

÷ 100

$200 \times 0.01 = 2$ $200 \div 100 = 2$

- Example 2 – dividend is a three-digit multiple of ten

× 0.01 × 0.01

Divide 120 by 100. ↓ ÷ 100

1,000s	100s	10s	1s	0.1s	0.01s
	1	2	0		
			1	2	0

÷ 100 ÷ 100

$120 \times 0.01 = 1.2$ $120 \div 100 = 1.2$

- Example 3 – dividend is a two-digit number

× 0.01 × 0.01

Divide 12 by 100. ↓ ÷ 100

1,000s	100s	10s	1s	0.1s	0.01s
		1	2		
			0	1	2

÷ 100 ÷ 100

$12 \times 0.01 = 0.12$ $12 \div 100 = 0.12$

- Example 4 – dividend is a single-digit number

× 0.01

Divide 4 by 100. ↓ ÷ 100

1,000s	100s	10s	1s	0.1s	0.01s
			4		
			0	0	4

÷ 100

$4 \times 0.01 = 0.04$ $4 \div 100 = 0.04$

2:5

Complete this teaching point by providing children with varied practice, including:

- multiplying by 0.1, dividing by 10, and connecting the two operations
- multiplying by 0.01, dividing by 100, and connecting the two operations.

Also use missing-number problems, and contextual problems such as:

- *'Francesca has saved £75. Marley has saved one-tenth that amount. How much money has Marley saved?'*
- *'Nia measures the height of her desk. It is 64 cm tall. How tall is Nia's desk in metres?'*

Missing-number problems:

'Fill in the missing numbers.'

$150 \div 10 = \square$ $150 \div 100 = \square$

$15 \div 10 = \square$ $15 \div 100 = \square$

$700 \times 0.1 = \square$ $700 \times 0.01 = \square$

$70 \times 0.1 = \square$ $70 \times 0.01 = \square$

$7 \times 0.1 = \square$ $7 \times 0.01 = \square$

$0.7 \times 0.1 = \square$ $0.7 \times 0.01 = \square$

$50 \times \square = 5$ $750 \div \square = 75$

$5 \times \square = 0.5$ $75 \div \square = 7.5$

$0.5 \times \square = 0.05$ $750 \div \square = 7.5$

$80 \div \square = 80 \times 0.1$ $63 \div 100 = 63 \times \square$

$80 \div \square = 80 \times 0.01$ $\square \div 10 = 63 \times 0.1$

Teaching point 3:

To multiply a single-digit number by a decimal fraction with up to two decimal places, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the product by dividing by 10 or 100.

Steps in learning

3:1

Guidance

This teaching point builds on known facts and strategies in order to multiply a single-digit number by any decimal fraction with up to two decimal places (not just those with a whole number of tenths between 0.1 and 0.9, or with a whole number of hundredths between 0.01 and 0.09).

Begin with an example that doesn't require application of short multiplication. For example, children should already know that *4 × 25 = 100* (see *Spine 1: Number, Addition and Subtraction*, segment *1.17, Teaching point 1* to recap, and practice counting up to and beyond 100 in multiples of 25, supported by a number line). Use this fact, and unitising language to solve the related problems *4 × 2.5* and *4 × 0.25*, as shown opposite. Then compare each of these calculations to the whole-number calculation (*4 × 25*), using the following stem sentences:

- ***'___ is one-tenth the size of ___, so ___ times ___ is one-tenth the size of ___ times ___.'***
- ***'___ is one-hundredth the size of ___, so ___ times ___ is one-hundredth the size of ___ times ___.'***

Generalise:

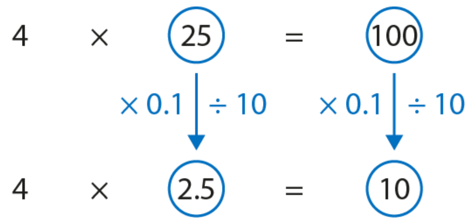
- ***'If one factor is made one-tenth times the size, the product will be one-tenth times the size.'***
- ***'If one factor is made one-hundredth times the size, the product will be one-hundredth times the size.'***

Representations

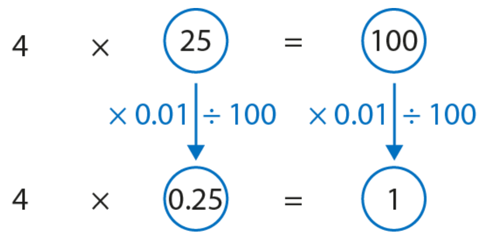
Using known facts and unitising:

4 × 25 = 100	4 × 25 ones = 100 ones
4 × 2.5 = 10	4 × 25 tenths = 100 tenths; 10 tenths = 1 so 100 tenths = 10
4 × 0.25 = 1	4 × 25 hundredths = 100 hundredths 100 hundredths = 1

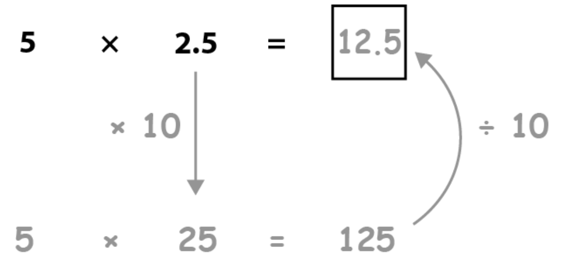
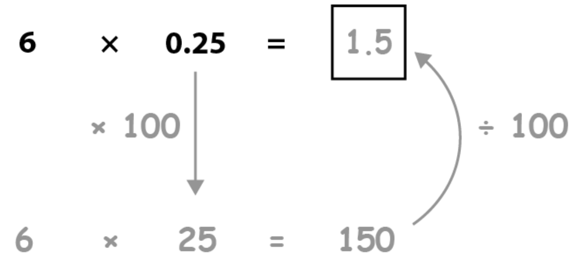
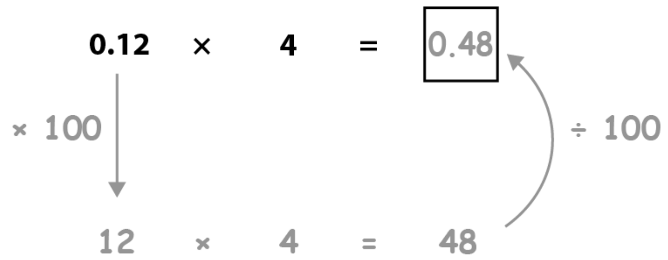
Comparing the equations:



- *'Two-point-five is one-tenth the size of twenty-five, so four times two-point-five is one-tenth the size of four times twenty-five.'*



- *'Zero-point-two-five is one-hundredth the size of twenty-five, so four times zero-point-two-five is one-hundredth the size of four times twenty-five.'*

3:2	Present a similar problem, such as *5 × 2.5*, and ask children to identify what fact we can use to solve this (*5 × 25*). Draw attention the process involved: • We are multiplying by 2.5; to convert this into a whole number, we multiply by 10, resulting in 25. • We calculate using whole numbers. • We then need to divide the product by 10 to solve the original calculation. Similarly, present a calculation such as *6 × 0.25*, and work through the process of converting the multiplier to a whole number (multiplying by 100), performing the calculation using whole numbers, then adjusting the product (dividing by 100). Before continuing, check children's understanding of the process by providing them with a contextual problem connected to multiplication by 25; for example: '*I buy three raffle tickets that cost £0.25 each. How much do they cost altogether?*'	Single-digit number multiplied by a number with one decimal place:  5 × 25 ones = 125 ones so 5 × 25 tenths = 125 tenths Single-digit number multiplied by a number with two decimal places:  6 × 25 ones = 150 ones so 6 × 25 hundredths = 150 hundredths
3:3	Now explore other examples related to known facts or mental strategies. Vary the order of the numbers, including both *whole number × decimal fraction* and *decimal fraction × whole number* calculations. Note that there are often several reasonable strategies for a given calculation. For *Example 2* on the next page, as well as the strategy shown, alternatives include: • using the doubling/halving approach that children learnt in segment *2.18 Using equivalence to calculate*: $4 \times 1.5 = 2 \times 3$ $= 6$	Example 1 – using a known times-table fact: 

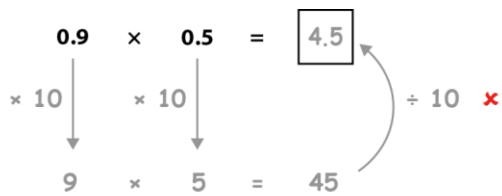
- using the distributive law:

$$\begin{aligned} 4 \times 1.5 &= 4 \times 1 \; + \; 4 \times 0.5 \\ &= \;\; 4 \;\; + \;\; 2 \\ &= 6 \end{aligned}$$

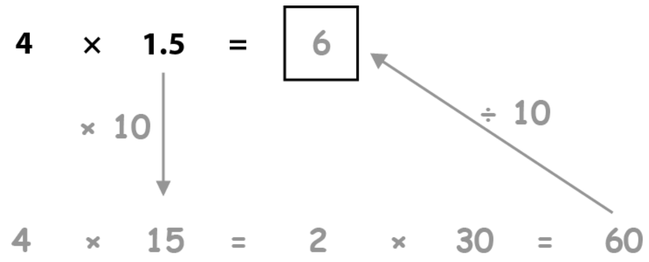
Always encourage children to work flexibly and to choose sensible strategies.

Continue using the stem sentences and generalisations from step *3:1* until children have a solid understanding of the underlying mathematics. You can begin to use the following stem sentence: ***'I move the digits of the number being multiplied ___ places to the left until I get a whole number; then I multiply; then I move the digits of the product ___ places to the right.'***

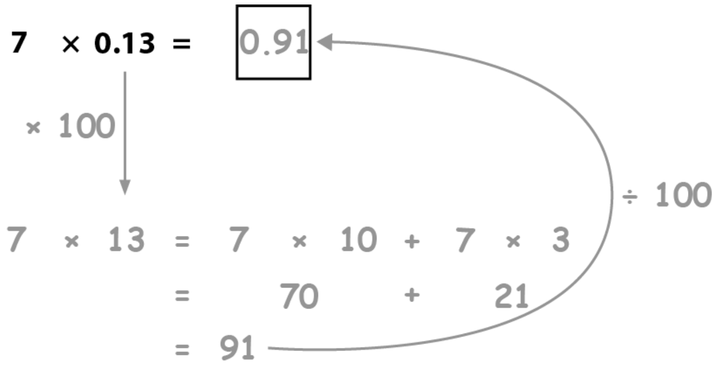
Irrespective of which stem sentences and notation you use, ensure that children do not 'overapply' the concept when they encounter calculations that involve multiplication of one decimal fraction by another (e.g. *0.9* × *0.5*) i.e.:



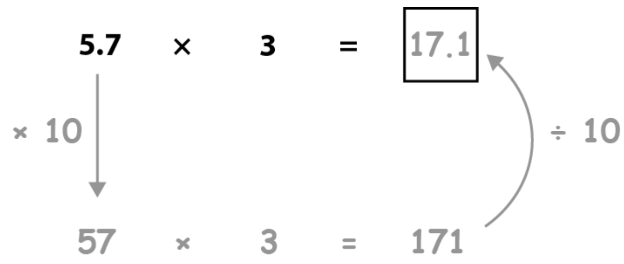
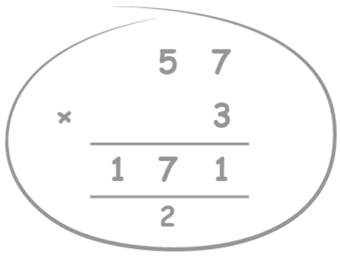
Example 2 – using a repeated-doubling strategy:

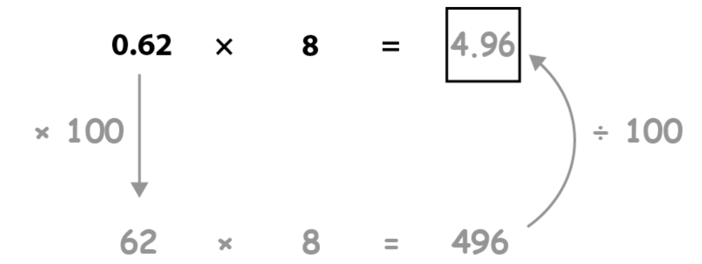
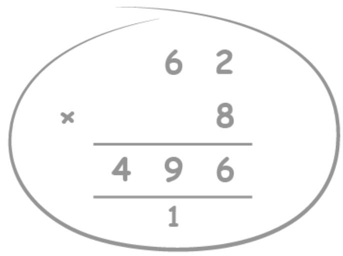


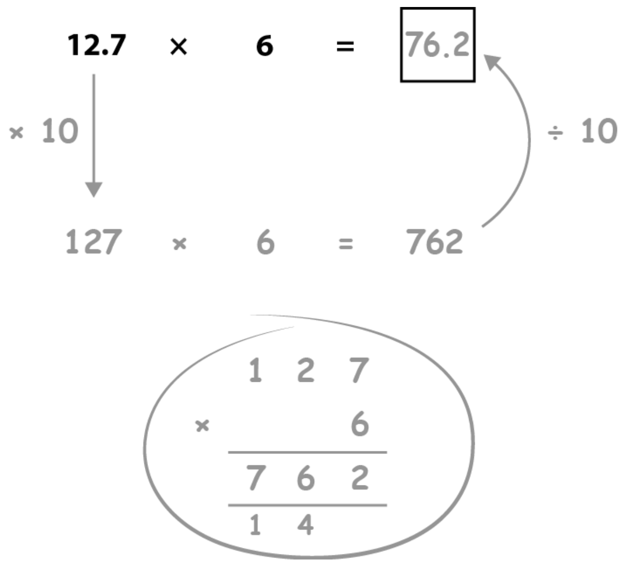
Example 3 – using the distributive law:



3:4	Provide children with some practice, initially scaffolded as calculation sequences, as shown opposite. Keep to calculations for which the relevant whole-number calculation can be solved using known facts or mental strategies.	Missing-number problems: *'Fill in the missing numbers.'* $6 \times 15 = \square$ $6 \times 1.5 = \square$ $6 \times 0.15 = \square$ $9 \times \square = 99$ $9 \times \square = 9.9$ $9 \times \square = 0.99$ $\square \times 12 = 36$ $\square \times 1.2 = 3.6$ $\square \times 0.12 = 0.36$ $2 \times 0.75 = \square$ $0.05 \times 8 = \square$ $5 \times 1.3 = \square$

3:5	Now apply the process to cases where the whole-number calculation is performed using the short multiplication algorithm. First consider a calculation with scaling by ten, e.g. *5.7 × 3*: • To convert 5.7 into a whole number, we multiply by 10, resulting in 57. • Calculate 57 × 3 using short multiplication. • We then need to divide the product by 10 to solve the original calculation. Establish the routine of checking that the answer is reasonable, using estimation to sense-check the result. For example, 5.7 is slightly less than 6 and *6 × 3 = 18*, so the result should be slightly less than 18. Then repeat for a calculation with scaling by 100, for example *0.62 × 8*. Note that most children should be able to multiply a two-digit number by a single-digit number using informal methods (simply partitioning the two-digit number into tens and ones, multiplying, then adding the partial products). The short multiplication algorithm is also a viable strategy, and in the next step the process will be extended to cases where short multiplication is the most efficient strategy.	Example 1:   57 ones × 3 = 171 ones so 57 tenths × 3 = 171 tenths Check by estimation • 5.7 is slightly less than 6. • 6 × 3 = 18 • So, 5.7 × 3 is slightly less than 18.

		Example 2:   62 ones × 8 = 496 ones so 62 hundredths × 8 = 496 hundredths Check by estimation • 0.62 is slightly greater than 0.6. • 6 tenths × 8 = 48 tenths = 4.8 • So, 0.62 × 8 is slightly greater than 4.8.

3:6	Now extend to examples where the decimal fraction has three-significant figures. Vary the order in which the numbers are presented in the initial calculation, including both *whole number* × *decimal fraction* and *decimal fraction* × *whole number* calculations.	Example 1:  127 ones × 6 = 762 ones so 127 tenths × 6 = 762 tenths Check by estimation • 12.7 is slightly less than 13. • 13 × 6 = 10 × 6 + 3 × 6 = 60 + 18 = 78 • So, 12.7 × 6 is slightly less than 78.

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<td></td>
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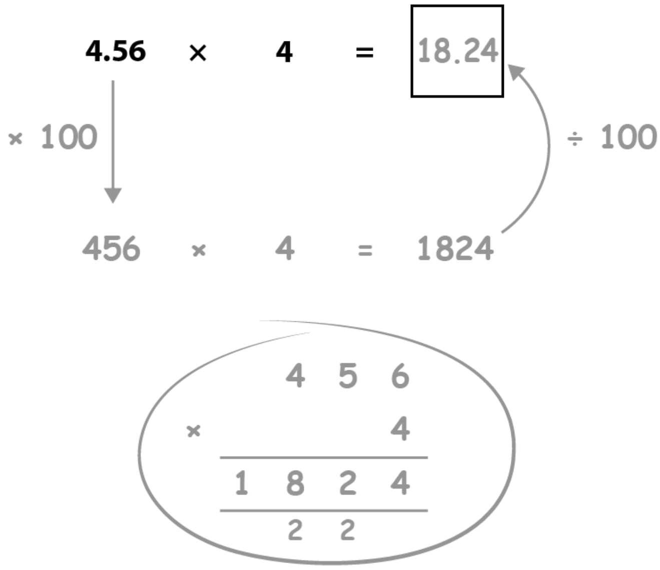
<td>Example 2:

456 ones × 4 = 1824 ones
so
456 hundredths × 4 = 1824 hundredths

Check by estimation
• 4.56 is slightly greater than 4.5.
• 4.5 × 4 = 9 × 2
= 18
• So, 4.56 × 4 is slightly greater than 18.</td>
</tr>
<tr>
<td>3:7</td>
<td colspan="2">Look at the calculations completed so far in steps 3:5 and 3:6, and draw attention to the position of the decimal point. Insert the decimal points into the completed short multiplication calculations, and ask children what they notice about the position of the decimal point in the product (it is aligned with the decimal point in the decimal fraction being multiplied). When a decimal point is placed before the first digit, draw attention to the use of a zero in the ones place as a place-value holder (62 → .62 → 0.62).</td>
</tr>
</table>

	$\begin{array}{cccc} & & 5 & 7 \\ \times & & & 3 \\ \hline & 1 & 7 & 1 \\ \hline & & {\scriptstyle 2} & \end{array}$ 57 ones × 3 = 171 ones so 57 tenths × 3 = 171 tenths 5.7 × 3 = 17.1	$\begin{array}{cccc} & & 5. & 7 \\ \times & & & 3 \\ \hline & 1 & 7. & 1 \\ \hline & & {\scriptstyle 2} & \end{array}$
	$\begin{array}{cccc} & 0 & 6 & 2 \\ \times & & & 8 \\ \hline & 4 & 9 & 6 \\ \hline & & {\scriptstyle 1} & \end{array}$ 62 ones × 8 = 496 ones so 62 hundredths × 8 = 496 hundredths 0.62 × 8 = 4.96	$\begin{array}{cccc} & 0. & 6 & 2 \\ \times & & & 8 \\ \hline & 4. & 9 & 6 \\ \hline & & {\scriptstyle 1} & \end{array}$
	$\begin{array}{cccc} & 1 & 2 & 7 \\ \times & & & 6 \\ \hline & 7 & 6 & 2 \\ \hline & {\scriptstyle 1} & {\scriptstyle 4} & \end{array}$ 127 ones × 6 = 762 ones so 127 tenths × 6 = 762 tenths 12.7 × 6 = 76.2	$\begin{array}{cccc} & 1 & 2. & 7 \\ \times & & & 6 \\ \hline & 7 & 6. & 2 \\ \hline & {\scriptstyle 1} & {\scriptstyle 4} & \end{array}$
	$\begin{array}{ccccc} & & 4 & 5 & 6 \\ \times & & & & 4 \\ \hline & 1 & 8 & 2 & 4 \\ \hline & & {\scriptstyle 2} & {\scriptstyle 2} & \end{array}$ 456 ones × 4 = 1,824 ones so 456 hundredths × 4 = 1,824 hundredths 4.56 × 4 = 18.24	$\begin{array}{ccccc} & & 4. & 5 & 6 \\ \times & & & & 4 \\ \hline & 1 & 8. & 2 & 4 \\ \hline & & {\scriptstyle 2} & {\scriptstyle 2} & \end{array}$

3:8	Now present a new problem, such as *2.46* $\times$ *3*, and ask children whether we need to find a related integer calculation to complete it. Work through the process of short multiplication with the decimal point already in place. Repeat for several examples, working towards the generalisation: ***'In short multiplication, if there is a decimal point in the number being multiplied, put a decimal point in the product; line it up with the decimal point in the number being multiplied.'***	Step 1 – lay out the calculation: 2 . 4 6 × 3 ——— ——— Step 2 – write the decimal point for the product: 2 . 4 6 × 3 ——— . ——— Step 3 – perform the calculation, with unitising: 2 . 4 6 × 3 ——— 7 . 3 8 ——— 1 1 • 3 × 6 hundredths = 18 hundredths 18 hundredths = 1 tenth and 8 hundredths *'Write "1" below the tenths column and "8" in the hundredths column.'* • 3 × 4 tenths = 12 tenths 12 tenths + 1 tenth = 13 tenths 13 tenths = 1 one + 3 tenths *'Write "1" below the ones column and "3" in the tenths column.'* • 3 × 2 ones = 6 ones 6 ones + 1 one = 7 ones *'Write "7" in the ones column.'*
3:9	Provide children with practice similar to that in step *3:4*, but now for examples for which the relevant whole-number calculation is 'best' solved using short multiplication. Vary the order in which the numbers are presented (i.e. sometimes present the decimal fraction first, and sometimes present the whole number first).	*'Complete these calculations.'* $238 \times 3 =$ $6 \times 423 =$ $23.8 \times 3 =$ $6 \times 42.3 =$ $2.38 \times 3 =$ $6 \times 4.23 =$ $43.7 \times 5 =$ $4 \times 3.96 =$

3:10	To complete this teaching point, provide children with contextual practice, including both the grouping and scaling structures of multiplication. Continue to vary the order in which the numbers are presented (including both *whole number × decimal fraction* and *decimal fraction × whole number* calculations):
	• *'Matteo walks six times the distance that Lorenzo walks. If Lorenzo walks 2.3 km, how far does Matteo walk?'*
	• *'Strawberries cost £4.23 per kilogram. Yulia buys five kilograms of strawberries to make some jam. How much does she spend?'*
	Ensure that children are able to solve such calculations as part of multistep problems:
	• *'A shop sells shampoo in two different bottle sizes. A large bottle contains 0.35 litres of shampoo, and a small bottle contains 0.25 litres. Megan buys two large bottles and five small bottles. How much shampoo does she have altogether (in litres)?'*
	• *'Joseph buys four magazines.'*
	  ◦ *'Each magazine costs £2.35. How much does Joseph spend?'*
	  ◦ *'How much change does Joseph get from a £20 note?'*
	• *'Becca has three times as much flour as Unber. Unber has 0.8 kg of flour.'*
	  ◦ *'How much flour does Becca have?'*
	  ◦ *'Becca needs 3 kg of flour to make enough cakes for the school fete. How much more does she need?'*	

Teaching point 4:

If the multiplier is less than one, the product is less than the multiplicand; if the multiplier is greater than one, the product is greater than the multiplicand.

Steps in learning

4:1 Since children have now learnt to multiply whole numbers by decimal fractions, they should already be experientially aware of the fact that when the multiplier is less than one, the product is less than the multiplicand (and, conversely, that if the multiplier is greater than one, the product is greater than the multiplicand); however, it is worth spending a little time formalising this idea. These rules can be used as another layer of answer-checking, and also challenge the common misconception that multiplication always 'makes things bigger'/increases the value.

Use the scaling structure of multiplication to expose these rules. Children have already learnt what happens to the product when the multiplier is made:

- 100 times the size or 10 times the size (segment *2.13 Calculation: multiplying and dividing by 10 or 100*)
- one times the size (segment *2.17 Structures: using measures and comparison to understand scaling*)
- one-tenth the size or one-hundredth the size (segment *2.17* and this segment).

Starting with a known fact (e.g. *2 × 6 = 12*), build a sequence of related calculations, involving multiplication of the same number by a multiple of 100, 10, 1, 0.1 and 0.01, e.g.:

- 2 × 600 = 1,200
- 2 × 60 = 120
- **2 × 6 = 12**
- 2 × 0.6 = 1.2
- 2 × 0.06 = 0.12

Use the stem sentence: ***'If one factor is made ___ times the size, the product will be ___ times the size.'***

You can present the calculations using place-value charts, as shown below, to draw attention to the structure.

	1,000s	100s	10s	1s	0.1s	0.01s		1,000s	100s	10s	1s	0.1s	0.01s
2 ×		6	0	0 •			=	1	2	0	0 •		
2 ×			6	0 •			=		1	2	0 •		
2 ×				**6** •			=			**1**	**2** •		
2 ×				0 •	6		=				1 •	2	
2 ×				0 •	0	6	=				0 •	1	2

$2 \times 600 = 1{,}200$	• *'If one factor is made one hundred times the size, the product will be one hundred times the size.'*
$2 \times 60 = 120$	• *'If one factor is made ten times the size, the product will be ten times the size.'*
$2 \times 6 = 12$	
$2 \times 0.6 = 1.2$	• *'If one factor is made one tenth times the size, the product will be one tenth times the size.'*
$2 \times 0.06 = 0.12$	• *'If one factor is made one hundredth times the size, the product will be one hundredth times the size.'*

4:2

Now ask children to compare the product with the multiplicand (here, 2) in each equation, prompting them to notice that when the multiplier is greater than one, the product is larger than the multiplicand, and vice versa.

Repeat with a different sequence of calculations, for example:

- $57 \times 300 = 17{,}100$
- $57 \times 30 = 1{,}710$
- **$57 \times 3 = 171$**
- $57 \times 0.3 = 17.1$
- $57 \times 0.03 = 1.71$

Work towards the following generalisations:

- ***'When a number is multiplied by a value greater than one, the product is greater than the original number.'***
- ***'When a number is multiplied by a value less than one, the product is less than the original number.'***

You may also wish to remind children of the generalisation introduced in segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*: ***'When one is a factor, the product is equal to the other factor.'***

		> 1		> 2
2	×	600	=	1,200
2	×	60	=	120
2	×	6	=	12
2	×	0.6	=	1.2
2	×	0.06	=	0.12
		< 1		< 2

Summary:

2 × 'number > 1' = 'number > 2'

2 × 1 = 2

2 × 'number < 1' = 'number < 2'

4:3	Now investigate whether the rules still hold for decimal fractions greater than one, to address the misconception that multiplication by a decimal always makes a number smaller. Again, use known facts and strategies to develop a sequence of equations, as illustrated opposite, and check as a class whether the generalisations from step *4:2* apply here.	< 1 < 7 7×7 tenths $= 49$ tenths $7 \times 0.7 = 4.9$ 7×8 tenths $= 56$ tenths $7 \times 0.8 = 5.6$ 7×9 tenths $= 63$ tenths $7 \times 0.9 = 6.3$ 7×10 tenths $= 70$ tenths $7 \times 1 = 7$ 7×11 tenths $= 77$ tenths $7 \times 1.1 = 7.7$ 7×12 tenths $= 84$ tenths $7 \times 1.2 = 8.4$ > 1 > 7
4:4	To complete this teaching point, provide children with practice completing sequences of missing-number/symbol problems Encourage children to explain their answers using unitising language and the generalisations from step *4:2*.	Missing-number problems: *'Fill in the missing numbers.'* $9 \times \square = 36$ $5 \times 0.23 = 1.15$ $9 \times 40 = \square$ $5 \times \square = 115$ $9 \times \square = 3.6$ $5 \times 2.3 = \square$ $9 \times 0.04 = \square$ $5 \times 230 = \square$ Missing-symbol problems: *'Fill in the missing symbols (>, < and =).'* $8 \times 0.7 \bigcirc 8$ $3 \bigcirc 3 \times 0.09$ $8 \times 1 \bigcirc 8$ $3 \bigcirc 3 \times 0.9$ $8 \times 1.7 \bigcirc 8$ $3 \bigcirc 3 \times 9$ Dòng nǎo jīn: *'Write a single digit in each box to make this equation correct.'* $2.5 > 0.\square \times \square$ *'How many different solutions are there?'*

Teaching point 5:

To divide any decimal fraction with up to two decimal places by a single-digit number, convert the decimal fraction to an integer by multiplying by 10 or 100, perform the resulting calculation using an appropriate strategy, then adjust the quotient by dividing by 10 or 100.

Steps in learning

5:1

Guidance

This teaching point follows a similar progression to *Teaching point 3*, but now explores strategies for dividing a decimal fraction by a single-digit number. Before beginning, you may wish to briefly review the effect on the quotient of making the dividend 10 or 100 times the size (see segment *2.13 Calculation: multiplying and dividing by 10 or 100*).

Begin by comparing *decimal fraction ÷ single-digit* calculations that can be related to known facts, as shown opposite. Use unitising language to solve the decimal fraction calculations based on the known integer calculation, or use the related multiplication facts. Then compare the resulting division equations.

Use the following stem sentences:

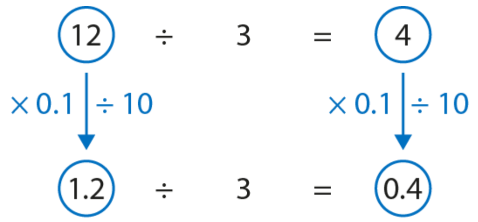
- ***'___ is one-tenth the size of ___, so ___ divided by ___ is one-tenth the size of ___ divided by ___.'***
- ***'___ is one-hundredth the size of ___, so ___ divided by ___ is one-hundredth the size of ___ divided by ___.'***

Representations

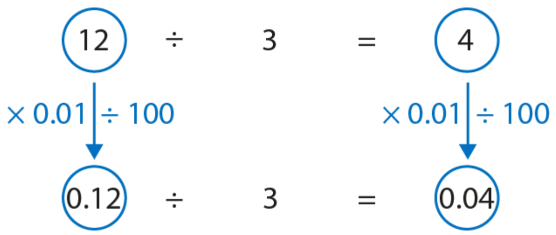
Using known facts and unitising:

12 ÷ 3 = 4	12 ones ÷ 3 = 4 ones
1.2 ÷ 3 = 0.4	12 tenths ÷ 3 = 4 tenths
0.12 ÷ 3 = 0.04	12 hundredths ÷ 3 = 4 hundredths

Comparing the equations:



- *'One-point-two is one-tenth the size of twelve, so one-point-two divided by three is one-tenth the size of twelve divided by three.'*



- *'Zero-point-one-two is one-hundredth the size of twelve, so zero-point-one-two divided by three is one-hundredth the size of twelve divided by three.'*

<table>
<tr>
<td>5:2</td>
<td>
Present a similar problem, such as 5.6 ÷ 8, and ask children to identify what fact we can use to solve this (56 ÷ 8). Draw attention to the process involved:

Multiply the dividend by ten to convert it to a whole number.
Calculate using whole numbers.
Then divide the quotient by ten to solve the original calculation.

Similarly, present a calculation such as 0.56 ÷ 8, and work through the process of converting the dividend to a whole number (multiplying by 100), performing the calculation using whole numbers, then adjusting the quotient (dividing by 100).

Generalise:

'If the dividend is made one-tenth times the size, the quotient will be one-tenth times the size.'
'If the dividend is made one-hundredth times the size, the quotient will be one-hundredth times the size.'

Explore other examples related to known facts or mental strategies. Once children have a solid understanding of the process, you can begin to use the following stem sentence: 'I move the digits of the dividend ___ places to the left until I get a whole number; then I divide; then I move the digits of the quotient ___ places to the right.'
</td>
<td>
Decimal fraction with one decimal place divided by a single-digit number:

5.6 ÷ 8 = [0.7]
× 10 ↓   ↑ ÷ 10
56 ÷ 8 = 7

56 ones ÷ 8 = 7 ones
so
56 tenths ÷ 8 = 7 tenths

Decimal fraction with two decimal places divided by a single-digit number:

0.56 ÷ 8 = [0.07]
× 100 ↓   ↑ ÷ 100
56 ÷ 8 = 7

56 ones ÷ 8 = 7 ones
so
56 hundredths ÷ 8 = 7 hundredths
</td>
</tr>
<tr>
<td>5:3</td>
<td>
Now explore an example such as 0.3 ÷ 6. Following the same process as in the previous step would result in multiplying 0.3 by ten to transform to the related whole-number calculation 3 ÷ 6. We could recognise that 3 ÷ 6 = 0.5 (so 0.3 ÷ 6 = 0.05); however, children have not explicitly worked with problems where the divisor is greater than the dividend. Instead, we can multiply 0.3 by 100 to transform to a known fact (30 ÷ 6).
</td>
<td>
0.3 ÷ 6 = [0.05]
× 100 ↓   ↑ ÷ 100
30 ÷ 6 = 5
</td>
</tr>
</table>

	Work through several problems until children can confidently transform calculations to whole-number calculations where the dividend is greater than the divisor. The process is the same, but now the 'first' whole-number calculation we reach is not a known fact, so we must effectively multiply the dividend by ten again.	
5:4	Provide children with some practice, initially scaffolded as calculation sequences, as shown opposite. Keep to calculations for which the relevant whole-number calculation can be solved using known facts or mental strategies.	Missing-number problems: *'Fill in the missing numbers.'* $6 \times 4 = \square$ $24 \div 6 = \square$ $6 \times 0.4 = \square$ $2.4 \div 6 = \square$ $6 \times 0.04 = \square$ $0.24 \div 6 = \square$ $99 \div 9 = \square$ $9.9 \div 9 = \square$ $0.99 \div 9 = \square$ $15 \div 5 = \square$ $\square \div 5 = 6$ $1.5 \div 5 = \square$ $\square \div 5 = 0.6$ $0.15 \div 5 = \square$ $\square \div 5 = 0.06$ $45 \div \square = 9$ $4.5 \div \square = 0.9$ $0.45 \div \square = 0.09$ $0.6 \div 2 = \square$ $0.36 \div 3 = \square$ $0.4 \div 8 = \square$ $4.2 \div 7 = \square$

5:5

Now apply the process to cases where the whole-number calculation is performed using the short division algorithm.

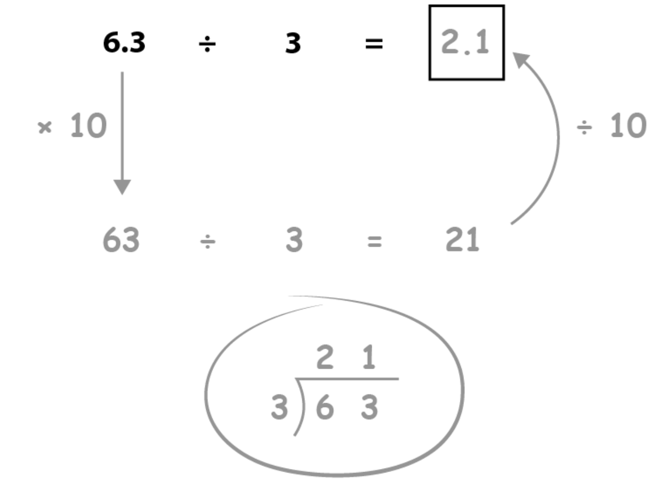
First consider calculations where the dividend is a decimal fraction with two significant figures, including:

- problems where we scale by 10 to reach a whole-number calculation (e.g. *6.3 ÷ 3*)
- problems where we scale by 100 to reach a whole-number calculation (e.g. *0.85 ÷ 5*).

Note that most children should be able to perform the related integer calculations using mental strategies, but short division is used here to exemplify the process.

As with multiplication in *Teaching point 3*, encourage children to use estimation to check that their answers are reasonable.

Decimal fraction with one decimal place and two significant figures divided by a single-digit number:



63 ones ÷ 3 = 21 ones

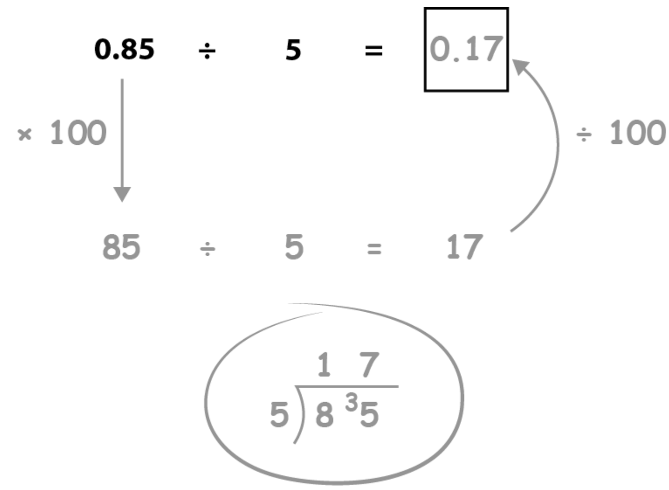
so

63 tenths ÷ 3 = 21 tenths

Check by estimation

- 6.3 is slightly greater than 6.
- 6 ÷ 3 = 2
- So, 6.3 ÷ 3 is slightly greater than 2.

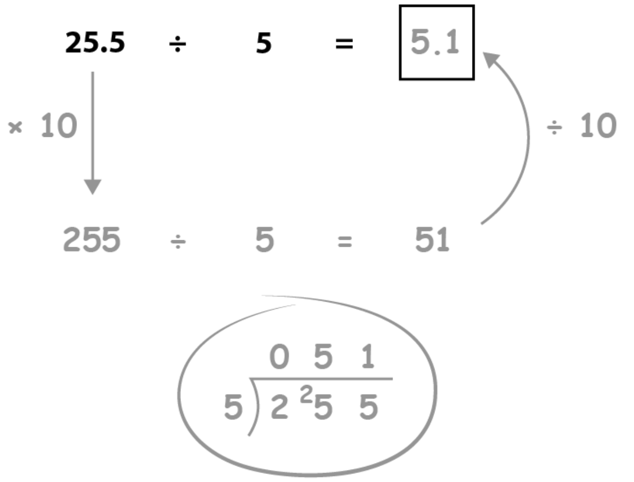
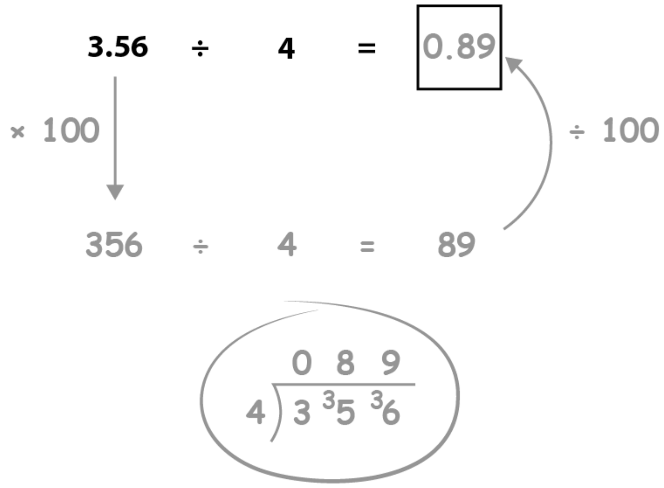
Decimal fraction with two decimal places and two significant figures divided by a single-digit number:



85 ones ÷ 5 = 17 ones

so

85 hundredths ÷ 5 = 17 hundredths

		Check by estimation • 0.85 is slightly less than 1 • 1 ÷ 5 = 0.2 (because 10 ÷ 5 = 2) • So, 0.85 ÷ 5 is slightly less than 0.2.
5:6	Now progress to calculations where the dividend is a decimal fraction with *three* significant figures, again including: • problems where we scale by 10 to reach a whole-number calculation (e.g. *25.5 ÷ 5*) • problems where we scale by 100 to reach a whole-number calculation (e.g. *3.56 ÷ 4*).	Decimal fraction with one decimal place and three significant figures divided by a single-digit number:  255 ones ÷ 5 = 51 ones so 255 tenths ÷ 5 = 51 tenths Check by estimation • 25.5 is slightly greater than 25. • 25 ÷ 5 = 5 • So, 25.5 ÷ 5 is slightly greater than 5. Decimal fraction with two decimal places and three significant figures divided by a single-digit number: 

		356 ones ÷ 4 = 89 ones so 356 hundredths ÷ 4 = 89 hundredths Check by estimation • 3.56 is slightly less than 4. • 4 ÷ 4 = 1 • So, 3.56 ÷ 4 is slightly less than 1.

5:7 Look at the calculations completed so far in steps *5:5* and *5:6*, and draw attention to the position of the decimal point. Put the decimal points into position on the completed short division calculations, and ask children what they notice about the position of the decimal point in the quotient (it is aligned with the decimal point in the dividend).

When a decimal point is placed before the first digit of the dividend, draw attention to the use of a zero in the ones place as a place-value holder (e.g. 85 → .85 → 0.85).

$\begin{array}{r} 2\ \ 1 \\ 3\overline{)6\ \ 3} \end{array}$ 63 ones ÷ 3 = 21 ones so 63 tenths ÷ 3 = 21 tenths 6.3 ÷ 3 = 2.1	$\begin{array}{r} 2\ .\ 1 \\ 3\overline{)6\ .\ 3} \end{array}$
$\begin{array}{r} 1\ \ \ 7 \\ 5\overline{)\ \ \ 8\ \ {}^{3}5} \end{array}$ 85 ones ÷ 5 = 17 ones so 85 hundredths ÷ 5 = 17 hundredths 0.85 ÷ 5 = 0.17	$\begin{array}{r} 0\ .\ 1\ \ \ 7 \\ 5\overline{)0\ .\ 8\ \ {}^{3}5} \end{array}$
$\begin{array}{r} 0\ \ 5\ \ 1 \\ 5\overline{)2\ \ {}^{2}5\ \ 5} \end{array}$ 255 ones ÷ 5 = 51 ones so 255 tenths ÷ 5 = 51 tenths 25.5 ÷ 5 = 5.1	$\begin{array}{r} 0\ \ 5\ .\ 1 \\ 5\overline{)2\ \ {}^{2}5\ .\ 5} \end{array}$

	$\begin{array}{r} 0\quad 8\quad 9 \\ 4\overline{)\,3\quad {}^{3}5\quad {}^{3}6} \end{array}$ 356 ones ÷ 4 = 89 ones so 356 hundredths ÷ 4 = 89 hundredths 3.56 ÷ 4 = 0.89	$\begin{array}{r} 0\ .\ 8\quad 9 \\ 4\overline{)\,3\ .\ {}^{3}5\quad {}^{3}6} \end{array}$
5:8	Now present a new problem, such as *2.46 ÷ 6*, and ask children whether we need to find a related integer calculation to complete it. Then work through the process of short division with the decimal point already in place. Work through several examples, including: • dividends with a zero in the ones place • dividends with a zero in the tenths place. Use the generalisation: ***'In short division, if there is a decimal point in the dividend, put a decimal point in the quotient; line it up with the decimal point in the dividend.'***	Step 1 – write the divisor, dividend and frame: $6\overline{)\,2\ .\ 4\quad 6}$ Step 2 – write the decimal point for the quotient: $\begin{array}{r} . \qquad \\ 6\overline{)\,2\ .\ 4\quad 6} \end{array}$ Step 3 – perform the calculation, with unitising: $\begin{array}{r} 0\ .\ 4\quad 1 \\ 6\overline{)\,2\ .\ {}^{2}4\quad 6} \end{array}$ • 2 ones ÷ 6 = 0 ones r 2 ones *'Write "0" in the ones column and write "2" to the left of the tenths column of the dividend to make twenty-four tenths.'* • 24 tenths ÷ 6 = 4 tenths *'Write "4" in the tenths column.'* • 6 hundredths ÷ 6 = 1 hundredth *'Write "1" in the hundredths column.'*
5:9	Provide children with practice similar to that in step *5:4*, but now for examples that may require application of the short division algorithm.	*'Complete these calculations.'* 714 ÷ 3 = ☐ 456 ÷ 8 = ☐ 71.4 ÷ 3 = ☐ 45.6 ÷ 8 = ☐ 7.14 ÷ 3 = ☐ 4.56 ÷ 8 = ☐ 31.5 ÷ 5 = ☐ 7.92 ÷ 4 = ☐

		Dòng nǎo jīn: • *'Explain how you could use this fact:'* $432 \div 10 = 43.2$ *'to calculate:'* 11×43.2 • *'Explain how you could use this fact:'* $432 \div 5 = 86.4$ *'to calculate:'* $432 \div 6$
5:10	To complete this teaching point, provide children with contextual practice, including the quotitive, partitive and scaling structures of division, for example: • *'Elizabeth shares 2.85 kg of flour equally between three containers. How much flour is in each container?'* (partitive division) • *'Sid grows some potatoes. He has 27.5 kg of potatoes altogether. How many 5 kg bags can he make?'* (quotitive division) • *'Mateo lives 6.4 km away from school. Felicity lives one-quarter times that distance from school. How far from school does Felicity live?'* (scaling) Ensure that children are able to solve such calculations as part of multistep problems: • *'Catalina has an 8.52 m length of cloth to make some costumes for the school play. For each costume she uses 3 m of cloth. How many costumes can she make with this piece of cloth, and how much is left over?'* • *'A can of soup costs £1.22. How many whole cans can you buy with a £10 note? How much money would you have left?'*	