MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development Materials

5 Statistics and probability

Theme overview

Guidance document | Key Stage 3

Making connections

'Teaching for mastery' describes the elements of classroom practice and school organisation that combine to give students the best chance of developing a deep, connected, embedded and sustainable understanding of mathematics.

At any one point in a student's journey through school, achieving mastery means acquiring a secure understanding of the mathematics that has been taught to enable them to move on to more advanced material.

To achieve this, students need to understand the interconnected nature of mathematics and how one idea builds on and develops from other ideas. To this end, the NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of connected ideas or 'core concepts'.

The theme *Statistics and probability* covers the following interconnected core concepts:

5.1 Statistical representations and measures

5.2 Statistical analysis

5.3 Probability

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Why is this mathematical theme important?

We live in an information and data-rich age. The ability to interpret, analyse and critically evaluate all that we are presented with is vitally important if our students are to be engaged, thoughtful and aware citizens.

Key underpinning knowledge

Several important considerations are key to students gaining a secure and deep understanding of the mathematics within this theme, namely:

- that the purpose of statistical diagrams is to summarise data in such a way that clarifies and pulls out the important aspects. Different diagrams emphasise different aspects, and it is important to know which diagrams to choose and why
- that statistical measures (for example, measures of average and spread) are calculated in order to emphasise certain aspects of the data. An understanding of what quality they are measuring, and how they are doing that, is just as important as being able to carry out the relevant calculation
- that, while acquiring the skills of drawing diagrams and performing statistical calculations is important, it is also vital that students use these skills as an integral part of a problem-solving process, where there is a question to answer or hypothesis to test and some data to collect, interrogate, analyse and interpret, in order to answer the question or shed light on an issue
- that all statistical diagrams and analyses are ways of interpreting data and that this can sometimes be done in ways that might mislead. It is important to recognise this and to approach such analysis critically
- that probabilities can be quantified to predict the likelihood of an event happening, and that outcomes will tend to these predictions given enough trials.

Statements of knowledge, skills and understanding

Each of the three core concepts within the theme *Statistics and probability* has been broken down further into a set of statements of knowledge, skills and understanding, as listed below.

5.1 Statistical representations and measures

5.1.1 Understand and calculate accurately measures of central tendency and spread

5.1.2 Construct accurately statistical representations

5.2 Statistical analysis

5.2.1 Interpret reasonably statistical measures and representations

5.2.2 Choose appropriately statistical measures and representations

5.3 Probability

5.3.1 Explore, describe and analyse the frequency of outcomes in a range of situations

5.3.2 Systematically record outcomes to find theoretical probabilities

5.3.3 Calculate and use probabilities of single and combined events

We have produced guidance documents that offer an overview of each core concept, as well as an overview of the content of each statement of knowledge, skills and understanding. We have also broken down each of the latter into a series of key ideas to support planning, with some of the key ideas exemplified as to what teaching for mastery may look like.

We make no suggestion that each key idea represents a lesson. Rather, the fine-grained distinctions we offer in these key ideas are intended to help you think about the learning journey irrespective of the number of lessons taught.

Not all key ideas are of equal weight and the amount of classroom time required for them to be mastered will vary, but each step is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

These materials are designed for teachers to use collaboratively when planning how they will teach for a secure and deep understanding of mathematics throughout Key Stage 3. They are underpinned by a clear set of pedagogical principles and practices.

The *Statistics and probability* core concept guidance documents[1] can be downloaded from the NCETM website.

Links to the national curriculum

A mapping[2] of all statements of knowledge skills and understanding to the national curriculum Key Stage 3 programme of study is available on the NCETM website.

Previous learning

From Upper Key Stage 2, students will bring experience of:

- interpreting and presenting discrete and continuous data using appropriate graphical methods, including bar charts, pictograms and time graphs
- solving comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs
- interpreting and constructing pie charts and line graphs, and using these to solve problems
- encountering and drawing graphs relating two variables, arising from their own enquiry and in other subjects (non-statutory guidance)
- calculating and interpreting the mean as an average
- knowing when it is appropriate to find the mean of a data set (non-statutory guidance).

Future learning

In Key Stage 4, students will build on the core concepts in this mathematical theme to:

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- {construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use}
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
    - appropriate graphical representation involving discrete, continuous and grouped data, {including box plots}

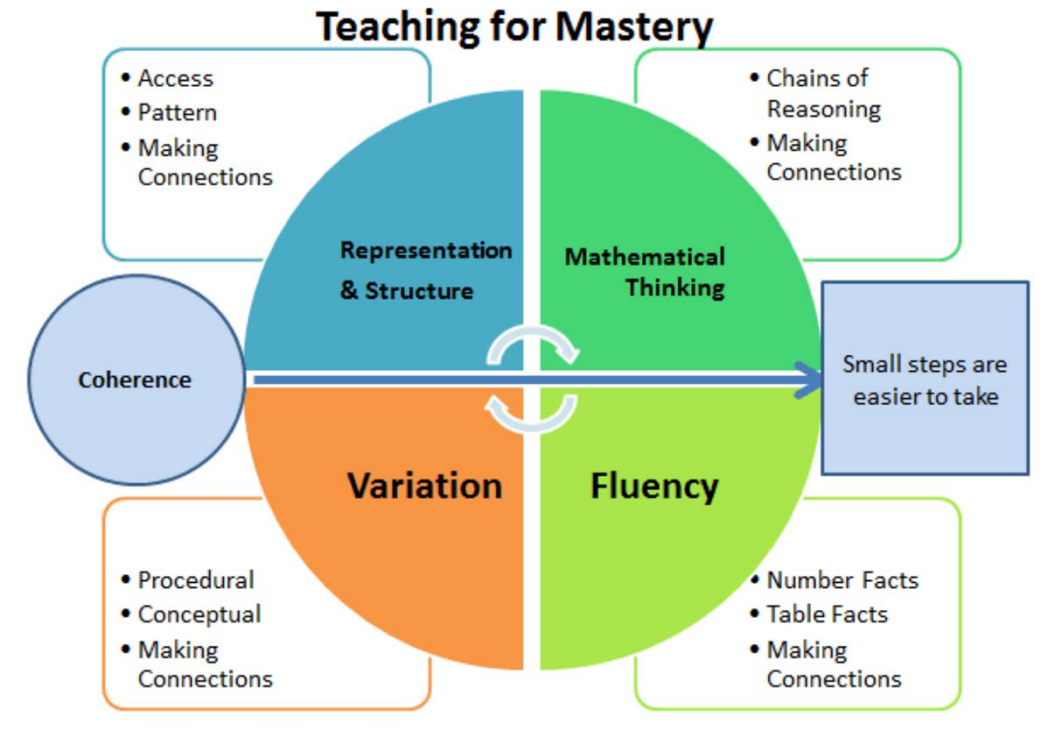
    - appropriate measures of central tendency (including modal class) and spread {including quartiles and inter-quartile range}
- apply statistics to describe a population
- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- {calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}.

Please note: Braces { } indicate additional mathematical content to be taught to more highly attaining students.

Teaching for mastery

A central component in the NCETM/Maths Hubs programmes to support the development of teaching for mastery has been discussion of Five Big Ideas[3] underpinning teaching for mastery. These are:

- Coherence
- Representation and structure
- Variation
- Fluency
- Mathematical thinking

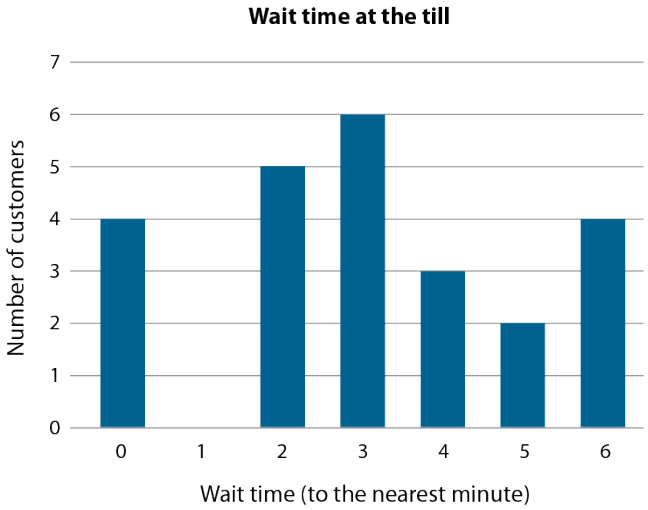


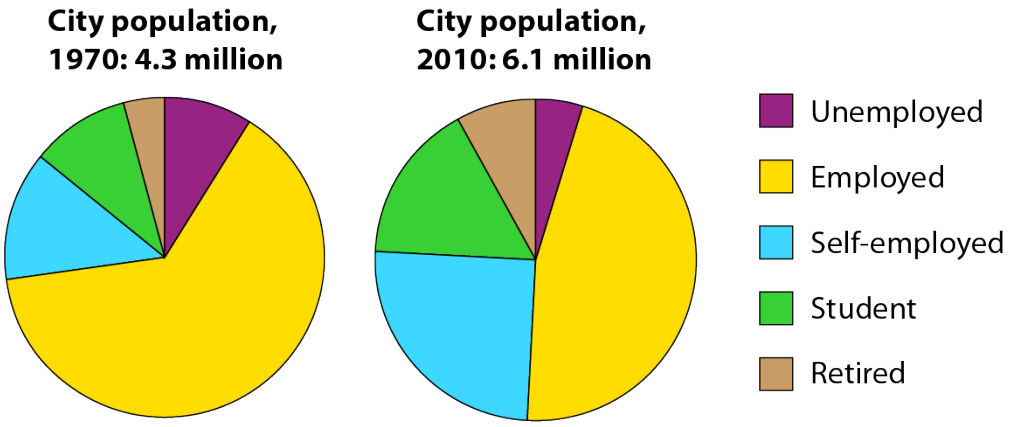
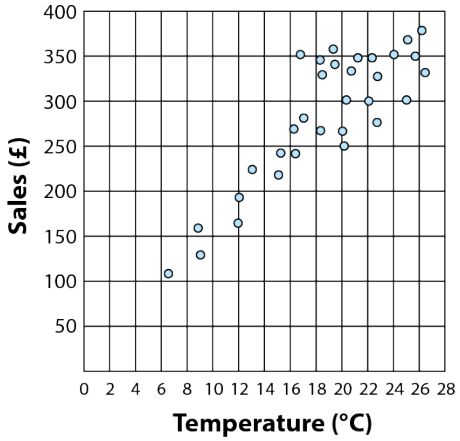
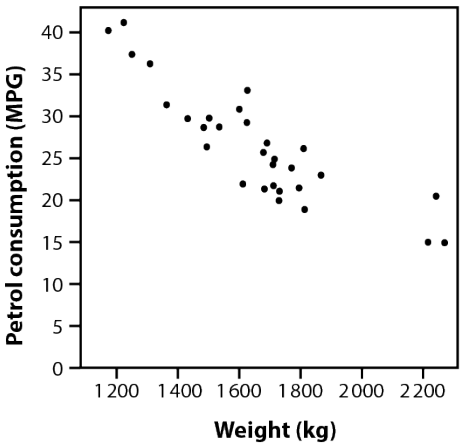
The sections below offer guidance about how these ideas relate to *Statistics and probability*.

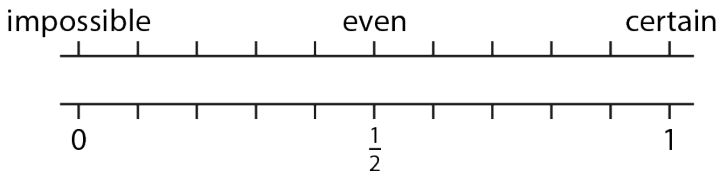
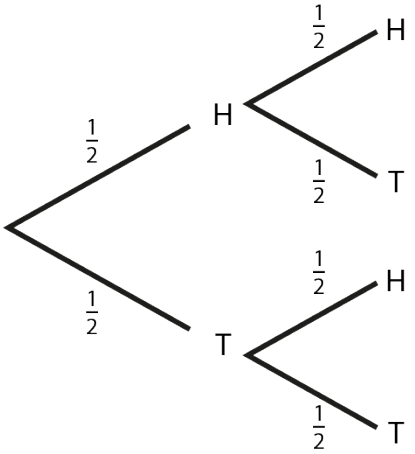
Coherence

It is important to find a balance between focusing in on important elements of this theme where it is useful to plan a coherent set of small steps (for example, when introducing statistical measures, to do so with discrete data presented as a list, then in a frequency table, then in a diagram) and an appreciation of how each idea is connected to others in the theme. For example, when looking at data collected to help answer a particular question or interrogate a particular hypothesis, to consider all measures (or central tendency and range) and diagrams, and make sound choices based on the original question or hypothesis.

Representation and structure

<table>
<tr><th>Representations</th><th>Structural understanding</th></tr>
<tr><td>Pictograms</td><td>Pictograms are a simple way of recording frequencies.
<table>
<tr><th>Attitude to school uniform</th><th>☺ represents two students</th></tr>
<tr><td>In favour of a strict uniform policy</td><td>☺☺☺☺☺☺</td></tr>
<tr><td>In favour of a dress code</td><td>☺☺☺☺☺☺☺☺☺</td></tr>
<tr><td>In favour of no uniform or dress code</td><td>☺☺☺☺☺☺☺☺☺☺</td></tr>
<tr><td>Undecided</td><td>☺☺☺☺☺</td></tr>
</table></td></tr>
<tr><td>Bar charts</td><td>Bar charts can be used give students a sense of the ‘shape’ of the distribution of the data across a sample, to identify and compare frequencies. They give an opportunity for comparisons to be made using absolute values.

</td></tr>
</table>

Pie charts	Pie charts allow for comparisons between proportions. They are particularly relevant for students when they are working with populations of different sizes, or for making multiplicative comparisons within the same population. 
Scatter graphs	Scatter graphs show the relationship between two variables. They allow students to: • make statements about the extent to which they are correlated (for example, the higher the temperature, the more ice creams we sell; the heavier the car, the worse the petrol consumption) • identify trends over time • use the relationship between the variables to make predictions about data points that have not been gathered (for example, climate change, predicting future temperatures given data about temperature change over the centuries). **Scatter graph showing sales of ice cream at a shop against temperature**  **Scatter graph showing vehicle weight against petrol consumption** 

Probability scale	A probability scale supports students in quantifying between everyday terms such as 'likely', 'impossible' and 'certain'. 
Probability space	A probability space is a useful representation to identify all outcomes where events are being combined. For example, when throwing two dice and adding the totals, the probability space can help students identify that the most likely outcome is 7 and quantify it as having a probability of one sixth, as well as helping to get a sense of the range of possible outcomes. See table below.
Tree diagrams	Although not a requirement of the national curriculum Key Stage 3 programme of study, students may find a tree diagram helpful in identifying and classifying outcomes of more than one event. For example, when two coins are flipped, a tree diagram can show the distinction between HT and TH, and so support understanding of why the probability of two heads is $\frac{1}{4}$ rather than $\frac{1}{3}$ as is often assumed. 

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Further guidance on using representations[4] in Key Stage 3 is available on the NCETM website.

Variation

Three aspects of variation that can be usefully employed:

1. Careful **choice of exercises** to 'home in' on the important concept. For example, when working on finding the mean average of a set of data, choosing sets of data where adding or subtracting the same amount to each data point results in the mean increasing (or decreasing) by that amount.
2. Careful **choice of examples** to include *'what it is'* (using non-standard as well as standard examples) and *'what it is not'*. For example, offering non-standard examples where the mean is given but some data points are absent, or offering examples of incorrect or misleading representations of data.
3. Rather than focusing on the answer and asking only that students solve a problem, inviting students to see **in how many different ways they can solve a problem** can prompt important discussions about methods and processes, and support students' development of increasingly efficient, creative and elegant approaches. For example, finding the mean of a relatively large set of data by constructing a frequency table and considering different ways of grouping the data.

Fluency

A key aspect of fluency is the ability to move between different representations and compare and contrast them. It is useful to engage students in activities where they evaluate a range of different statistical representations for the same data and make decisions as to which one most effectively conveys the information required.

Mathematical thinking

Throughout all the work that falls within *Statistics and probability*, the emphasis is on understanding what certain statistical calculations (for example, calculating the mean) convey, as well the ability to carry out such calculations. It is vital that students are prompted to reason and explain, conjecture and prove, through carefully planned teacher–student and student–student discussion (for example, *'What would happen if some data points were removed; would the mean go up or down? Why?'*) and not merely to listen to and follow carefully constructed teacher demonstrations and explanations.

Further reading

NCETM secondary assessment materials[5]
Exemplar questions, tasks and activities, which may be used to support teaching and assessment. The assessment materials are mapped against the key mathematical skills and concepts within the national curriculum Key Stage 3 programme of study. Of particular relevance to *Statistics and probability* are the sections focusing on: probability (pages 47–50) and statistics (pages 51–53).

Weblinks

1 Theme 5: *Statistics and probability* – core concept guidance documents
https://www.ncetm.org.uk/resources/53534

2 NCETM Key Stage 3 mastery curriculum structure, including national curriculum mapping
https://www.ncetm.org.uk/secondarymasterypd#curriculum_structure

3 Five Big Ideas in teaching for mastery
https://www.ncetm.org.uk/resources/50042

[4] Representations in Key Stage 3 – guidance documents
https://www.ncetm.org.uk/resources/53609

[5] NCETM secondary assessment materials
https://www.ncetm.org.uk/resources/51246