



Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for 85 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the [Primary Magazine Archive](#).

Contents

This month we have the [second of four articles](#), which looks at developing and demonstrating understanding of mathematics in the National Curriculum; this month the focus is on representing in a variety of ways.

[Maths in the Staff Room](#) suggests ways in which collective teacher discussions - both formal and informal - can form part of the ongoing process of professional learning, and help increase the effectiveness of maths teaching across the school. This month's article looks at the use of elicitation tasks to shape teaching and assess progress.

[Seen and Heard](#) provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. This month children in a Year 4 class respond to some data, prompting us to think about what children understand about the relationship between large numbers and how they can be supported to make sense of scales.

If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please [email](#) it to us. If we publish your suggestion, we'll put a £20 voucher in the post.

But first, as always, we have a [News](#) section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

Image credit

[Page header](#) by [Scott Akerman](#) (adapted), [some rights reserved](#)



News



The Schools Minister Nick Gibb has given schools more time to submit KS1 and KS2 teacher assessments this summer. The deadline, for both KS1 and KS2, will now be 30 June, as the Minister detailed in a [letter to the NAHT](#) [headteachers' union] in February.



On the same day as the DfE published the Minister's letter, they also offered schools what they called [Five things you need to know about changes to primary assessment](#).



In the latest edition of [Bespoke](#), the newsletter of the Maths Hubs programme, there are accounts of how eight primary schools are embedding teaching for mastery approaches into their lessons and school organisation.



Have you looked through the records of our [weekly online discussions](#) held using Twitter? You don't have to be a Twitter user to benefit from these, as they're all archived on the NCETM website. Recent chats have centred on [classroom organisation](#), [CPD](#), and [how to help children memorise mathematical facts](#) - and you can find topics for the rest of this terms discussions [here](#).

Image credit

[Page header](#) from [Negative Space](#) (adapted), [in the public domain](#)



National Curriculum in Focus

National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series [here](#)

Developing and demonstrating understanding: Part Two - Representing in a variety of ways

This is the second of four articles focused on how teaching can support the development of understanding and how pupils can demonstrate understanding of the mathematics within the National Curriculum

In the [NCETM Teaching for Mastery booklets](#) it states that:

“Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills.”

One of the phrases within this statement, with which many teachers are currently grappling, is ‘a deep understanding’, provoking the questions: ‘What does it mean to have a deep understanding of the mathematics?’ and ‘How do I know if a child has a deep understanding of the mathematics?’ If teachers are to focus on supporting children to develop a deep understanding, then finding answers to these questions is crucial.

The Teaching for Mastery booklets provide some support with this, with the inclusion of the following section in the introduction:

A pupil really understands a mathematical concept, idea or technique if he or she can:

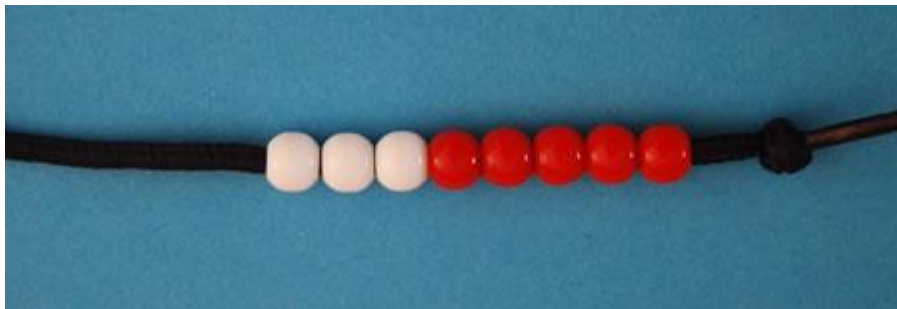
- describe it in his or her own words;
- represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)¹;
- explain it to someone else;
- make up his or her own examples (and non-examples) of it;
- see connections between it and other facts or ideas;
- recognise it in new situations and contexts;
- make use of it in various ways, including in new situations.

Here we will focus on the second of these, represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)¹. This highlights the importance of a multidimensional approach to mathematics.

For some children, ‘doing’ the maths might be quite straightforward but communicating understanding by representing the maths with pictures, mathematical resources, symbols and in a context can be challenging, as this requires a deep understanding of mathematical structures. Different representations of the mathematics can accompany and illustrate explanations and children need to demonstrate how the different representations are connected, presenting a further challenge.

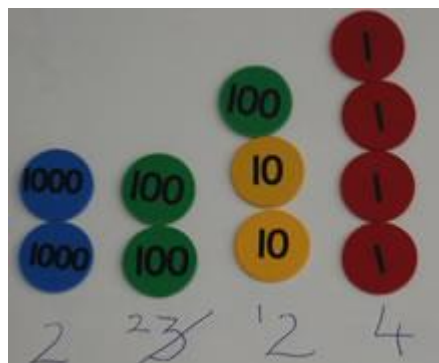
Examples

In year two pupils are expected to be able to “show that addition of two numbers can be done in any order”. They might use a bead string to show $5+3$ and explain that moving the bead string so that the three is first doesn't change the number of beads, there are still eight in total:



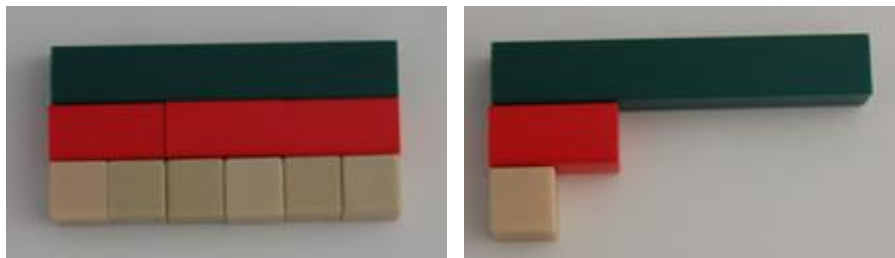
They might draw two bars to represent five and three and again show that by turning the image, it becomes three and five, record $5+3=3+5$. and explain that if you put £5 in your piggy bank and then add £3 it will be the same amount as £3 in the piggy bank plus £5.

In year four pupils are expected to be able to ‘add and subtract numbers with up to four digits using the formal written methods of columnar addition and subtraction where appropriate’. For the calculation $2324 - 1172$ the children might use a written method to solve and then explain and show with place value counters that when they crossed out the 3 hundreds and wrote a 1 by the 2 tens they were simply partitioning 2324 in a different way to make it easier to subtract. They use the image to show that the number is still 2324 and that there are now two hundreds and twelve tens because one hundred, which is equivalent to ten tens, has been moved to the tens:

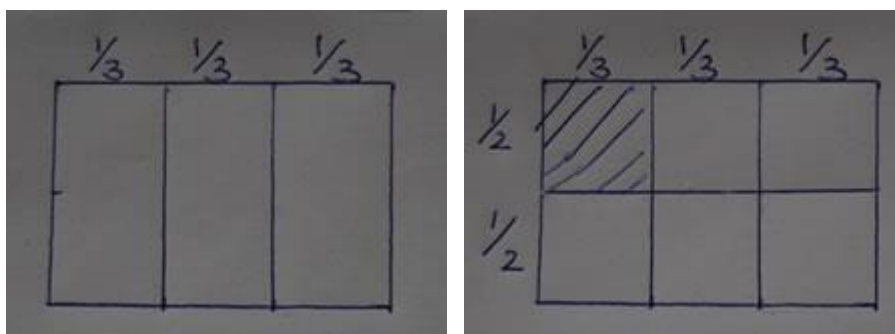


A context for this calculation might be £2324 in the bank and a family pays for a holiday which costs £1172, or the difference between the number of visitors to a museum on a Saturday compared with a Sunday.

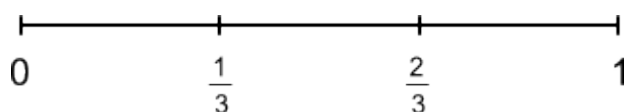
In year six pupils are expected to be able to 'divide proper fractions by whole numbers'. For the calculation $\frac{1}{3} \div 2 = \frac{1}{6}$ the children might use Cuisenaire rods to show why this works, explaining that dividing thirds into two parts means there will be twice as many rods, which is why they will each have the value of one sixth, so one sixth is half of one third:



The children might then relate this to an array and draw a rectangle which represents one, divide it into thirds and then split these in half, showing that there are six pieces and so each is one sixth:

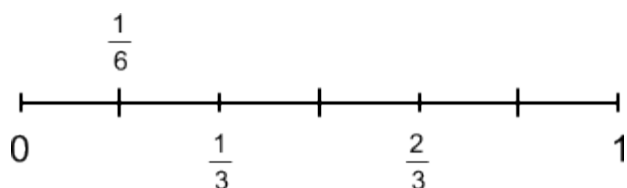


Or they might represent and explain it using a number line:



I divided my number line 0 to 1 into three and labelled the points 0, $\frac{1}{3}$, $\frac{2}{3}$ and 1.

I then halved each section of the line:



They may record this as $\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ and suggest a context for this calculation could be that when cooking, a recipe requires $\frac{1}{3}$ of a cup of margarine and they decide to make half the quantity.

¹ The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner's conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since the early 1980s. See [here](#) for an introduction to this approach

Image credit

[Page header](#) by [Romain Vignes](#) (adapted), [in the public domain](#)



Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series [here](#)

Assessing Progress

Meeting aims

- To explore the use of elicitation and application tasks
- To plan elicitation tasks.

Timings

- Sixty minutes initial input
- Forty-five minutes minutes follow-up after three or four weeks.

Resources

- Copies of the paper [Elicitation tasks in mathematics](#) (Babcock LDP)
- Variety of mathematical resources (counters, Numicon, Cuisenaire, bead strings etc).

Session 1

- Explain that you are going to look at one way that teachers can see progress across a teaching sequence. Introduce the idea of elicitation and application tasks; elicitation tasks take place before a teaching sequence starts, and the purpose is to find out what the children already know and understand about the focus for the sequence, and what they don't understand yet. Application tasks take place at the end of a sequence and allow the children to demonstrate what they now understand.
- Say that you are going to start by reading a short paper; give everyone a copy of the Elicitation tasks in mathematics paper and ask them to read just the first page.
- Discuss the key points from this first page and make sure that everyone attends to the part which says:
Elicitation tasks are most productive when they provide the opportunity to expose misconceptions and the children appreciate that they are expected to explain and demonstrate what they do and don't understand.
Say that it is important that the children understand that this is not like other maths they do, where one efficient, elegant solution is sought, it is an opportunity to demonstrate thinking in many different ways and to raise and ask questions.
- Say that you are now going to look at a couple of children's elicitation tasks which are in the paper but first they will have a go at the maths. Do the following activities either in individually, in small groups or altogether:
 - On a piece of paper, write 'What is a quarter?' in the middle and invite people to represent a quarter in different ways – remind them of the list on the first page of the paper and see if they can use different concrete materials, pictures, language, symbols and contexts. Once you have a collection, ask different people to explain how one of the representations shows a quarter.

- Look at pages 6, 7 and 8. The first two pages are from Y1 children; discuss what you think they understand about quarters. Page 8 is from Y2; again what does this child understand about quarters. For all of the children, what would be the key understanding you would want to focus on in your teaching?
 - Write up the following statement for everyone to consider:
“ $\frac{1}{2}$ is equivalent to 0.5, $\frac{5}{10}$, $\frac{50}{100}$ and $5 \div 10$. Is this correct?” Can you prove or disprove this statement? Write down your thinking using maths talk.
 - Tell them that this was used as a Y4 elicitation task and ask them to discuss how you might expect a Y4 child to prove this statement.
 - Look at pages 9 and 10 which show one child’s response to this question. Ask: What do you notice about how this child has responded? Does this give you useful information, i.e. would you be able to shape your teaching in response to this information?
 - Ask everyone to draw a number line from 0 to 1 and say that a Y6 elicitation task was to use this to ‘Show what you understand about decimals, fractions and percentages on a number line’. Give them a few minutes to have a go, thinking about what they would expect Y6 children to do.
 - Look at page 13 and ask: What do you notice about how this child has responded? Where would you focus your teaching in the teaching sequence?
-
- Remind everyone that the paper says there is very little that needs to be standard about the tasks and say that pages 2 to 4 give examples of different sources and types of elicitation task. Allow everyone time to look at the different types.
 - In teaching teams, ask teachers to discuss the next teaching sequence and which type of task would be best suited, for their children for that area of maths. Using either one of the sources suggested in the paper or other ideas, each teaching team plans the elicitation task for the next sequence.
 - Say that you will come back together at the end of the sequence when the children have completed the application task. This can be either the same as the elicitation task or a similar task; suggest that for this first time everyone repeats the elicitation task.

Follow up meeting after the application tasks have been completed

- Ask everyone to bring copies of one or two children’s elicitation and application tasks. Remember, these may not be from books; they may be observations and photographs, annotated pictures etc.
- Depending on the size of your school, either all work together or start in age-group teams. Share what you found out from the elicitation task, how this fed into the planning and teaching and what progress you can see from the application task. In larger schools then mix up so that teachers are sharing the elicitation tasks across age groups and ask them to discuss whether or not the task used was a useful elicitation task and how it could have been improved.
- Plan next steps.

Image credit

Page header by [Startup Stock Photos](#) (adapted), [in the public domain](#)



Seen and Heard

Seen and Heard will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom

At the start of a Y4 lesson the children considered the following:



During the discussion that followed, one child said that the tiger population fell by one thousand between 1980 and 1990, and by 500 between 1990 and 2000. Another child said that in the year 2000 the tiger population was around 17 000 and a half.

- What do the two children understand about the numbers and what are their misconceptions?
- How can the children be supported to make sense of large numbers?
- Which mathematical images would help the children to make sense of the numbers?
- How can the children be supported to understand how to find the number that is half way between two large numbers, for example halfway between 15 000 and 20 000?
- How did the choice of graph provide the teacher with an opportunity to assess understanding of number?

With thanks to Sam Malyn from St Margaret's C of E Junior School, Devon, for sending us this example.

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.

Image credit

Page header by [Anthony Delanoix](#) (adapted), [in the public domain](#)