NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Core concept 6.3: Transforming shapes

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

6.3.1 Understand and use translations

How confident are you that you understand the nature of a translation, including what is variant and invariant and the minimum information required to describe it, and can complete a translation given in a variety of forms including in vector form?

1 ☐ 2 ☐ 3 ☐ 4 ☐

When an object undergoes a translation, the size of its angles and the lengths of its lines are maintained so that the object and image are congruent. This property is shared with both rotation and reflection, but a translation, uniquely, always maintains the orientation of the object in the image. The use of notation to record a translation may follow from a need to describe it accurately and succinctly. Initially, students are likely to use informal language as they develop their understanding of the transformation, describing, for example, a move of 'three across' and 'two down'. While the formal use of vectors is part of the national curriculum Key Stage 4 programme of study, translation offers students a natural opportunity to formalise their intuitive understanding about the distinction between movement and position. As a result, formal vector notation (e.g. $\binom{+3}{-2}$) could be introduced at Key Stage 3.

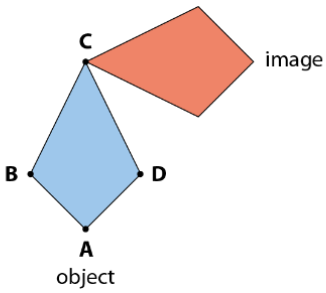
6.3.2 Understand and use rotations

How confident are you that you understand the nature of rotation, including what is variant and invariant and the minimum information required, and can complete and describe a rotation?

1 ☐ 2 ☐ 3 ☐ 4 ☐

As with translations, rotations maintain congruence but offer a further degree of change between the object and the image, since the orientation of the object is not necessarily maintained. In Key Stage 2, students will have worked with objects rotated through a half, a quarter and three quarters of a turn, and this is generalised to any angle at Key Stage 3 (specifying the size and direction of turn). In addition, more attention is paid to the centre of rotation (the one point that does not move under the rotation) and the fact that the position of the image changes with different centres of rotation, even though the orientation may not.

Here, the image is a 90° anticlockwise rotation of the object about point C.



When constructing examples to support students' understanding, it is important to vary the position of the centre of rotation to include:

- on a vertex of the object
- lying within the object
- lying outside of the object.

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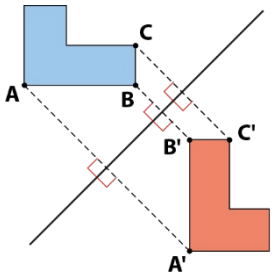
6.3.3 Understand and use reflections

How confident are you that you understand the nature of reflection, including what is variant and invariant and the minimum information required, and can complete and describe a reflection?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Transforming an object by reflecting it offers the full range of possible congruent shapes, and a context in which congruence may be explored further. Reflection in lines that are neither horizontal nor vertical presents increased challenge and requires students to have a sense of where the image will be. Using a range of tools, such as dynamic geometry software, alongside pencil and paper methods, gives students a greater depth of understanding.



It will be important to offer students a wide range of non-standard, as well as standard, examples of lines of reflection. Draw students' attention to the fact that every line that joins a point on the object to its image is **perpendicular** to the line of reflection.

Further support links

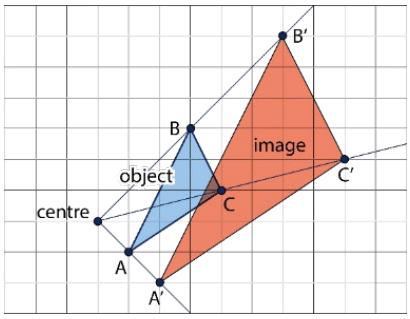
- NCETM Secondary Professional Development materials: 6.3 Transforming shapes, page 13

6.3.4 Understand and use enlargements

How confident are you that you understand the nature of enlargement, including what is variant and invariant and the minimum information required, and can complete and describe an enlargement?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Students are likely to be familiar with enlargements through their work on similar shapes in Key Stage 2. At Key Stage 3, students are introduced to the idea of a centre of enlargement and that the position of this in relation to the object affects the image's position. In this set of key ideas, students consider the range of possible outcomes with an enlargement. They should come to appreciate that enlargement is the only transformation that does not guarantee a congruent shape.



At Key Stage 3, the focus is on enlargements with a scale factor ≥1, but the use of dynamic geometry software offers students an opportunity to reason mathematically about the images that will result if a scale factor outside of this range is used (as it is in Key Stage 4), and to then test and refine their conjectures.

Notes

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