Mastery Professional Development Materials

4 Sequences and graphs

Theme overview

Guidance document | Key Stage 3

Making connections

'Teaching for mastery' describes the elements of classroom practice and school organisation that combine to give students the best chance of developing a deep, connected, embedded and sustainable understanding of mathematics.

At any one point in a student's journey through school, achieving mastery means acquiring a secure understanding of the mathematics that has been taught to enable them to move on to more advanced material.

To achieve this, students need to understand the interconnected nature of mathematics and how one idea builds on and develops from other ideas. To this end, the NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of connected ideas or 'core concepts'.

The theme *Sequences and graphs* covers the following interconnected core concepts:

4.1   Sequences

4.2   Graphical representations

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Why is this mathematical theme important?

A fundamental feature of algebra and algebraic thinking is the recognition and expression of generality. When studying sequences, students are asked to discern what is the same about a collection of different terms, what links them together and what makes them examples of the same thing. As such, it is a context that provides an accessible and powerful introduction to using algebraic symbolism which builds on intuitive ideas of generality, many of which have been laid down at primary school. While introduced in Year 6, algebra is developed further at Key Stage 3 and is a key element of conceptual development essential for success at Key Stage 4.

When working with sequences, a powerful and important distinction that students will make is figuring out what is needed to generate the next term in the sequence and describing any term in the sequence in relation to its position in that sequence. Such a generalisation requires seeing and expressing a common structure behind each term of a sequence and is an important precursor to proof.

Coordinates can be plotted to represent a sequence, with the *x*-value representing the position of a term in the sequence and the corresponding *y*-value representing the value of that term. When representing sequences in this way, it is important to be aware that the points are discrete (each one representing a term in the sequence) and a continuous line should not be drawn through them. Later, students will experience graphs as a way of representing a more general mathematical relationship (or function) connecting the *x*- and *y*-values where the *x*-values are not restricted to integers. The move from seeing a graph as a finite collection of coordinates that satisfy a relationship to a single mathematical object which encapsulates an infinite set of points, is an important one. Being able to move freely between different forms of representation (numerical values, algebraic symbolism and graphs) and to make links between them is an important skill that can support students in clear thinking, reasoning and problem-solving.

Key underpinning knowledge

Several important considerations are key to students gaining a secure and deep understanding of the mathematics within this theme, namely:

- that an ordered sequence of pictures or numbers follows a pattern and can be continued according to that pattern
- that it is important to distinguish between pattern and structure. A pattern exists because there is some underlying structure (or rule) that is producing it; following a pattern without knowing what is producing it is not enough. To be able to generalise, students must be able to describe the structure (or know the rule)
- that plotting coordinates is another way of representing a sequence
- that when a line is drawn joining the coordinates (where this is appropriate) to produce a graph, this describes the relationship between the infinite set of coordinates that are captured, and the equation of the graph is the representation of that relationship.

Statements of knowledge, skills and understanding

Each of the two core concepts within the theme *Sequences and graphs* has been broken down further into a set of statements of knowledge, skills and understanding, as listed below.

4.1   Sequences

4.1.1   Understand the features of a sequence

4.1.2   Recognise and describe arithmetic sequences

4.1.3   Recognise and describe other types of sequences (non-arithmetic)

4.2   Graphical representations

4.2.1   Connect coordinates, equations and graphs

4.2.2   Explore linear relationships

4.2.3   Model and interpret a range of situations graphically

We have produced guidance documents that offer an overview of each core concept, as well as an overview of the content of each statement of knowledge, skills and understanding. We have also broken down each of the latter into a series of key ideas to support planning, with some of the key ideas exemplified as to what teaching for mastery may look like.

We make no suggestion that each key idea represents a lesson. Rather, the fine-grained distinctions we offer in these key ideas are intended to help you think about the learning journey irrespective of the number of lessons taught.

Not all key ideas are of equal weight and the amount of classroom time required for them to be mastered will vary, but each step is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

These materials are designed for teachers to use collaboratively when planning how they will teach for a secure and deep understanding of mathematics throughout Key Stage 3. They are underpinned by a clear set of pedagogical principles and practices.

The *Sequences and graphs* core concept guidance documents[1] can be downloaded from the NCETM website.

Links to the national curriculum

A mapping[2] of all statements of knowledge skills and understanding to the national curriculum Key Stage 3 programme of study is available on the NCETM website.

Previous learning

From Upper Key Stage 2, students will bring experience of:

- using simple formulae
- generating and describing linear number sequences
- describing positions on the full coordinate grid (all four quadrants).

Future learning

In Key Stage 4, students will build on the core concepts in this mathematical theme to:

- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci-type sequences, quadratic sequences, and simple geometric

progressions (r^n where n is an integer, and r is a positive rational number {or a surd}) {and other sequences}
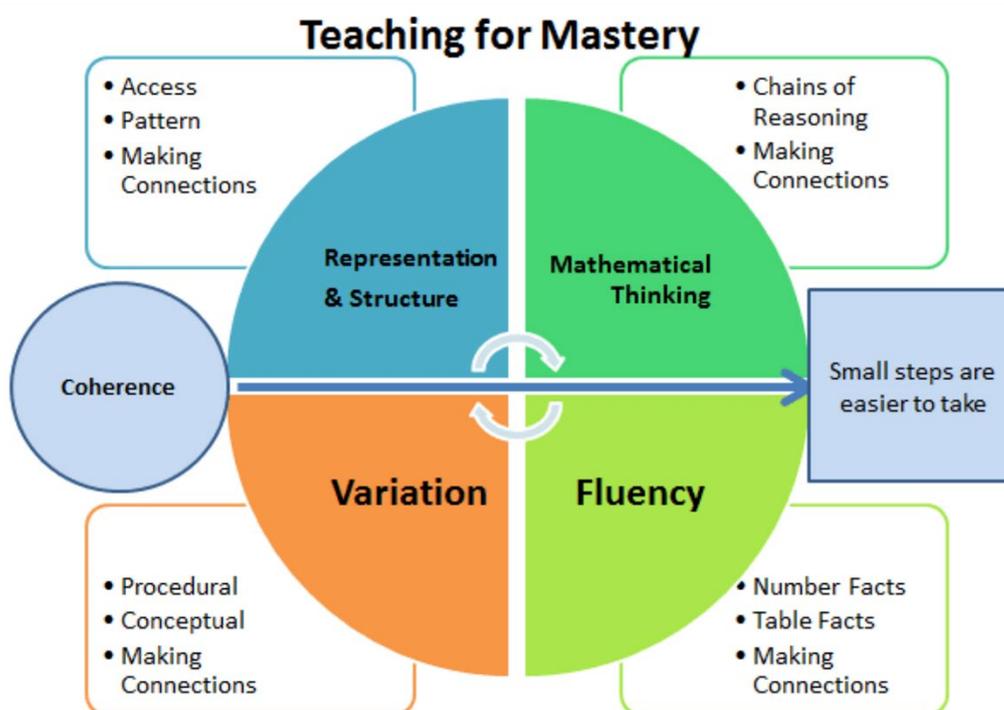
- deduce expressions to calculate the nth term of linear {and quadratic} sequences
- use the form $y = mx + c$ to identify parallel {and perpendicular} lines; find the equation of the line through two given points, or through one point with a given gradient
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \dfrac{1}{x}$ with $x \neq 0$, {the exponential function $y = kx$ for positive values of k, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$, $y = \tan x$ for angles of any size}
- {sketch translations and reflections of the graph of a given function}
- plot and interpret graphs (including reciprocal graphs {and exponential graphs}) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration.

Please note: Braces { } indicates additional mathematical content to be taught to more highly attaining students.

Teaching for mastery

A central component in the NCETM/Maths Hubs programmes to support the development of teaching for mastery has been discussion of <u>Five Big Ideas</u>[3] underpinning teaching for mastery. These are:

- Coherence
- Representation and structure
- Variation
- Fluency
- Mathematical thinking

Teaching for Mastery

- Access
- Pattern
- Making Connections

- Chains of Reasoning
- Making Connections

Representation & Structure

Mathematical Thinking

Coherence

Small steps are easier to take

Variation

Fluency

- Procedural
- Conceptual
- Making Connections

- Number Facts
- Table Facts
- Making Connections

The sections below offer guidance about how these ideas relate to *Sequences and graphs*.

Coherence

It is important to find a balance between focusing on important elements of this theme where it is useful to plan a coherent set of small steps (for example, when drawing linear graphs, considering equations of the form $y = mx + c$ before equations in other forms, focusing on the intercept and its meaning separately from the slope and before integrating the two, and so on) and appreciating how each idea is connected to others in the theme. For example, relating the drawing of graphs with sequences and linking the idea of the nth term of a sequence to the equation of a graph.

Representation and structure

Representations	Structural understanding
Tables of values, graphs, sequences and structured pictorial arrangements	

It is important for students to see multiple representations of the same structure, as in the diagram above, and to connect them. Prompts such as *'Where is the 3 in this representation?'* and *'Where is the 1?'* can support students in making these connections and understanding that each representation has the same underpinning structure.

It can be helpful to challenge students to supply some of these representations when given others; for example, draw a graph for a given table of values, state the sequence and an algebraic generalisation given a pictorial arrangement, and so on.

Note, it will be important for students to be aware that while certain points on a graph may be used to represent a sequence, points in between may not (as above) even though the straight line joining the points does represent the relation $y = 3x + 1$ (for example).

Further guidance on using [representations](#)[4] in Key Stage 3 is available on the NCETM website.

Variation

Three aspects of variation that can be usefully employed:

1. Careful **choice of exercises** to 'home in' on the important concept. For example, when drawing graphs of the form $y = mx + c$, engineering examples so that the value of c changes while keeping the m term the same will allow students' attention to be drawn to its effect.

2. Careful **choice of examples** to include *'what it is'* (using non-standard as well as standard examples) and *'what it is not'*. For example, offering sequences that are not linear (perhaps sequences that increase by one and then by two and then by three, etc.) and using prompts to challenge students to think carefully about what a linear sequence is.

3. Rather than focusing on the answer and asking only that students solve a problem, inviting students to see **in how many different ways they can solve a problem** can prompt important discussions about methods and processes, and support students' development of increasingly efficient, creative and elegant approaches. For example, finding the missing term in a sequence by looking at a table of values and by examining the graph.

Fluency

A key aspect of fluency is the ability to choose the most efficient strategy for a problem or calculation. Understanding that only two points are necessary to uniquely define a linear graph, coupled with reasoning that putting in a value of zero requires little effort in calculation, leads to the efficient strategy of choosing $x = 0$ and calculating y and choosing $y = 0$ and calculating x.

Mathematical thinking

Throughout all the work that falls within *Sequences and graphs*, the emphasis is on expressing generality, understanding the connection between sequences and graphs, and seeing graphs as a representation of a function that links the two numbers in an infinite number of coordinates. It is vital that students are prompted to reason, explain, conjecture and prove through carefully planned teacher–student and student–student discussion and not merely to listen to and follow carefully constructed teacher demonstrations and explanations.

Further reading

[NCETM secondary assessment materials](#)[5]
Exemplar questions, tasks and activities, which may be used to support teaching and assessment. The assessment materials are mapped against the key mathematical skills and concepts within the national curriculum Key Stage 3 programme of study. Of particular relevance to *Sequences and graphs* are the sections focusing on: graphical representation (pages 18–20) and sequences (pages 21–22).

Weblinks

[1] Theme 4: *Sequences and graphs* – core concept guidance documents
https://www.ncetm.org.uk/resources/53533

[2] NCETM Key Stage 3 mastery curriculum structure, including national curriculum mapping
https://www.ncetm.org.uk/secondarymasterypd#curriculum_structure

[3] Five Big Ideas in Teaching for Mastery
https://www.ncetm.org.uk/resources/50042

4   Representations in Key Stage 3 – guidance documents
     https://www.ncetm.org.uk/resources/53609

5   NCETM secondary assessment materials
     https://www.ncetm.org.uk/resources/51246