Primary & Early Years Magazine 80

National Centre for Excellence in the Teaching of Mathematics

Welcome to another issue of our Primary Magazine. This magazine has been serving primary teachers for 79 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are accessible through the Primary Magazine Archive.

Contents

This issue marks the start of a modest re-shape of the magazine's content. Each issue will still have a central focus on the National Curriculum, but, given that it's now in its second year, we've dropped the word 'new' from the section title. This month we look at planning for misconceptions.

Maths in the Staff Room remains as a vehicle for suggesting ways in which collective teacher discussions - both formal and informal - can form part of the ongoing process of professional learning, and help increase the effectiveness of maths lessons across the school. This may also occasionally stray into cross-curricular themes. This month is the first in a series focusing on understanding key mathematical structures.

A new monthly feature, Seen and Heard, will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience - the aim being to stimulate thought and questions about how you would react to similar events in your own classroom. This month, a written response from a year 5 pupil makes us think about how children understand improper fractions. If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please email it to us. If we publish your suggestion, we'll put a £20 voucher in the post.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

NCETM Calculation Guidance

The NCETM has published new calculation guidance for primary schools, drawing on the first year of schools in Maths Hubs adopting elements of a teaching for mastery approach in maths lessons.

The schools, all of which hosted teachers from Shanghai for a month during the last school year, worked with the NCETM to pool their experiences and distil what they considered to be the key elements of a successful approach to teaching calculation.

The resulting document, divided into 15 sections, can be downloaded and/or printed from the NCETM and Maths Hubs programme websites.

The Commission on Assessment Without Levels

The final report of the Commission on Assessment Without Levels, and the government response, have both been published since the last Primary Magazine came out.

The Commission's report provides advice to schools about how to develop new approaches to pupil assessment that:

- are based on needs of the pupils
- follow the school's curriculum
- support effective teaching.

It includes a section on 'Mastery in assessment' which concludes that 'The new national curriculum is premised on this kind of understanding of mastery, as something which every child can aspire to and every teacher should promote' and presents as 'good practice' the focus on wrong answers and misconceptions in maths lessons.

Interim teacher assessment frameworks at the end of key stage 1 and key stage 2

Two interim frameworks have been published by the Standards and Testing Agency (STA), to support teachers in making assessments at the end of key stage 1 and key stage 2 in 2016. The frameworks are valid for one year only, while the Department for Education evaluates options for future years. Each framework sets out the standard(s) a child will be assessed against at the end of the key stage for reading, writing, mathematics and science. In mathematics at the end of key stage 1, there are three standards: working towards the expected standard, working at the expected standard, and working at greater depth within the expected standard. In mathematics at the end of key stage 2 there is one standard: working at the expected standard.

Introductory video for NCETM Assessment Materials

We've recently posted a short introductory video by Debbie Morgan, NCETM Director for Primary, in which she gives some background to the production and use of the assessment materials, which were published just before the start of term.

The Secret Rules of Modern Living: Algorithms
This BBC programme, broadcast on 24 September and available through the BBC iPlayer, sees Professor Marcus du Sautoy uncover how algorithms are essential to modern life and delve into their history.

Image credit
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National Curriculum in Focus

National Curriculum in Focus *is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series here*

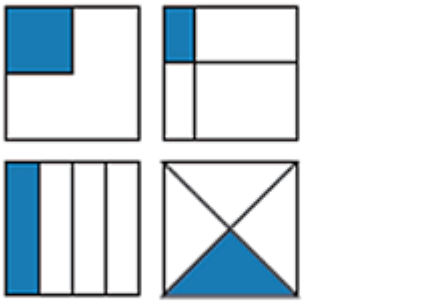
Planning for misconceptions – Teach, Learn, Confuse

The focus on mastery in the National Curriculum is a focus on understanding. Understanding builds from experiencing a concept in lots of different ways in different contexts. One of the most striking observations about the lessons taught by teachers from Shanghai, as part of the national England-China project, has been that the teachers focus on provoking misconceptions. One of the visiting teachers explained this as: 'Teach, Learn, Confuse.'

For some teachers, planning to confuse and expecting all children to struggle within maths lessons will be a new experience. The importance of struggle in learning has been highlighted by numerous educationalists (including Stigler, Dweck and Boaler) and can be represented by the Learning Pit (Nottingham) where cognitive conflict or 'wobble' is encouraged in order to make learners think deeply.

This focus on provoking misconceptions is part of a focus on reasoning and allows learners to develop sound generalisations. Misconceptions, which can lead to incorrect generalisations, are often the result of limited experience and a limited diet of questions which have provided a one-dimensional view of a concept. Planning for misconceptions can be supported by using the Teaching for Mastery booklets produced by the NCETM. The questions in each domain give a sense of the breadth of understanding expected and many are deliberately shaped to expose misconceptions. For example the following question from the Y2 booklet could expose a number of misconceptions, all arising from a limited experience of finding quarters:

Which of these diagrams have $\frac{1}{4}$ of the whole shaded?



Explain your reasoning.

Depending on the children's experiences they may consider that:

- the first does not have a quarter shaded because they can only see two parts
- the second does have a quarter shaded because it is one of four parts

- the third and fourth ones do not have a quarter shaded because they have not seen a square split like this for quarters before (it has always been split into four squares) and they do not think that the rectangle or triangle can be a quarter of the square. They may also think that one of them is correct and that the other cannot be because they look different to each other so can't both be a quarter of the same sized square
- the fifth one does not have a quarter shaded because it is not split into four parts or because the parts shaded are not together.

The starting point for any teaching sequence in maths is to be clear about what it is the children need to understand about the concepts included and, therefore, what they should be able to generalise at the end of the teaching sequence. This then leads to considering potential misconceptions; it will be important to plan for children to explore concepts in a variety of contexts and expect them to use a variety of representations in order to expose and address misconceptions and build a more complete understanding. This is explained in the Teaching for Mastery booklets as follows:

> *"A pupil really understands a mathematical concept, idea or technique if he or she can:*
> - *describe it in his or her own words;*
> - *represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach);*
> - *explain it to someone else;*
> - *make up his or her own examples (and non-examples) of it;*
> - *see connections between it and other facts or ideas;*
> - *recognise it in new situations and contexts;*
> - *make use of it in various ways, including in new situations."*

This list of ways to demonstrate understanding provides teachers and pupils with different ways to challenge thinking and explore concepts. It is important to note that sometimes teachers will expose misconceptions which they had not anticipated (for an example of this, see the Seen and Heard article this month); we are always interested in unusual and unexpected misconceptions so please email these to us for inclusion in future issues of the magazine.

References

Stigler, J NPR interview 2012
Dweck, C
Boaler, J Unlocking Children's Math Potential 2014
Nottingham, J A Guide to the Learning Pit 2014
Holt J, How Children Fail 1964

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Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room *provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series here*

Understanding key mathematical structures. Part one: 'Doing and undoing

Meeting aims

- To consider the importance of structure in mathematics
- To explore understanding of a key mathematical structure, which has relevance across the curriculum
- To make explicit opportunities to embed the aims of the National Curriculum.

Timings

- Ten minutes initial input
- Ten minutes, thirty minutes, sixty minutes or ninety minutes follow up after two weeks.

Resources

- Asking Mathematical Questions Mathematically by John Mason
- Key understandings in mathematics learning. Paper 2: Understanding whole numbers by Terezinha Nunes and Peter Bryant
- Large sheet of paper for display in the staffroom with 'Doing and Undoing' in the middle.

Ten minute introduction

1. Explain that understanding structure is an important part of mathematics and asking children to attend to structure is part of the role of the teacher in mathematics lessons. Say that John Mason explains this as the difference between 'working-through' some maths and 'working-on' some maths and share the following extract from the John Mason paper:

Working through exercises and working on exercises
The first describes the student who does a few questions, takes a break, does a bit more on the bus, copies a bit from a friend, and ends up with no overall sense of the exercises as examples of anything or what they are about. Contrast this with the student who in doing the exercises asks themself what is similar about the questions and what different, what it is about the context which enables the technique to work, what sorts of difficulties might the technique encounter in different situations, etc. That student is working-on the exercises.
The two states of working-through and working-on are completely different, and in particular they involve different energies. Working-through minimises effort through minimum involvement. It is unreflective and unmathematical. Working-on minimises effort mathematically, by trying to locate underlying structure and so reduce memory demands.

2. Explain that you are going to begin to look what this means, by exploring one particular example, 'Doing and undoing'. Understanding how to undo what has been done in mathematics requires an understanding of how the mathematics works and mathematical relationships and involves generalising (part of the aims of the National Curriculum). Doing and undoing includes things such as the inverse relationship between addition and subtraction which has been identified as a key understanding in KS1; reference the Key Understandings paper (page 5).

"If children are assessed in their understanding of the inverse relation between addition and subtraction, of additive composition, and of one-to-many correspondence in their first year of school, this provides us with a good way of anticipating whether they will have difficulties in learning mathematics in school."

3. Introduce the large sheet of paper with 'Doing and Undoing' in the centre. Invite everyone to think about this theme across the maths curriculum over the next two weeks and whenever they think of somewhere it applies, to add it to the sheet, which you will come back to during a future meeting. Write 'adding and subtracting in KS1' as the first idea on the sheet.

Follow-up meeting two weeks later (you may need to prompt people to add to the sheet and model this by adding ideas during the two weeks):

- Have the large sheet which has ideas connected to 'Doing and Undoing'. Choose from the following to explore, depending on the length of your meeting:
  - Look at the mathematical ideas. Ask: *Are there any other examples of doing and undoing in maths which you can think of?*
    Make sure the following are on the sheet: filling and emptying
    - doubling and halving
    - adding and subtracting
    - multiplying and dividing, including with fractions (for example dividing one to make a unit fraction and then multiplying the unit fraction to make one)
    - square and square root
    - turning clockwise and anticlockwise
    - reflecting and reflecting again
    - translating a shape and returning it to its original position
    - converting (for example from g to kg and back to g)
    - sequences (such as think of a number)

  - Choose one idea relevant to your children and discuss how to support understanding. *Consider:*
    - *What might the children physically do to demonstrate this relationship?*
    - *What contexts make sense of the relationship?*
    - *How could it be modelled with different resources/pictures/drawings?*
    - *How could it be recorded symbolically?*
    - *What would you want the children to notice and be able to explain?*
    - *What sort of questions would show if the children have understood?*

  - Asking the children to do things physically and modelling with resources and drawings will prompt different explanations and support the children to focus on what has changed and what has stayed the same.

    An example connected to adding and subtracting:
    - The children move forwards four on a large number track on the floor and then discuss what they would need to do to return to the number they started on; this would generate the language of forwards and backwards which would be matched to adding and subtracting.
    - Contexts would include board games (for example, throwing four in a game, moving

forwards and the square landed on says 'move backwards four places'), saving money and then spending it, children joining a class and children leaving it, making some cakes and then eating them etc.
- Bead strings used to model adding then subtracting the same beads, which could be matched to a number line; money put in the savings box and then taken out; or a tin of counters being added to a pile of counters and then taken off the pile.
- $10 + 4 - 4 = 10$, $x + 4 - 4 = x$ and $4 + x - x = 4$
- It is important that the children notice that it doesn't matter which number you start from or what you add, as long as you subtract what you have added you return to the number you started with.
- Understanding is applied in questions such as: $34 + 17 + 65 - 17 + 1 =$

- Ask: *Does undoing what you have done always take you back to where you started? Can you think of any examples in maths where it doesn't?*
  - Sometimes, doing and undoing will not always take you back to the original number. For example:
    - When using some calculators $1 \div 3 \times 3 = 0.9999999$ and $(\sqrt{7})^2 = 6.9999999$
    - Undoing to solve 'I think of a number, subtract five then multiply my new number by itself and reach four. What number did I start with?' which may take you to 7, when the original number was actually 2.
    - Situations where it is not possible to know the original number by undoing; for example undoing multiplying by 0.

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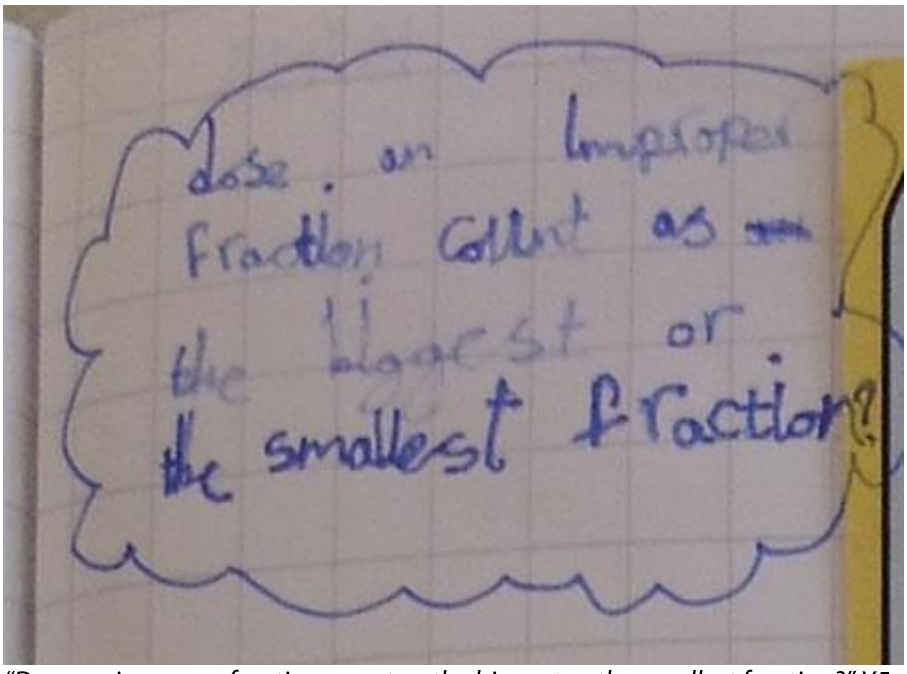
Seen and Heard

Seen and Heard *will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

The photo below was taken from a response to an elicitation task, used at the beginning of a Y5 teaching sequence which focused on fractions. The task included ordering the numbers $^{15}/_{10}$, $^{5}/_{10}$, $^{1}/_{2}$, $^{15}/_{20}$, $^{1}/_{5}$ and $^{37}/_{100}$ with the children asked to explain their thinking.



"Does an improper fraction count as the biggest or the smallest fraction?" Y5 child

- As a teacher, what would you think if a child wrote this?
- What else would you need to ask to appreciate what the child understands about fractions? What might you ask them to draw to help them communicate their understanding?
- If a teacher thought the child believes that fractions do not fit throughout the number system but exist only between 0 and 1, so that when they reach 1 they start again at 0, would you agree? Why?
- What would you do next with the child?

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.