Core concept 3.1: Understanding multiplicative relationships

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

3.1.1 Understand the concept of multiplicative relationships

How confident are you that you can explain how any two numbers are connected via a multiplicative relationship?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can recognise multiplicative relationships in the contexts such as proportion, similarity and conversion rates?

1 ☐ 2 ☐ 3 ☐ 4 ☐

A key concept here is that any two quantities can be linked multiplicatively.

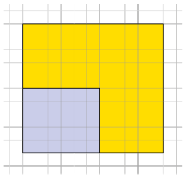
×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

1) Look at the 4s row in the multiplication square.
a) What does 4 need to be multiplied by to move to 8?
b) What does the 8 need to be multiplied by to move to 16?
c) What does the 8 need to be multiplied by to move to 20?
d) What does the 8 need to be multiplied by to move to 4?

2) Consider corresponding entries in the 4s row and the 6s row.
a) What is the relationship between 8 and 12; 20 and 30; 40 and 60, etc.?
b) Picture where 10 might be in the 4s row. What number would be in the corresponding position in the 6s row?

Later, students will appreciate that the multiplier can be expressed as the fraction comprising the two numbers (for example, $3 \times \frac{5}{3} = 5$ and $5 \times \frac{3}{5} = 3$).

This example shows multiplicative relationships in the context of similarity. Are these rectangles similar shapes? How do you know?



Further support links

- NCETM Secondary Professional Development materials: 3.1 Understanding multiplicative relationships, pages 8–15
- NCETM: Using mathematical representations at KS3: Cuisenaire rods

3.1.2 Understand that multiplicative relationships can be represented in a number of ways and connect and move between those different representations

How confident are you that you can explain how represent a multiplicative relationship using a double number line, ratio table, a graph and a scaling diagram?

1 ☐ 2 ☐ 3 ☐ 4 ☐

These images and representations can be used to build an understanding of the different interpretations of multiplicative structures and so make the connections between seemingly distinct topics explicit. Keep in mind that the purpose of these different representations is to reveal the underpinning mathematical structure, rather than to provide a method to achieve an answer. For example:

Ali buys eight identical packets of cakes for a birthday party. The total cost is £12.

Which line in this double number line represents the number of packets of cakes Ali buys and which represents the total cost? Explain how you know.

a) Use this diagram to write down the cost of some other quantities of packets of cakes.

b) Can you write down roughly how much nine packets of cakes would cost? Can you be precise? Explain how.

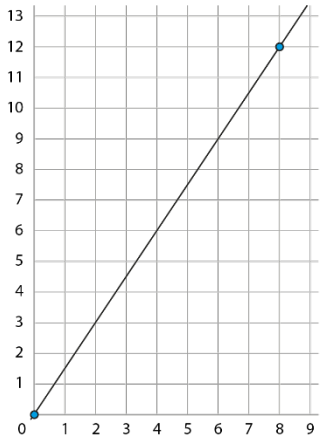
The double number line can be a useful image to support understanding of the underlying mathematical structure of a multiplicative relationship. It may not always be an efficient representation with which to calculate an answer, but it is an important representation to think with.

This ratio table shows the same information as the double number line in the previous question:

12	
8	9

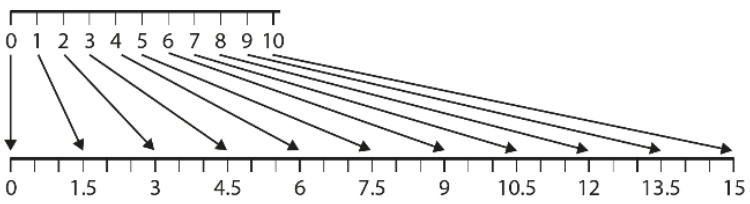
A ratio table can be thought of as a compressed version of the double number line and is likely to be more efficient.

The information can also be represented using a line graph:



By comparing the double number line and the line graph, we can see that every pair of numbers that align on the double number line are represented by coordinates on the graph.

By visualising a number line being stretched, the important sense of multiplication as scaling can be introduced.



Further support links

- NCETM Secondary Professional Development materials: 3.1 Understanding multiplicative relationships, pages 23–27
- NCETM: Using mathematical representations at KS3: Double number line and ratio tables

3.1.3 Understand that fractions are an example of a multiplicative relationship and apply this understanding to a range of contexts

How confident are you that you can explain how to express one number as a fraction of another?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can explain how to find a fraction of a given amount?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Fraction notation holds within it a multiplicative relationship. In a fraction such as $\frac{2}{3}$, the numerator will be two-thirds of the denominator, and the denominator will be three-halves of the numerator. However, a particular focus here is the use of a fraction as a multiplier. Students should view a relationship of the form $ab = c$ (where a and/or b is a fraction) from different perspectives and in different contexts. For example:

- $\frac{2}{3}$ of $6 = 4$
- 7 is $\frac{1}{3}$ of 21
- And 5 as a fraction of 15 is $\frac{1}{3}$

Further support links

- NRICH: Teaching fractions with understanding: https://nrich.maths.org/2550

3.1.4 Understand that ratios are an example of a multiplicative relationship and apply this understanding to a range of contexts

How confident are you that you understand and can calculate with ratio in a range of contexts?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Here, ratios are used to describe and explore multiplicative relationships.

For example: 'Some money is shared between Alan and Layla in the ratio 2 : 3. If Alan receives £10, how much does Layla receive?'

Students should have the awareness that Alan and Layla's money is linked by multiplicative relationships: Layla has $\frac{3}{2}$ of Alan's share, and Alan has $\frac{2}{3}$ of Layla's share. Also, Alan has $\frac{2}{5}$ and Layla has $\frac{3}{5}$ of the total.

The double number line and ratio table representations can be key in seeing a rate as representing different multiplicative relationships. It is important to emphasise that there are two multiplicative relationships evident in each situation:

- one that scales a value or quantity to the next (for example, if £3 = $4, then £6 = $8, doubling each value)
- one that converts a value or quantity to another (if £3 = $4, then £7.50 = $10 as 7.5 is $\frac{3}{4}$ of 10).

Further support links

- NCETM Secondary Professional Development materials: 3.1 Understanding multiplicative relationships, pages 28–33
- NCETM: Using mathematical representations at KS3: Double number line and ratio tables

3.1.5 Understand that percentages are an example of a multiplicative relationship and apply this understanding to a range of contexts

How confident are you that you understand and can explain how to calculate with percentages in a range of contexts?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Here the use of percentages to represent multiplicative relationships is explored.

As with ratio, the double number line and ratio table representations are useful in identifying and working with the multiplier, and consistent use of these representations through ratio, percentages and proportion may make the connections between these apparently different topics more apparent.

Students are again working on the relationship $ab = c$, where a or b is written as, or interpreted as, a percentage, and are exploring this in different contexts and with different representations.

Further support links

- NCETM Secondary Professional Development materials: 3.1 Understanding multiplicative relationships, pages 34–37
- NCETM: Using mathematical representations at KS3: Double number line and ratio tables

3.1.6 Understand proportionality

How confident are you that you understand and can explain direct and inverse proportion in a range of contexts?

1 ☐ 2 ☐ 3 ☐ 4 ☐

An important awareness is that there is one unifying structure which connects fractions, percentages and ratio, and that this one structure can be described by the algebraic formulae $x \times k = y$ or alternatively $k = \frac{y}{x}$, where x and y are the quantities in proportion and k is the constant of proportionality.

While exploring a wide range of examples of proportionality (including examples of *'what it's not'*) it is important to make the distinction between linear relationships that are not proportional (i.e., of the form $y = mx + c$ rather than $y = kx$) and also to become aware of situations where the variables are inversely proportional (i.e. $y = k \times = \frac{1}{x}$, or $y = \frac{k}{x}$). In formalising this generalisation, students are able to use the underlying structure to develop an awareness that there are different types of proportionality – particularly inverse proportionality.

Further support links

- NRICH: Ratio or proportion? https://nrich.maths.org/4825

Notes