Secondary Magazine 119

National Centre for Excellence in the Teaching of Mathematics

For those of us teaching further Mechanics, Spring is probably already here – in the form of Hooke's Law and/or Simple Harmonic Motion (ok, not our best maths-joke)…but for the rest of us, it still seems a long way off as we scrape frost from the windscreen and travel to work and back in the gloom. So we hope that this latest issue of the Secondary and FE magazine will act like an upside-down bucket in the rhubarb shed of your mathematical mind, and soon zingy fresh ideas will be bursting into your thinking and teaching like Yorkshire's proudest pinkest finest! Please do share with us what comes forth, by email to info@ncetm.org.uk or on Twitter, @NCETMsecondary.

Let us take a moment to give our huge thanks – and congratulations – to the Maths team in the White Rose Maths Hub, who organised the stadium-filling jamboree that was the Celebration of Maths on 7 February. You know you're at a high-profile Maths event when you're behind Marcus du Sautoy and Rachel Riley in the queue for the pi and chips! It was great fun, hugely stimulating, and wildly popular: well done White Rose – and thank you for giving us another winter treat from Yorkshire!

Contents

Image credit

Heads Up

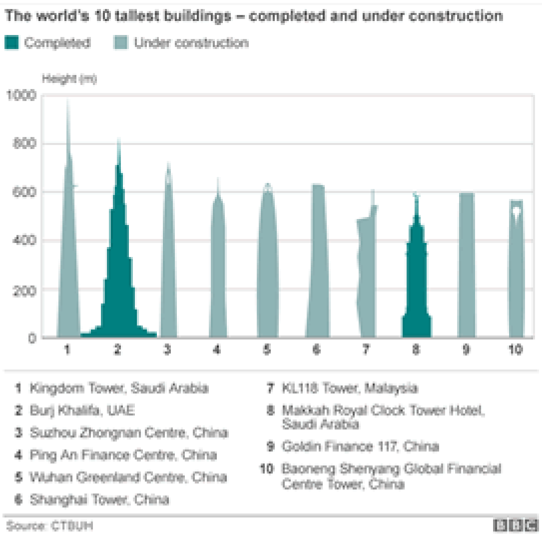
Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.

If you're in any way involved in, or interested in, the new Core Maths qualifications, you might want to attend one of two open events being run in connection with the Core Maths Support Programme. There's one event in London on 2 March, and one in York on 4 March, and both events are free.

There's less of the wonderful *More or Less* on BBC Radio 4 at the moment, but you can download podcasts of editions from the World Service on the programme's homepage. You can also follow the presenter on Twitter, @TimHarford: lots of great ideas to take into lessons, especially but not only about the misunderstanding and misuse of data.

In January, Ofqual (Twitter @ofqual) published details of a number of measures they are taking to review the level of demand of the sample GSCE assessment materials published by exam boards in preparation for the introduction of the new GCSE (first teaching September 2015, first exams Summer 2017). The three strands to the research programme, and the proposed timeline, are set out here. We will keep you updated, of course.

The Institute of Mathematics and its Applications (IMA) is offering Mathematics Teacher Training Scholarships of £25 000. The IMA says that "a Maths teacher training scholarship will recognise your potential to be an inspirational teacher and future leader in maths education. It not only brings with it a tax free grant of £25 000 (replacing any teacher training bursary entitlement) but membership of a community where you can share your highs and lows, experiences and queries". Do share this with aspiring mathematics teachers that you know: it's never been a better time to train to teach Maths.



You may have noticed the recent news item, Lift me higher: Building the world's tallest lift. The graphic representation of the heights of these buildings reminded us of the resource Fox scales Shard.

Three new publications have caught our eyes this month:

- Exploring Geometry with a 9 Pin Circular Geoboard by Geoff Faux,
- Algebra Project - Introduction to Algebra and Solving Equations, written by a group of teachers in North Yorkshire, and
- Bigger Ideas by Christopher Martin.

Thinking about spring cleaning? Start with the attic - at Bletchley Park, they've just found some of Turing's code-breaking research being used to keep the draughts (as well as the Germans!) at bay. Maybe Fermat's own proof of his last theorem is currently patching up your loft insulation?!

Looking ahead, by the time you're reading issue 120, we will have celebrated the numerological maths-gasm that is Pi Day, Stephen Hawking is basking in Oscar glory (as might have Alan Turing), and the new film X + Y (about an ASD mathematician finding love at the Taiwanese IMO – no, honestly!) will be in the multiplex. Right now, my "Maths" and "Zeitgeist" Venn diagram is pretty much two concentric congruent circles! What will you, your students or your school/college be doing to join in?

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Secondary Magazine 119

National Centre for Excellence in the Teaching of Mathematics

Building Bridges

In the last issue, we considered how the area model for multiplication helps pupils visualise and understand multiplicative invariance: that A × B = (kA) × (B÷K). The area model is one of the best examples of a bridge that can stretch all the way from KS1 to KS5, and so for this issue we thought we'd pull together some uses of it, and invite you to tell us others – if you have pictures you can tweet @NCETMsecondary or send to info@ncetm.org.uk, we'll share them in the next issue.

Most children will, in KS1, use arrays of concrete objects to represent multiplication, for example arranging 15 counters into 3 rows of 5 (and 5 rows of 3). The extension to the area model – often called the grid method – in KS2 comes naturally, perhaps thinking about a farmer with a field that is 16m by 37m who wants to grow four crops – one of which, of course, is rhubarb! Dividing the field into four smaller sub-fields enables the calculation of 16 × 37 to be broken into four easier calculations:

×	30	7	
10	300	70	
6	180	42	
			592

Part of the power of this visualisation is that it can indeed be visualised, and pupils' mental multiplication of two digit numbers can be practised and improved – passers-by of many of my KS3 lessons will have heard the slightly odd instructions "right, close your eyes, think, ok, top left, top right, bottom left, bottom right, what have you got?" Handheld whiteboards might well be useful at first.

The grid also explains the steps of column multiplication. This formal algorithm is certainly more efficient with larger numbers and must be taught and practised to ensure procedural fluency – pupils mustn't be left **only** able to use the grid method – but it doesn't develop conceptual understanding as well as thinking about fields (of rhubarb!) does.

Once pupils are confident with negative numbers, the efficiency of the grid method can be refined: to evaluate 21 × 19, ask pupils to compare

×	20	1	
10	200	10	
9	180	9	
			399

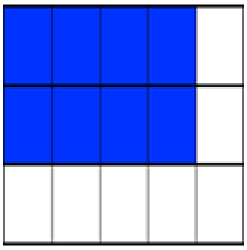
and

×	20	1	
20	400	20	
-1	-20	-1	
			399

This enables pupils to make and justify conjectures equivalent to the difference of two squares without having to use algebraic symbols, if they would obfuscate the reasoning.

The area model can be adapted throughout KS2 to 5:

- $\frac{4}{5} \times \frac{2}{3}$



The grid shows **why** the algorithm is to multiply the numerators and the denominators.

- $2\frac{2}{3} \times 5\frac{1}{4}$

×	5	¼	
2	10	½	
⅔	3⅓	⅙	
			14

This is certainly not as efficient as the algorithm "make the fractions top-heavy and then multiply the numerators and the denominators", but the connection it makes to pupils' earlier understanding of multiplication is clear, and thus develops their secure conceptual understanding.

- Expanding brackets

×	$3x$	2	
x	$3x^2$	$2x$	
-5	$-15x$	-10	
			$3x^2 - 13x - 10$

in particular

×	x	6	
x	x^2	**$6x$**	
6	**$6x$**	36	
			x^2 **$+ 12x$** $+ 36$

and

×	x	6	
x	x^2	$6x$	
-6	$-6x$	-36	
			$x^2 - 36$

and then rich challenges such as $(a + b + c)^2$

- Pupils can teach themselves to factorise quadratics, from a well-designed sequence of "guess and check" intelligent practice questions such as:

×	x	?
x	x^2	?
?	?	5

$x^2 + 6x + 5$

then

×	x	?
x	x^2	?
?	?	-8

$x^2 + 2x - 8$

then

×	$?x$	?
x	$3x^2$	?
?	?	?

$3x^2 + 15x$

then

×	x	?
x	x^2	?
?	?	?

$x^2 - 25$

then

×	$?x$	?
$?x$	$9x^2$	?
?	?	?

$9x^2 - 4$

then

×	$?x$	?
x	$3x^2$	?
?	?	?

$3x^2 - 8x - 3$

then

×	$?x$	?
$?x$	?	?
?	?	?

$4x^2 + 7x - 15$

then

×	$?x$	?
$?x$	?	?
?	?	?

$4x^2 + 12x + 9$

and so on.

- And, in KS5,

×	5	$2j$	
3	15	$6j$	
$-4j$	$-20j$	$-8j^2$	
			$23 - 14j$

and

×	?	?	
?	a^2	?	
?	?	?	
			$a^2 + b^2$

and a personal favourite: factorising $x^4 + 4y^4$ (hint, this farmer grows 9 crops).

The area model is, therefore, a Humber-spanning bridge that takes pupils all the way from their first experiences of positive integer multiplication to their exploration of the complex plane in Further Pure Maths: a route well worth their following.

Image credit

Secondary Magazine 119

National Centre for Excellence in the Teaching of Mathematics

Sixth Sense
Making More Sense of Probability

This article continues from Making Sense of Probability in Issue 118

One of the key issues with using probabilities in the real world is the way in which probabilities are stated as percentages and the way in which this information is then transmitted to the person needing to assess the probabilities, usually in some sort of assessment of risk (Gigerenzer 2003).

As an example, this might be a common statement of probabilities around a medical test for a disease:

> *'On historical evidence, 0.8% of the population will develop a particular kind of cancer. When tested, 90% of those with cancer will get a positive result and 7% of those without cancer will get a positive result'.*

For statisticians and medical practitioners who frequently work with such statements and fully understand them, the percentages are a clear assessment of both the risk of this type of cancer and the likely outcomes of the test. For the non-specialist this is probably unclear and using frequencies would probably be more helpful.

Compare the above with the following.

> *'Out of 1000 people, on historical evidence, 8 are very likely to develop this type of cancer.*
> *Of these 8 people, the test will successfully identify about 7 people and leave roughly one person with cancer undiagnosed.*
> *Of the remaining 992 people, the test will be clear for 922 people but there are likely to be 70 people, who, as a result of the screening, get a positive test but do not have this type of cancer.'*

or with this diagram.

Which do you think conveys the information in the simplest way to a non-specialist?

Now consider the following scenario.

> *'In a court case, a witness sees a crime in their local town involving a taxi. This witness says the taxi was green. It is known from previous research by the police that witnesses are correct 80% of the time when making such statements. The police also know that 85% of taxis locally are blue and the other 15% are green. What is the probability that a green taxi was actually involved the crime?'*

This scenario opens up a discussion around probabilities of combined events linked to the use of tree diagrams and the calculation of conditional probabilities.

Here, we want P (the taxi was green given that it was identified as green) which is the same as P (green and identified as green)/P(identified as green) = (0.15x0.8)/(0.15x0.8 + 0.85x0.2) = 0.41.

This is a lower probability than many jury members might expect and this would need careful explanation during the trial in relation to the low relative frequency of green taxis and the potentially poor quality of witness colour identification.

Are all your Core Maths/A Level Maths students using all the visual tools available to them in solving probability problems and could they all operate in a real world setting and explain probabilities to a non-specialist audience?

Further reading

Gigerenzer, G. “Reckoning With Risk: Learning To Live With Uncertainty” (Penguin 2003)
Schneps and Colmez “Maths on Trial” (Basic Books 2013).

Image credit

From the Library
Shh! No Talking!

Our regular feature highlighting an article or research paper that will, we hope, have a helpful bearing on your teaching of mathematics

The equation, the whole equation and nothing but the equation by Susan Pirie gives us the theme for this issue. In the article, Dr Pirie reflects on approaches to solving linear equations. The paper was presented at the British Society for Research into Learning Mathematics day conference held at the University of Birmingham, on Saturday 25 March 1995. The article talks about the balance method and then the alternative inverse operation method, stating that “Both these methods lead to the pupils "solving" an equation that is not the original given one, but some altered, in some sense equivalent, equation”. At the start, the article is insightful about how learners read “=”: it’s well worth asking your pupils / students what they think the difference is, if any, between $2x + 7 = 29$ and $29 = 2x + 7$.

It is worth persevering with the slightly odd symbolisation in this print version of the article to uncover how the pupils use reasoning to solve the equations by thinking of the idea of a fence (rather than a balance), and how this extends into solving equations with unknowns on both sides. In particular, on page 5 of the paper there is some dialogue between a ‘low ability’ (to use what is now out-dated language) child (Joan) and her teacher about solving the equation $11 + 5b = 3b + 25$. The conversation reveals that Joan is clearly thinking through the logic of the equation and not relying on a formulaic method. How could we help our pupils develop the confidence of their mathematical reasoning so that more – all – of them have this insight.

We think that you will, having read this paper, have a deeper understanding of why some pupils find equations difficult, and also have some ideas how to respond to this in order to help your pupils develop more secure conceptual understanding and procedural fluency. The paper is now 20 years old: how much of it has stood the test of time, do you think? Would the findings be the same if the research were carried out today? Have you taught successfully a method for solving equations that is not based upon the very common “balance” model? If so, what was it and why was it effective? Let us know, @NCETMsecondary or info@ncetm.org.uk.

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Secondary Magazine 119

National Centre for Excellence in the Teaching of Mathematics

It Stands to Reason

Solving Equations

In this regular feature, an element of the mathematics curriculum is chosen and we collate for you some teaching ideas and resources that we think will help your pupils develop their reasoning skills. If you'd like to suggest a future topic, please do so to info@ncetm.org.uk or @NCETMsecondary.

So how do you think about equations? What do you 'see' in your head when there are symbols such as

$$y + 3 = 5$$

on paper? The chances are that you have stopped 'seeing' anything because you have secure grasp of the mathematics and you have not considered this for a while. But what does this set of symbols mean to pupils, or what images do you provide to help their understanding?

You could start by doing some work on expressions. You could use a set of digit cards or ask pupils to work on mini white boards to construct some expressions representing statements such as:

- *'I'm thinking of a number and I add one'*
- *'I'm thinking of a number and I double it'*
- *'I'm thinking of a number and I take away one'*

Having ironed out any possible errors in what can be called 'symbol sense', you are ready to move onto that significant moment when you say:

- *'I'm thinking of a number, I add 5 and the answer is 12'.*

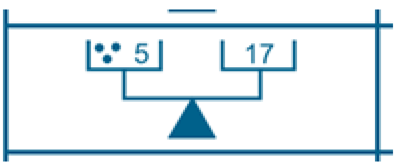
It is worth asking pupils to tell you what is different now.

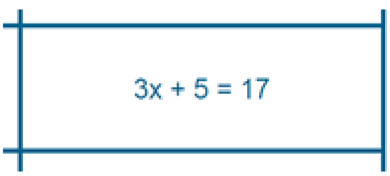
It is important soon to extend your equations to something like:

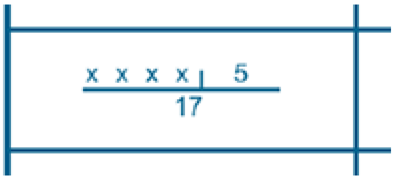
- *'I'm thinking of a number, I double it and add 3. This gives the same result as thinking of the number and adding 5'.*

This immediately introduces the idea of a variable on both sides of the equation but also reinforces the idea that the equals sign means 'is the same as'.

The NCETM resource Solving Equations will give you some ideas how to represent equations in alternative forms. Developing with pupils a range of representations increases the likelihood of each pupil seeing one that resonates strongly with him or her, and which then unlocks his or her understanding.







Alternatively, you could start with some puzzles that don't seem to require formal algebra at all, but which will develop algebraic reasoning as they are solved. These are worthwhile.

Further resources include:

- the ATM publication Introduction to Algebra and Solving Equations
- Constructing and Solving Linear Equations, available from the National STEM Centre eLibrary
- the NCETM Departmental Workshop Constructing Equations will give you food for thought
- these interesting Fibonacci equations.

You or your pupils could share your thinking on a Pinterest board.

How do the other teachers in your department teach pupils to solve linear equations? Do they develop pupils' reasoning using a "balance" image, or do they use an inverse method? Is there something that you prefer to use? Is there a department policy? If you are all using different models to develop pupils' conceptual understanding and procedural fluency, what happens when a pupil changes teacher?

Image credit

Eyes Down

Our monthly picture that you could use with your pupils, or your department, or just by yourself, to make you think about something in a different way

This lovely tiling picture was snapped in a shop doorway – aren't the colours great?

You could have this picture displayed as pupils enter the classroom and ask them, as they settle down, to write down some things they notice about the picture, and some questions they'd like to think further about. In this way, every pupil will make a contribution. Questions to consider include:

- What polygons can you see?
- Are the "squares" and octagons regular polygons? Justify your answer.
- Are the blue squares and the white squares congruent? Explain your answer.
- Would it be possible to create a similar picture with only regular polygons?
- If you think you can, draw your design(s). How many different designs can you draw?
- Imagine halving the edge length of the blue squares. What stays the same? What would be different?

- Can you prove that three hexagons fit around a point leaving no gap? What sets of three regular polygons (which don't all have to have the same number of sides) fit together around a point leaving no gaps?

If other questions prove fruitful, do let us know them.

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk, with a note of where and when it was taken, and any comments on it you may have.

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