

Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 2

How confident are you that you understand and can support children to identify the principles of equal grouping and scaling in multiplication?

1

2

3

4

How would you respond ...?

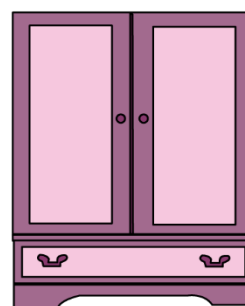
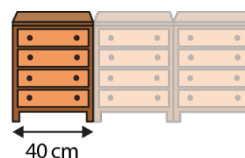
- a. Explain how repeated addition is linked to multiplication, identifying the structure it exposes.



5	5	5
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$$5 + 5 + 5$$

- b. Using the image, create a scaling multiplication problem.



Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that...?

- The multiplication structure of equal grouping is based on repeated addition of equal groups. In this question, there are three groups (the three buckets) of five (the water in the bucket). Repeated addition ($5 + 5 + 5$) is used to describe the image. This can be developed into a multiplication equation, e.g. three groups of five or 3×5 .
- When writing a scaling problem, focus on the language used. 'Charlotte is arranging the furniture in her room. Her wardrobe is **three times** the width of her cabinet. How wide is her wardrobe?' Notice the use of 'three times'. This is different to multiplication as equal grouping, where the focus is on group size.

Structures of multiplication

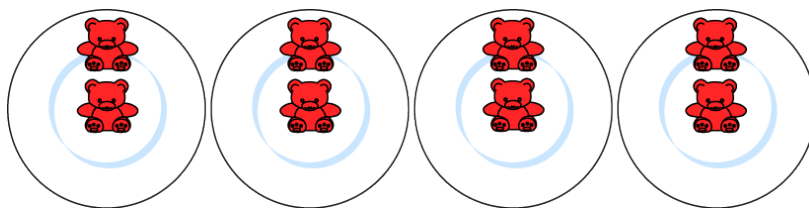
This section explores multiplication as repeated addition and multiplication as scaling, within the context of measure, as scaling often involves an action onto an object rather than repeating groups of objects.

Multiplication as equal groups

Before introducing the structures of multiplication, children should have experience of identifying equal and unequal groups. When they are able to accurately identify these, they can then begin to focus on the size and number of equal groups.

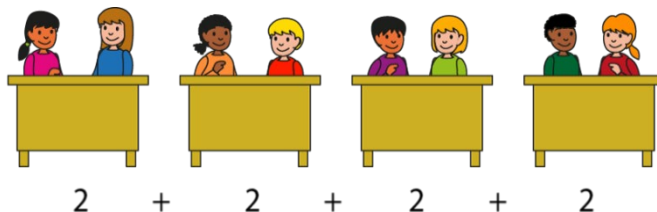
For example:

- 'There are four equal groups of bears.'
- 'There are two bears in each group.'
- 'There are four groups of two.'



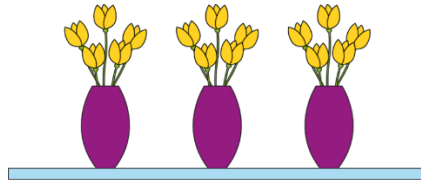
To build children's understanding further, concrete objects can be used, with the children asked to arrange them into groups. For example, 12 cubes arranged into 3 groups of 4.

As children develop this concept, a repeated addition can be introduced. It is important to take time to describe these stages.



- 'There are four groups of two.'
- 'There are two and two and two and two.'
- 'We can write this as two plus two plus two plus two.'
- 'What does each "2" represent?'
- 'Each "2" represents the number of children at each desk. There are two children at each desk.'
- 'Why are there four "2s"?''
- 'There are four "2s" because there are four groups of two.'

When the children can confidently identify the size and number of groups, it is a relatively simple matter to introduce the multiplication symbol. For example, 'three groups of four' can be written as ' 3×4 '. Since children can already represent equal groups using a repeated addition expression, they can also relate multiplication to repeated addition.



There are three groups of five.

'We can write this as five plus five plus five.'

$$5 + 5 + 5$$

'We can also write this as three times five.'

$$3 \times 5$$

'What does the '3' represent?'

'The '3' represents the number of groups.'

'What does the '5' represent?'

'The '5' represents the number of flowers in each group.'

Multiplication as scaling

Scaling is distinct from the grouping structure of multiplication (which has been connected to repeated addition) and from the quotitive and partitive structures of division (which have been connected to repeated subtraction). Multiplication as scaling considers an overall increase (when multiplying by a whole number) or decrease (when multiplying by a proper fraction) in value.

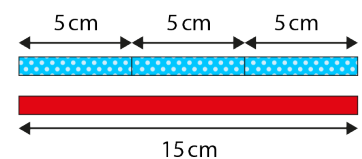
Scaling can represent multiplicative contexts that repeated addition cannot, such as decreasing the size of an object, or situations that do not represent multiple copies. For example, when a sunflower grows to ten times its original height, the outcome is not ten sunflowers but one taller sunflower.

In order for children to be able to focus on the scaling structure, it is important they are fluent with their times table facts (and appropriate mental/written methods for factors larger than 12). If children are still reliant on reciting facts up to the desired multiple, or on repeated addition, they will be less focused on the scaling structure.

Scaling as a comparison could be introduced using a practical situation.

**The plain ribbon is 15 centimetres long. The plain ribbon is three times the length of the spotty ribbon.
 $5\text{cm} \times 3 = 15\text{cm}$**

- The 5cm represents the length of one spotty ribbon.
- The 3 represents the number of spotty ribbons that are equal to the length of the plain ribbon.
- The 15cm represents the length of three spotty ribbons. It also represents the length of the plain ribbon.



Notice that the language used is very specific: '*... times the length*' and not '*... times longer*'.

Consider the question, 'I planted a twelve centimetre sunflower plant in the garden in April. By June, it was ten times the original height. What was the height of the sunflower in June?' The attribute being described is height; note that we say, 'ten times the original height' and not 'ten times taller'.

If we consider an abstract context, for example $7 \times 3 = 21$, it could be described as follows: '*twenty one is three times the size of seven*.'

Common errors in this area may include:

- children may not understand the continuous nature of scaling

What to look for

Can a child:

- identify when a problem is scaling or repeated addition and use an appropriate representation?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:

- Topic 2.17: Structures: using measures and comparison to understand scaling
- Topic 2.2: Structures: multiplication representing equal groups

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: