MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.21 Factors, multiples, prime numbers and composite numbers

Teacher guide | Year 5

Teaching point 1:

Factors are positive integers that can be multiplied together to equal a given number.

Teaching point 2:

Systematic methods can be used to find all factors of a number; factors come in pairs; all positive integers have an even number of factors apart from square numbers, which have an odd number of factors; numbers with more than two factors are called composite numbers.

Teaching point 3:

Prime numbers are positive integers that have exactly two factors.

Teaching point 4:

A common factor is a factor that is shared by two or more numbers. A prime factor is a factor that is also a prime number.

Teaching point 5:

A multiple of a number is the product of that number and an integer; a common multiple is a multiple that is shared by two or more numbers.

Teaching point 6:

The factor pairs of '100' can be used to support efficient calculation.

Overview of learning

In this segment children will:

- revisit the term 'factor' and learn to identify all factors of a positive integer
- learn that '1' is a factor of all positive integers, and every positive integer is a factor of itself
- use multiplication and division to find factors
- explore the link between square numbers and factors, learning that square numbers have an odd number of factors
- learn to identify composite numbers, prime numbers, common factors and prime factors
- be introduced to the term *'multiple'* and learn to identify multiples and common multiples
- identify the factor pairs of '100' and use this knowledge to solve problems efficiently.

Teaching point 1 begins by revisiting the term 'factor', which was covered in segment *2.3 Times tables: groups of 2 and commutativity (part 1)* in the form of the equation: *factor* × *factor* = *product*. Children will identify factors by making arrays using a given number of tiles and learn that the number of tiles in the rows/columns represent factors of the given number. Arrays are a useful tool to introduce the topic but should not be relied upon in later steps; once children are confident in the concept of factors they should apply their existing knowledge of multiplication and division.

In step *1:3* children learn that '1' is a factor of all positive integers and that every positive integer is a factor of itself. This is crucial for the understanding of later learning on prime numbers. Some children will begin to identify patterns and trends in the different types of arrays that can be made, such as prime numbers and square numbers, but these are not expanded upon at this point.

The multiplication chart is used to demonstrate the link between times tables and factors, and children are encouraged to use their known times table facts to identify factors. A link is also made between factors and division, using the language of divisor and quotient, preparing children for further learning in *Teaching point 2*.

In *Teaching point 2* children learn to work systematically to find factor pairs. It is important that children know when to stop checking for factors: this is covered in step *2:3*. Children also begin to think about common factors, but these are not defined until *Teaching point 4*. Next, square numbers are explored. Children are already familiar with square numbers (see segment *2.9 Times tables: 7 and patterns within/across times tables, Teaching point 3*), but here they extend this knowledge to learn that square numbers have an odd number of factors.

Prime numbers are defined in *Teaching point 3*, with a brief explanation as to why '1' is *not* a prime number. *Teaching point 4* explores common factors and prime factors.

Teaching point 5 introduces the term *'multiple'* and explores the link between factors and multiples. Children are encouraged to use known times table facts and divisibility rules to identify multiples and common multiples. In *Teaching point 6* the factor pairs of '100' are used to solve problems in real-life and abstract contexts.

Throughout the segment, children should be encouraged to use the correct mathematical terms to discuss and explain their answers, making links to multiplication and division.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Factors are positive integers that can be multiplied together to equal a given number.

Steps in learning

	Guidance	Representations
1:1	See guidance below	See representations below

Guidance

In this teaching point we revisit the word ‘factor’. Note that it is important in this segment to work only with *positive integers* when finding factors. This is because the factor of a number must be a whole number, and it is preferable to avoid using factors of zero at this stage.

In segment *2.3 Times tables: groups of 2 and commutativity (part 1)* children learnt that:

- *factor × factor = product*
- *product = factor × factor*

To refamiliarise children with the concept of factors, ask them to make arrays from a set number of objects such as square tiles. For the representation opposite, ask children *‘How many different ways can you arrange “12” tiles into a rectangle?’*

It is preferable to use square tiles rather than round counters if possible, so that children can clearly see when they have made a rectangle.

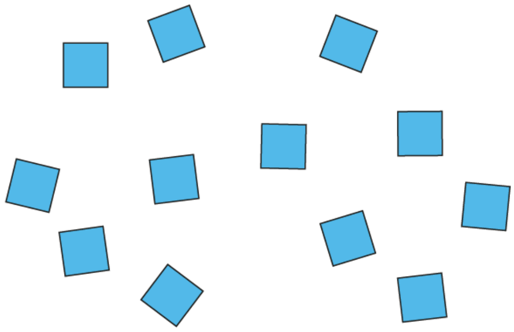
When working with the arrays, make links to commutativity (see segment *2.3*) to understand that a factor can be the length or the width of an array. For example, when rearranging the tiles for $1 \times 12 = 12 \times 1$, encourage children to use the following stem sentence: ***‘There are ___ tiles. There are ___ rows and ___ columns. So ___ and ___ are factors of ___.’***

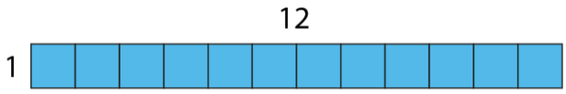
Children should understand that the array shows that ‘12’ can be divided into equal groups of ‘6’ and ‘2’, revealing that ‘6’ and ‘2’ are factors of ‘12’.

Representations

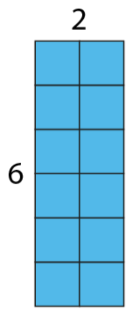
Arranging tiles into arrays:

‘How many different ways can you arrange “12” tiles into a rectangle?’

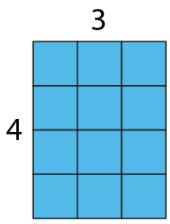




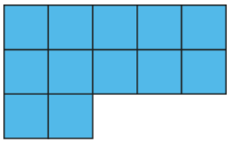
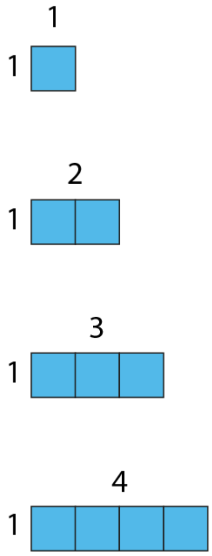
- *‘There are “12” tiles. There is “1” row and “12” columns. So “12” and “1” are factors of “12”.’*



- *‘There are “12” tiles. There are “6” rows and “2” columns. So “6” and “2” are factors of “12”.’*



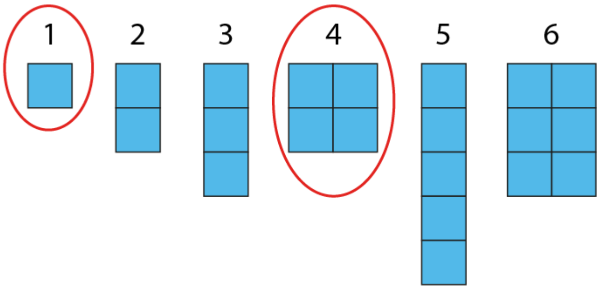
- *‘There are “12” tiles. There are “4” rows and “3” columns. So “4” and “3” are factors of “12”.’*

1:2	Some children may arrange the tiles incorrectly so that they are not true arrays. Remind children that in an array each column must contain the same number of objects as the other columns, and each row must have the same number as the other rows. Arrangements that are not arrays will not show correct factors.	An incorrect array: 
1:3	Repeat the exercise in step *1:1* using sets of 1–20 tiles. Make sure children are confident making all of the possible arrays for each set before moving on. Children will find that for each number it is always possible to have an array that has '1' as the row or column number. Link to segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1, Teaching point 4* in which children learnt that when '1' is a factor, the product is equal to the other factor. Develop this with reference to the arrays to reach the following generalisations: • ***'"1" is a factor of all positive integers.'*** • ***'Every positive integer is a factor of itself.'*** • ***'The smallest factor of a positive integer is always "1".'*** • ***'The largest factor of a positive integer is always itself.'*** If children are unfamiliar with the term 'positive integer', explain that a positive integer is a whole number that is greater than zero.	*'When "1" is a factor, the product is equal to the other factor.'*  $1 \times 1 = 1$ $2 \times 1 = 2$ $3 \times 1 = 3$ $4 \times 1 = 4$

<table>
<tr>
<td>**1:4**</td>
<td>This step begins to draw attention to the different types of rectangles that can be made with the arrays and what we can learn from these shapes.

Continue to use the word 'factor' and remind children that if you can make a rectangular array from a given number of objects, the factors are the number of objects in each row and column because they divide the array into equal groups.

Children have already learnt about square numbers (segment *2.9 Times tables: 7 and patterns within/across times tables, Teaching point 3*) and they should recognise these when creating the arrays.

They may start to see that some numbers can only be arranged in one rectangular array, and therefore only have two factors. Others can be arranged in more than one rectangular array. At this stage do not discuss the reason for this, as this will form teaching points later in the segment. This activity can be returned to as a reference point for later learning.</td>
<td>*'Which of these are square numbers?'*

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1:5	Next show children the completed multiplication chart (see segment *2.11 Times tables: 11 and 12*). Point out that we can use known multiplication facts to help us find factors. Use the following stem sentence: ***'___ is a factor of ___ because ___ is in the ___ times table.'*** Draw children's attention to the fact that there are other factors aside from those shown in the chart (e.g., $21 \times 2 = 42$), but we don't need to learn them all because we can work them out. We will move on to methods for finding factors in the next teaching point.
	Using the multiplication chart to find factors:

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

- *'"7" is a factor of "42" because "42" is in the "7" times table.'*
- *'$42 \div 7 = 6$ so I can make a rectangular array that is 6×7.'*
- *'"6" and "7" are factors of "42".'*

1:6	It is important for later steps that children are able to make links between factors and division. Using the starting number as the dividend, if the divisor gives a quotient that is a whole number, then we know both the divisor and the quotient are factors of the starting number. Note that we do not use the term 'factor'	

	within the division equation – the two factors are known as the divisor and the quotient. However, we can identify them as factors of the dividend.
	Provide children with some examples:
	• *'Is "2" a factor of "12"?'*
	  ◦ 12 ÷ 2 = 6
	  ◦ *'"2" and "6" are factors of "12".'*
	• *'Is "3" a factor of "24"?'*
	  ◦ 24 ÷ 3 = 8
	  ◦ *'"3" and "8" are factors of "24".'*	

1:7	In preparation for the next teaching point, provide children with practice identifying factors within the context of both multiplication and division. Use problems that are within the range of children's known times tables so that they can focus on the structure of the equations.	*'Complete the equations to find the factors.'* 2 × □ = 6 □ and □ are factors of '6'. 4 × □ = 20 □ and □ are factors of '20'. 6 ÷ 3 = □ □ and □ are factors of '6'. 20 ÷ 5 = □ □ and □ are factors of □ . 2 × □ = 10 □ and □ are factors of □ . 21 ÷ 7 = □ □ and □ are factors of □ .

Teaching point 2:

Systematic methods can be used to find all factors of a number; factors come in pairs; all positive integers have an even number of factors apart from square numbers, which have an odd number of factors; numbers with more than two factors are called composite numbers.

Steps in learning

2:1

Guidance

Once children are confident in identifying factors and linking these to multiplication and division, they can learn to find all of the factors of a number in a systematic way.

Show children a variety of numbers between '1' and '20' and ask how many factors each number has. Use tiles to begin with, repeating the process outlined in step *1:1*. Remind children that we can always create an array with '1' and the number itself as factors. Revisit the generalised statements from step *1:3* to reinforce this.

Some children may not initially identify all of the possible arrays for each number. You could prompt children by saying *'Some children have found two factors of six and some children have found four factors of six.'* Continue to prompt until all children have found all factors before moving on.

Record the factors of each number in a list or table so that children can compare them. Ask children:

- *'What's the same?'*
- *'What's different?'*

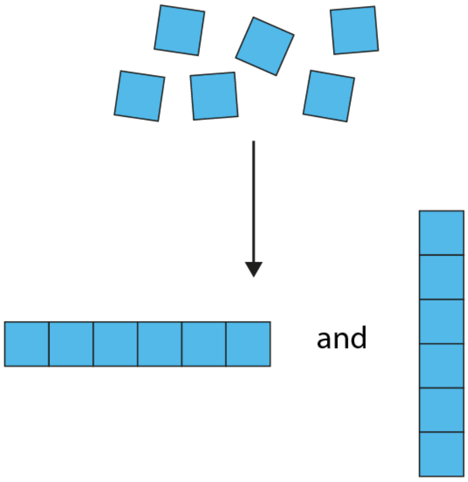
This will begin to identify common factors, which will be developed in *Teaching point 4*.

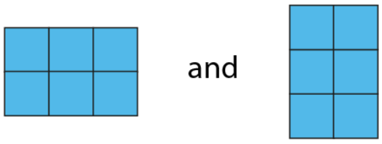
Encourage children to use their knowledge of times table facts to identify factors. For example:

- *'"14" is in the seven times table.'*
- *'"14" is 7 × 2'*
- *'"7" and "2" are factors of "14".'*

Representations

Arranging tiles into arrays to identify factors:





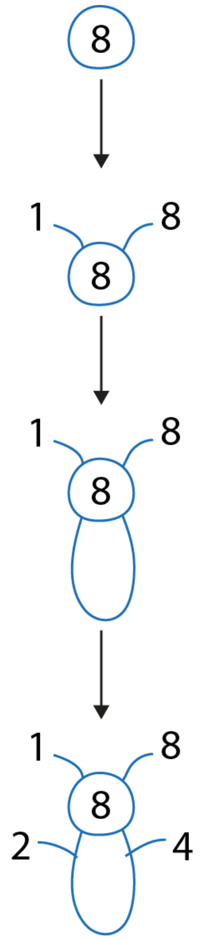
- *'Factors of "6" are "1", "2", "3" and "6".'*

Recording factors:

Number	Factors
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
14	1, 2, 7, 14

- *'What's the same?'*
  - *'All numbers have "1" and themselves as factors.'*
  - *'"6", "8" and "14" have "2" as a factor.'*
  - *'"7" and "14" have "7" as a factor.'*

		• *'What's different?'*
		  • *'Some factors only appear in one list.'*
		  • *'"7" has only two factors.'*
		  • *'"6", "8" and "14" have four factors.'*

2:2	It is important that children work systematically to ensure that they find all factors of a number. Children should also understand that factors come in pairs. This can be linked back to arrays, where both the width *and* the length are factors. One way to find and record factors is using factor bugs. These provide a structured visual method to record factors in pairs and to find all possible factors of a number. To begin, write a number and draw a circle around it. This will be the bug's head. Then draw two antennae and next to these write '1' and the number itself. These are always the first two factors to identify. Children should be confident in identifying these two factors from the learning covered in step *1:3*. They can use the visual prompt of the two antennae to remind them of this step. Now encourage children to work systematically to find all factors, adding 'legs' for each pair of factors. Ask children '*What number could I try next to make sure I find all factors?*' Encourage them to think of a logical approach, leading them to identify that '2' should be the next factor to check. Discuss what we know about the two times table, drawing out the conclusion that '2' will always be a factor if the number is even. Ensure that if '2' is identified, its factor pair is also identified. Continue to use the factor bug, adding 'legs' on opposite sides of the bug. After considering the factors '1' and '2', ask children whether '3' is a factor and discuss the reasons why or why not. For the example opposite, you could	Using factor bugs to find factors: 

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<td>discuss with the class whether ‘3’ is a factor of ‘8’, drawing out the response that ‘8’ is not in the three times table and therefore ‘3’ is not a factor of ‘8’. You could use skip counting to confirm this. You could also return to the earlier use of arrays and ask children to check whether an array can be made with three counters in each row.

Refer to the divisibility rules in segment 2.9 Times tables: 7 and patterns within/across times tables, Teaching point 4 to show children that you can use the rules to quickly identify factors.

Children should continue to identify whether the next number is a factor. For the example on the previous page, the next number to try will be ‘4’. Children should be able to identify that they have already written down ‘4’ as part of an earlier factor pair (‘2’ and ‘4’).</td>
<td></td>
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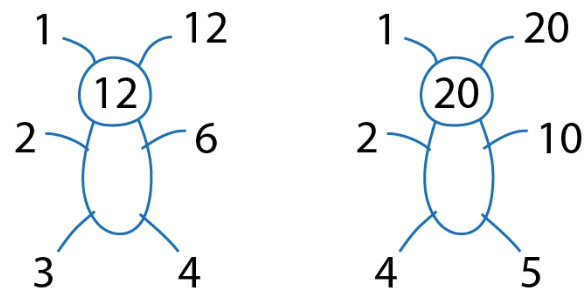
<tr>
<td>2:3</td>
<td>Now ask children ‘Do we need to check the next number?’

For this step, children will need to understand the link between factors and division, seen in step 1:6. Once we reach a point where the quotient is smaller than the divisor, we know we can stop checking factors. Use the question ‘Is the quotient smaller than the divisor?’ For the example in step 2:2:
• 8 ÷ 1 = 8 (carry on)
• 8 ÷ 2 = 4 (carry on)
• 8 ÷ 4 = 2 (the quotient is smaller; stop here)

Children should also make the connection that when we find a pair of factors we have already found, we can stop checking for more factors. In this example we have already found ‘2’ and ‘4’, so we can stop.

Provide children with the opportunity to find the factors of many numbers (begin with numbers that have more</td>
<td>Using factor bugs to find factors:

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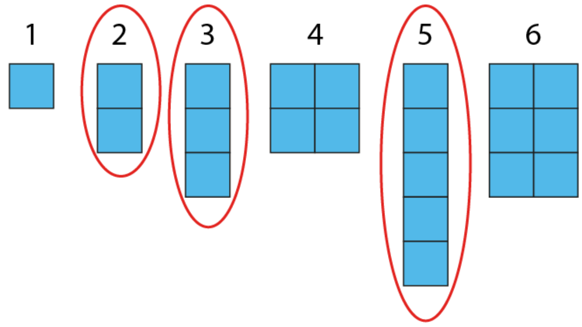
	than one pair of factors) and use the factor bugs as a way to record their working. Encourage children to consider the question *'Do we need to check the next number?'* as they work. Note that some of the numbers may be square numbers. Although we are adding factors in pairs, for square numbers one of the pairs will be a repeated factor (both 'legs' will be the same). This will be addressed on the next page in step *2:5*.	

2:4	Provide opportunities for children to practise finding factor pairs for a variety of numbers, linking to their knowledge of times tables and division. Use examples such as the ones below, making sure children talk through their solutions. They can continue to use the factor bugs to support their working out if needed.
	
	Identifying factor pairs:
	
	'Which of these numbers are factors of "12"? Identify each factor pair.'
	
	1 2 3 4 5 6 7 8 9 10 11 12
	
	'Now answer these questions:'
	• *'Is "12" in the one times table?'*
	• *'Is "12" in the two times table?'*
	• *'Is "12" in the three times table?'*
	• *'Is "12" in the four times table?'*
	• *'Is "12" in the five times table?'*
	• *'Is "12" in the six times table?'*
	• *'Why are there no factors between "6" and "12"?'*
	
	Ensure children know when to stop checking for factors, referring back to step *2:3* if needed.
	
	Using division to find factor pairs:
	• *'Using the following information, list the factors of "12".'*
	  • $12 \div 1 = 12$
	  • $12 \div 2 = 6$
	  • $12 \div 3 = 4$	

	• *'Do we need to write these equations as well? Explain your answer.'* • 12 ÷ 4 = 3 • 12 ÷ 6 = 2 • 12 ÷ 12 =1 Some children will notice that there are numbers that only have two factors. Explain that we will look at these later, as they have a special name.	
2:5	In this step we make a link between square numbers and factors. Square numbers were covered in detail in segment *2.9 Times tables: 7 and patterns within/across times tables, Teaching point 3*, and were recapped earlier (step *1:4*) using arrays for visualisation. We can identify square numbers when listing factors, because square numbers always have an odd number of factors. This is because one of the factors is multiplied by itself. Use examples such as the ones opposite to explore this.	• *'Identify the factor pairs for the number "16". What do you notice about the number of factors?'* 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 • *'The factor pairs are "1" and "16", "2" and "8", "4" and "4".'* • *'As "4" appears twice in the last pair we only use it once when listing the factors, so the factors for 16 are "1"," 16", "2", "8" and "4".'* • *'"16" has five factors.'* • *'Complete the table to identify factors and square numbers.'*

Number	Factors	Odd or even number of factors?	Square number?
1	1	Odd	✓
2	1, 2	Even	✗
3	1, 3	Even	✗
4	1, 2, 4	Odd	✓
…			

2:6	Finish this teaching point by introducing the term *'composite number'*. Ask children to use a list of factors, such as the table in step *2:5*, to identify any numbers that have more than two factors. You could also use the factor-bug method to identify numbers that have 'legs'. Work towards the generalisation: ***'Numbers that have more than two factors are composite numbers.'***	Dòng nǎo jīn: *'True or false?'* The larger the number, the more factors it has.

<table>
<tr><td colspan="3">Teaching point 3:
Prime numbers are positive integers that have exactly two factors.</td></tr>
<tr><td colspan="3">Steps in learning</td></tr>
<tr><td></td><td>Guidance</td><td>Representations</td></tr>
<tr><td>3:1</td><td>This teaching point introduces the definition of prime numbers. Return to the arrays used in Teaching point 1 and ask children to identify numbers that can only be arranged into one type of rectangle. Explain that these are called ‘prime numbers’. A prime number has exactly two factors. ‘1’ only has one factor and so is not prime.
You could also return to the factor bugs (step 2:2), identifying bugs that have no ‘legs’. Ask questions to deepen understanding, such as:
‘Using two factors, how many ways can you write an equation where “5” is the product?’
1 × 5 = 5 and 5 × 1 = 5
• ‘“5” has only two factors and is therefore a prime number.’
Work towards the following generalised statement: ‘Numbers that have exactly two factors are called “prime numbers”.’</td><td>Identifying prime numbers using arrays:
</td></tr>
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3:2

To make links between prime numbers and previous learning on times tables, show children the complete multiplication chart from step *1:5* and ask them whether it can be used for finding prime numbers. Draw out the point that:

- All numbers (prime or not) have '1' and themselves as factors, so the one times table is not useful for identifying prime numbers.
- All other squares in the multiplication chart contain numbers with at least three factors, so prime numbers will not appear.

'Can we use the multiplication chart to find prime numbers?'

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

3:3 To complete this teaching point provide children with varied practice including:

- Completing a table with information, such as the example shown below.
- 'Think of a number' games. For example, in pairs, one child thinks of a number and the other child asks questions to determine what the number is:
  - *'Does your number have more than two factors?'*
  - *'Is it greater than eleven?'*
  - *'Is it less than twenty?'*

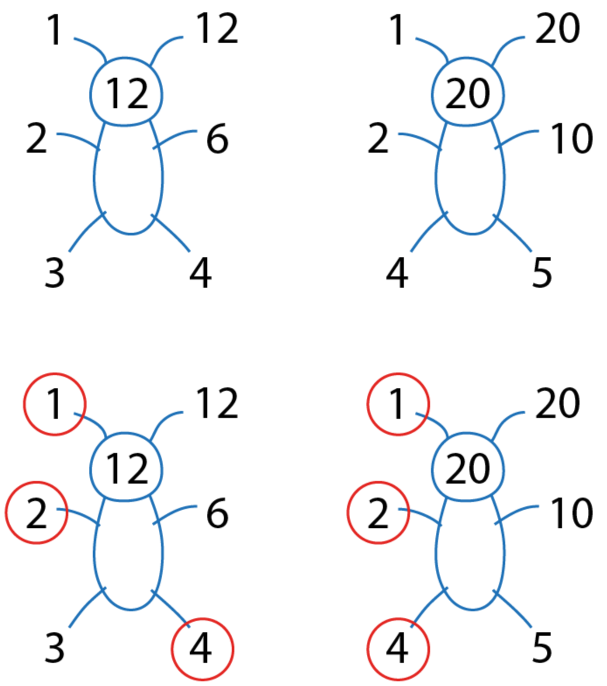
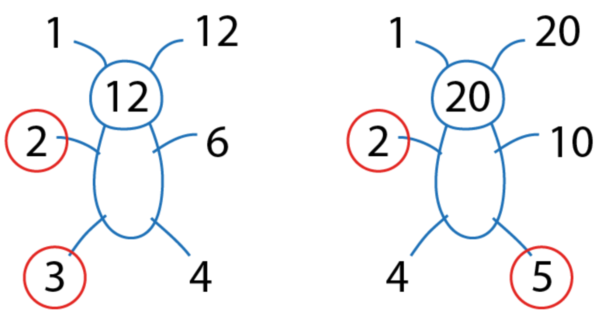
'Complete the table by ticking whether each number is a prime, square or composite number. Some numbers will have more than one column ticked.'

Number	Factors	Numbers of factors	Prime	Square	Composite
1	1	1		✓	
2	1, 2	2	✓		
3	1, 3	2	✓		
4	1, 2, 4	3		✓	✓
5	1, 5	2			
6	1, 2, 3, 6	4			
7	1, 7	2			
8	1, 2, 4, 8	4			
9	1, 3, 9	3			
10	1, 2, 5, 10	4			
11	1, 11	2			
12	1, 2, 3, 4, 6, 12	6			
13	1, 13	2			
14	1, 2, 7, 14	4			
15	1, 3, 5, 15	4			

Teaching point 4:

A common factor is a factor that is shared by two or more numbers. A prime factor is a factor that is also a prime number.

Steps in learning

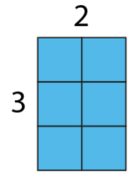
	Guidance	Representations
4:1	In this teaching point we will define common factors. To begin, children will need to find all factors of two numbers, and then compare them to see that they may share factors. Provide children with two numbers that have several common factors, for example '12' and '20'. Using factor bugs or lists, encourage children to work systematically to identify all of the factors of each number. Then ask: *'What do you notice about the factors of "12" and "20"?'* • Both numbers have an even number of factors. • Both numbers have the same number of factors. • '1', '2' and '4' are factors of both numbers. Explain that numbers that are factors of two numbers are called *'common factors'*. So the common factors of '12' and '20' are '1', '2' and '4'. Repeat this step using other numbers that have several common factors.	Finding common factors: *'What are the common factors of "12" and "20"?'*  • *'The common factors of "12" and "20" are "1", "2" and "4".'*
4:2	Return to the example of '12' and '20', and ask children if they can see any prime numbers in the factors. Explain that factors that are prime numbers are called *'prime factors'*. • '2' and '3' are prime factors of '12'. • '2' and '5' are prime factors of '20'. Repeat this step using other numbers that have prime factors.	Finding prime factors: 

4:3	Provide children with varied practice working with common and prime factors.	Reasoning problem: *'Is "1" always, sometimes or never a common factor of two numbers?'* Dòng nǎo jīn: *'The common factors of two numbers are "1", "3", "5" and "15". What could the two numbers be?'* • *'Alicia says that there are more than two numbers whose common factors are "1", "3", "5" and "15". Is she correct?'*

Teaching point 5:

A multiple of a number is the product of that number and an integer; a common multiple is a multiple that is shared by two or more numbers.

Steps in learning

	Guidance	Representations
5:1	Return to the equation from step *1:1*: *factor* × *factor* = *product* Explain that any number that has factors is called a multiple. For example, ‘6’ is a multiple of ‘2’ because ‘2’ multiplied by ‘3’ is ‘6’. Ask children to define factors and multiples using the following stem sentences: • **‘___ is a factor of ___ because ___ × ___ = ___.’** • **‘___ is a multiple of ___ because ___ × ___ = ___.’** • **‘___ is a factor of ___ because ___ ÷ ___ = ___.’** • **‘___ is a multiple of ___ because ___ ÷ ___ = ___.’**	Factors and multiples:  • *‘“2” is a factor of “6” because* $2 \times 3 = 6$*’* • *‘“6” is a multiple of “2” because* $2 \times 3 = 6$*’* • *‘“2” is a factor of “6” because* $6 \div 3 = 2$*’* • *‘“6” is a multiple of “2” because* $6 \div 3 = 2$*’*
5:2	Next link multiples to known times table facts. Ask children to find multiples of a number by skip counting, and record their answers in a ratio chart such as the one on the next page. Then ask what they notice about the chart. Children should identify that the multiples form a times table. Therefore, we can look for known times table facts when checking multiples.	

<table>
<tr>
<td></td>
<td></td>
<td>
Linking multiples to times tables using a ratio chart:
'There are seven players in a netball team. Look at the following ratio chart.'
<table>
<tr><th>Number of netball teams</th><th>Total number of players</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>7</td></tr>
<tr><td>2</td><td>14</td></tr>
<tr><td>3</td><td>21</td></tr>
<tr><td>4</td><td>28</td></tr>
<tr><td>5</td><td>35</td></tr>
<tr><td>6</td><td>42</td></tr>
<tr><td>7</td><td>49</td></tr>
<tr><td>8</td><td>56</td></tr>
<tr><td>9</td><td>63</td></tr>
<tr><td>10</td><td>70</td></tr>
<tr><td>11</td><td>77</td></tr>
<tr><td>12</td><td>84</td></tr>
</table>
'What do you notice about the total number of players?'

'They are all in the "7" times table.'
'They are all multiples of "7".'

</td>
</tr>
<tr>
<td>5:3</td>
<td>
Provide children with practice working with factors and multiples using skip counting, times table facts and divisibility rules to find the multiples of given numbers. You could use problems such as those given below, opposite and on the next page.

'Is "36" a multiple of "4"? Explain your answer.'
'Is "15" a multiple of "7"? Explain your answer.'
'Is "130" a multiple of "10"? Is it also a multiple of "5"? Explain your answer.'

</td>
<td>

'What is the next multiple of "8"? Explain your answer.'

16 24 32 ☐
</td>
</tr>
</table>

- *'Use a hundred square to help you solve the following problem:'*
  - *'I'm thinking of a whole number less than "100".'*
  - *'It is a multiple of "6".'*
  - *'"5" is a factor of this number.'*
  - *'It is also a multiple of "12".'*
  - *'What number am I thinking of?'*

Dòng nǎo jīn:

'Working in pairs, write instructions for your partner to work out a number that you are thinking of between "1" and "100". Use the words "factor" and "multiple" in your instructions.'

- *'Some multiples have been highlighted in this hundred square. Which number is a factor of all of these multiples? Is there more than one number that is a factor of all of these multiples?'*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5:4

In step *4:1*, children learnt that numbers that are factors of two numbers are called common factors. Now ask children *'What do you think a common multiple is?'* Draw out the answer that a common multiple is a multiple that is shared by two numbers.

Using a hundred square, ask children to highlight all of the multiples of '3' in one colour, and all of the multiples of '5' in another colour. Ask if there are any squares that have both colours. Explain that these are the common multiples, as they are multiples of both '3' and '5'.

Use practice questions to build on children's knowledge:

- *'Which of these are common multiples of "2" and "6"?'*

  6 9 12 20 22 24 30

- *'What are the first five common multiples of "3" and "8"?'*

- *'Use a hundred square to find the common multiples of "3" and "5".'*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5:5

Use the following 'factor and multiple chain' activity to consolidate children's understanding of factors and multiples. Provide all children with a hundred square. Children can work either collaboratively or competitively.

Choose a number between '1' and '100'. Children must find a number that is either a multiple or a factor of this number. They then find a multiple or factor of the new number, to create a chain of numbers. However, numbers may not be repeated; each link in the chain must be unique.

If playing collaboratively, children must try and make the chain as long as possible before they run out of unique numbers. If playing competitively, children take it in turns to add numbers to the chain and aim to make it impossible for their partner to continue without repeating a number.

Encourage children to explain their choices using the appropriate mathematical language: '*"12" can come after "2" in the chain because "12" is a multiple of "2".*'

Making a chain of factors and multiples:

1	~~2~~	3	4	~~5~~	~~6~~	7	8	9	~~10~~
11	~~12~~	13	14	15	16	17	~~18~~	19	20
21	22	23	24	~~25~~	26	27	28	29	30
31	32	33	34	35	~~36~~	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

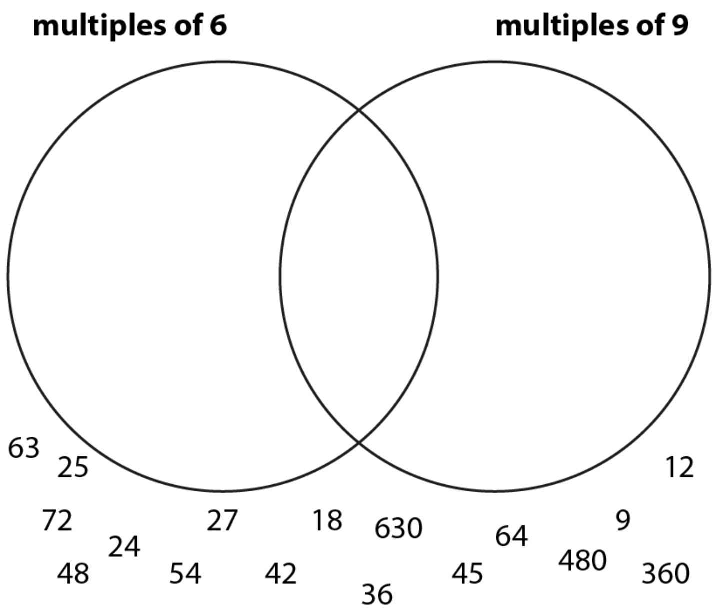
- Number chain: 25, 5, 10, 2, 12, 6, 18, 36…

5:6

To finish this teaching point provide children with practice, including asking them to sort numbers according to their properties using Venn diagrams or identify errors using Carroll diagrams.

Venn diagram:

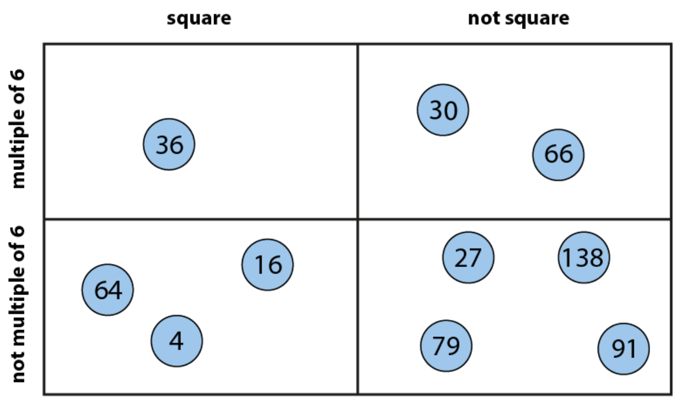
- *'Sort the numbers into multiples of "6", multiples of "9", both or neither.'*



- *'What do you notice about numbers in the intersection? Are they odd or even?'*

Carroll diagram:

'The numbers are sorted so that they are in the sections with the correct labels. One number is in the wrong place. Which number is it and why?'



Dòng nǎo jīn:

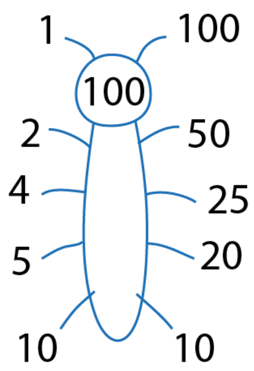
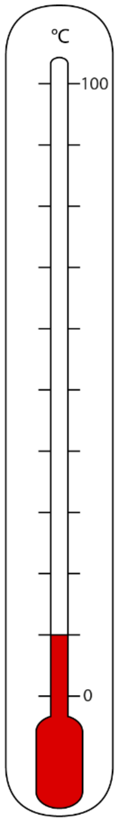
'True or False?'

All even multiples of 9 are also multiples of 6.

Teaching point 6:

The factor pairs of '100' can be used to support efficient calculation.

Steps in learning

	Guidance	**Representations**
6:1	In this teaching point children focus on using the factor pairs of '100' to help with calculations. This will build on their knowledge of multiplying by three factors, covered in segment *2.20 Multiplication with three factors and volume*. To begin, ask children to list the factor pairs of '100'. Ask them what the factor pairs show us about the number '100': • *'Is it a composite number?'* (Yes) • *'Is it a square number?'* (Yes) • *'Is it a prime number?'* (No) Ensure children can explain the reasons for their answers. Next consider how this information can be used in a real-life context, using thermometers as shown opposite and on the next page.	• *'List the factor pairs of "100".'*  • *'What is the temperature shown on this thermometer?'*  • *'There are "10" intervals on the thermometer. We know that "10" and "10" are factor pairs of "100", so there are "10" intervals of "10". The temperature is 10˚C.'*

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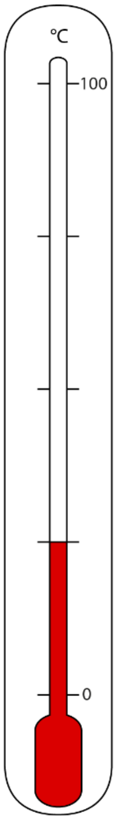
		• *'What is the temperature shown on this thermometer?'*  • *'There are "4" intervals on the thermometer. We know that "4" and "25" are factor pairs of "100", so there are "4" intervals of "25". The temperature is 25°C.'*
6:2	Next look at how using factor pairs of '100' can be used to simplify other expressions. • Start with the following equation: $4 \times 25 = 100$ • Ask children how you could use factor pairs to solve: 12×25 • Explain that because we know *$3 \times 4 = 12$*, we can write the problem as: $3 \times 4 \times 25$	Using factor pairs of '100' to solve equations: *'Fill in the missing numbers.'* $5 \times 20 = 100$ $15 \times 20 = 3 \times \square \times 20 = \square$ $25 \times 20 = \square \times \square \times 20 = \square$ $30 \times 20 = \square \times 5 \times 20 = \square$ $35 \times 20 = \square \times \square \times \square = \square$

	• Using the associative law (see segment *2.20 Multiplication with three factors and volume*) you can now rearrange the problem: $3 \times 4 \times 25 = 3 \times 100 = 300$ so $12 \times 25 = 300$ Work through another example that uses factor pairs of '100', for example: $4 \times 25 = 100$ $8 \times 25 = 2 \times 4 \times 25 = 2 \times 100 = 200$ Then ask children to practise using factor pairs to solve equations, using missing number problems such as those shown on the previous page.	
6:3	Finish this teaching point by ensuring that children can apply what they have learnt in a variety of contexts, using problems such as those shown opposite.	True/false style problems: • *'True or false? Tick or cross each answer.'* $20 \times 3 = 2 \times 10 \times 3$ $30 \times 3 = 3 \times 10 \times 2$ $3 \times 40 = 10 \times 4 \times 3$ $5 \times 75 = 5 \times 3 \times 25$ $5 \times 75 = 5 \times 7 \times 5$ • *'Mrs L. puts apples into bags to sell. She has 630 apples. She uses all of the apples, and all bags have the same number of apples. Which statements could be true?'* She can make 70 bags. She can make 80 bags. She can make 90 bags. She can make bags with 5 in each bag. She can make bags with 6 in each bag. She can make bags with 7 in each bag.