MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.21 Algorithms: column subtraction

Teacher guide | Year 3

Teaching point 1:

One number can be subtracted from another using an algorithm called '*column subtraction*'; the digits of the minuend and subtrahend must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left).

Teaching point 2:

If there is an insufficient number of any unit to subtract from in a given column, we must exchange from the column to the left.

Summer 2018 pilot

Overview of learning

In this segment children will:

- learn how to represent and apply the column-subtraction algorithm
- apply column subtraction to solve a range of questions with contexts representing different subtraction structures including partitioning, reduction ('taking away') and difference
- identify when exchange is required and how to represent and apply this.

Children should already have mastered column addition (see segment *1.20 Algorithms: column addition)*, so this teaching sequence assumes they have secured the following key principles:

- laying out a column calculation including aligning the digits correctly
- starting with the least significant digit (the right-hand side).

This segment builds on previous work on the concepts of place value, subtraction using horizontal written, as well as mental, strategies, and partitioning. For children to efficiently use the column method, subtraction of single digits, and subtraction facts which bridge ten, should have been mastered using appropriate strategies (see segments *1.7 Addition and subtraction: strategies within 10* and *1.11 Addition and subtraction: bridging 10*). Note that it is important for teachers to refer to the minus symbol as 'subtract' or 'minus' throughout (not 'take away') to allow the different structures (partitioning, reduction ('take away') and difference) to be understood as a result of the contexts.

The segment introduces the idea that we can apply column subtraction to any subtraction calculation. However, as the segment progresses, teaching can begin to discuss when this method is an efficient choice, and when other methods may be more suitable, based on the numbers involved. Estimation and reference to the inverse of subtraction should be included throughout as strategies to check whether answers are correct/reasonable.

Dienes will be used to support children as they learn to use the column-subtraction algorithm. However, as discussed in segment *1.20*, Dienes emphasise a different understanding of place value compared to column methods; for example, a Dienes representation of 43 shows 40 ones and three ones (40 and 3), while a column representation (for example, a place-value chart or column algorithm) of 43 shows four tens, indicated by a '4' in the tens column, and three ones. Continue to refer to whole Dienes tens rods with the language of unitising (as exemplified throughout this segment) to help children move towards the purely column understanding of place value, which underpins the column algorithms. Note that, throughout, the Dienes should only be used to support children's understanding of the *structure* of the algorithm and should not be used as a tool for finding the answer; children should be encouraged to use known facts to perform the calculation in each column.

Once children have mastered applying the column-subtraction algorithm without exchange (each digit of the subtrahend is smaller or equal to the corresponding digit of the minuend; *Teaching point 1*), they will build on their understanding of the equivalence between ten ones and one ten, and between ten tens and one hundred, to identify when exchange is necessary, and how it is recorded (*Teaching point 2*).

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

One number can be subtracted from another using an algorithm called '*column subtraction*'; the digits of the minuend and subtrahend must be aligned correctly; the algorithm is applied working from the least significant digit (on the right) to the most significant digit (on the left).

Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>1:1</td><td>Before beginning work on column subtraction, it is important to ensure that children have already mastered representing subtraction with horizontal expressions and that they are confident in using a range of mental strategies for subtraction (see segment 1.18 Securing mental strategies: calculation up to 999).

Review, and encourage children to use the generalised statement:

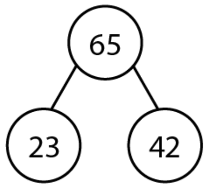
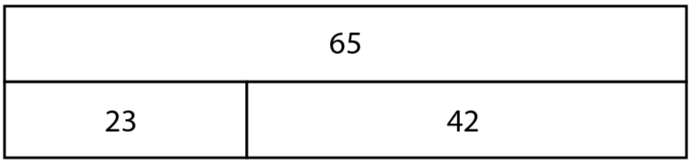
• 'Minuend minus subtrahend is equal to the difference.'

Make sure that children can identify the minuend and subtrahend in a range of horizontally written calculations.</td><td>65 – 23 = 65 – 20 – 3
= 45 – 3
= 42

65 – 23 = 42</td></tr>
<tr><td>1:2</td><td>Beginning with subtraction of one two-digit number from another, introduce the column subtraction algorithm. Show a completed calculation and ensure that children can identify the minuend, subtrahend and difference. Throughout this teaching point use problems:

• with only one subtrahend
• for which there is no need to exchange.

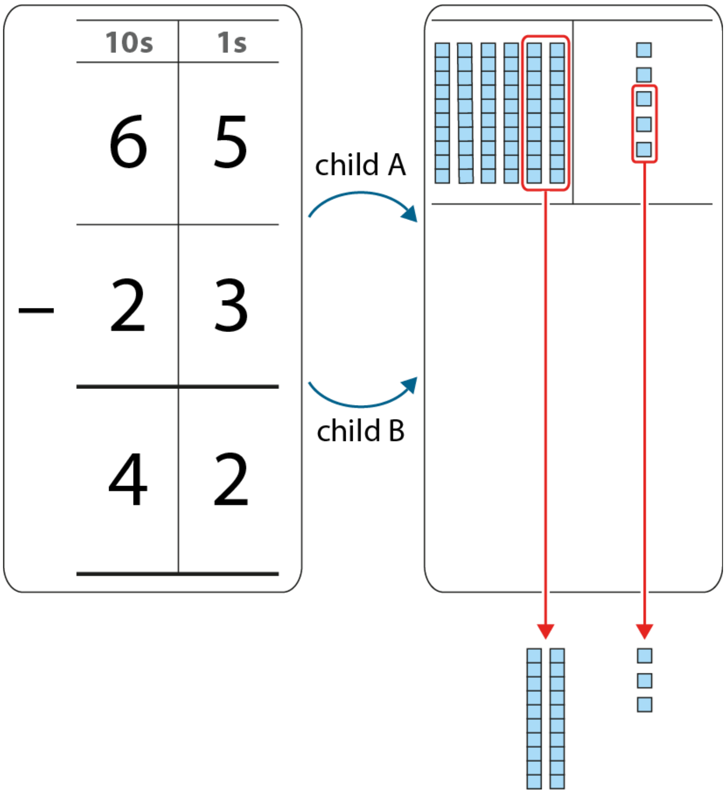
Use a range of familiar representations of a given calculation alongside the column-subtraction layout so that children see the relationship between the numbers, including:

• part–part–wholes (cherry or bar-model)
• Dienes.</td><td>Part–part–wholes:


Column subtraction:
$\begin{array}{rr} & 6\;5 \\ - & 2\;3 \\ \hline & 4\;2 \\ \hline \end{array}$</td></tr>
</table>

	To facilitate comparison of the representations, ask children: • *'What's the same?'* • *'What's different?'* • *'Which number is the minuend?'* (Draw attention to the fact that the minuend is always the number at the top of the algorithm.) • *'Which number is the subtrahend?'* • *'Which number is the difference?'* (To help children identify the difference in column subtraction, draw attention to the 'large equals symbol' that frames the difference.) Repeat the comparison of representations for a range two-digit subtraction problems. Then follow the same process for calculations involving varying digit sizes (e.g. *342 – 131*, *574 – 62*).	
1:3	Due to children's familiarity with the column addition calculation, in this step we combine correct alignment of digits with calculation of the answer; however, these steps could be taught separately as described for column addition in segment *1.20 Algorithms: column addition.* Remind children that, in column addition, we: • add like values: tens added to tens, ones added to ones (for more on alignment of digits, see segment *1.20*) • work from the least-significant digit (the right-hand side). Beginning with subtraction of one two-digit number from another, demonstrate, using Dienes, that we do the same for subtraction. Emphasise that we only make the *minuend* with the manipulatives, and then subtract the subtrahend from this. This is a significant difference to using	

	manipulatives during column addition; in that case both/all addends need to be made.
	Remember to use the language of unitising to help children make the link between the cardinality of the Dienes representation and the column representation of place-value:
	• For Dienes:
	  • ***'___ one(s) minus ___ one(s) is equal to ___ ones.'***
	  • ***'___ ten(s) minus ___ ten(s) is equal to ___ tens'.***
	• For the column addition calculation:
	  • ***'The ones column represents ___ one(s) minus ___ one(s) is equal to ___ ones.'***
	  • ***'The tens column represents ___ ten(s) minus ___ ten(s) is equal to ___ tens.'***
	With the Dienes representation, keep the subtrahend visible after it has been 'taken away' so that you can make a clear link to the inverse operation, demonstrating how this can be used to check the answer:
	$65 - 23 = 42$
	$23 + 42 = 65$	

1:4	After working through several examples as a class, ask children to work in pairs, moving between the concrete and abstract representations, laying out the calculation correctly and performing the calculation. Encourage them to continue using the language introduced in step *1:3* to describe the process. Then, when children have a secure understanding of how the algorithm works, remove the concrete apparatus. This should be done over a relatively short period of time, since the manipulatives only act as an aid to	

	laying out the calculation correctly and a link to earlier additive work; moving children to only the abstract representation early helps to avoid them using the manipulatives as a tool for calculation. Place-value headings have been included in the column algorithm opposite, and can be useful while children are learning the method; eventually, however, remove this scaffold and provide sufficient practice in the absence of column headings.	Child B writes out the calculation and says: • *'The ones column represents five ones minus three ones is equal to two ones.'* • *'The tens column represents six tens minus two tens is equal to four tens.'* Child A arranges the Dienes and says: • *'Five ones minus three ones is equal to two ones.'* • *'Six tens minus two tens is equal to four tens.'* 
1:5	Repeat steps *1:3* to *1:4* for calculations: • with three-digit minuends and subtrahends (e.g. *342 – 131*) • with three-digit minuends and two-digit subtrahends (e.g. *574 – 62*); emphasise that nothing is subtracted in the hundreds column • where some of the digits are zero (e.g. *140 – 120, 146 – 20, 146 – 103*). Even though the number of digits increases, the difficulty does not, providing that children understand the process that they are following.	

1:6

To complete this teaching point, present varied practice for column subtraction without exchange, including:

- completing column subtraction calculations, including different combinations of numbers as listed in step *1:5*
- laying out a given calculation as column subtraction
- real-life problems, including measures contexts, for example:
  - *'There are 76 books in Year 3's book corner. 32 are fiction, and the rest are non-fiction. How many are non-fiction?'* (partitioning)
  - *'Ali has £146. He spends £34 on a new skateboard. How much money does Ali have left?'* (reduction)
  - *'Zayn jumped 256 cm in the long jump and Joshua jumped 278 cm. How much further did Joshua jump?'* (difference)

Initially, encourage children to draw part–part–whole diagrams to facilitate interpretation of the word problems, before expressing them as column subtraction calculations. Part–part–whole diagrams can also be used to facilitate using the inverse addition calculation for answer checking; also encourage children to use estimation for answer checking, when appropriate.

To promote depth of understanding present a dòng nǎo jīn problem such as that shown opposite. For each missing number ask:

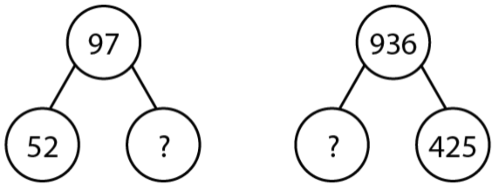
- *'What could the missing number be?'*
- *'What could it not be?'*
- *'How do you know?'*

'Complete the calculations.'

$$\begin{array}{r} 57 \\ -\ 32 \\ \hline \\ \hline \end{array} \quad \begin{array}{r} 462 \\ -\ 251 \\ \hline \\ \hline \end{array} \quad \begin{array}{r} 375 \\ -\ 42 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 87 \\ -\ 24 \\ \hline \\ \hline \end{array} \quad \begin{array}{r} 436 \\ -\ 204 \\ \hline \\ \hline \end{array} \quad \begin{array}{r} 395 \\ -\ 40 \\ \hline \\ \hline \end{array}$$

'Write these as column subtraction calculations.'



83 – 32

635 – 24

441 – 210

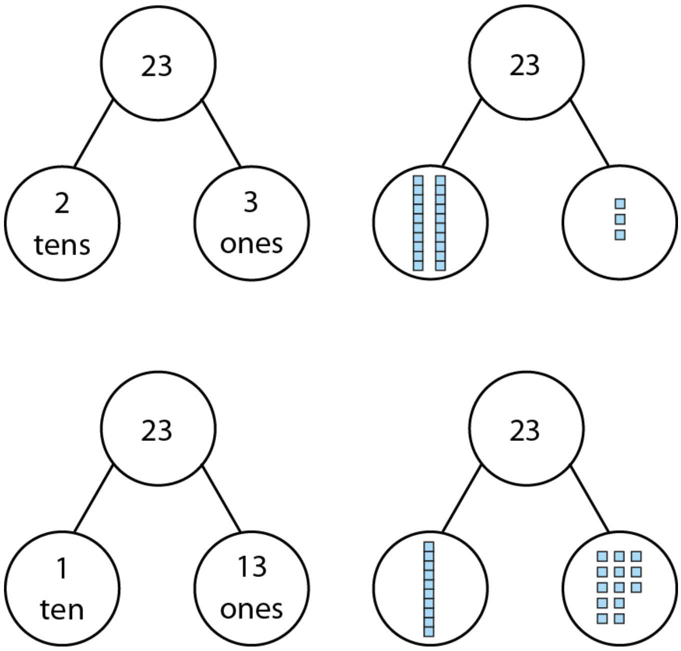
Dòng nǎo jīn:

$$\begin{array}{r} 3\ \square\ 5 \\ -\ \square\ 3\ \square \\ \hline 2\ 3\ 4 \\ \hline \end{array}$$

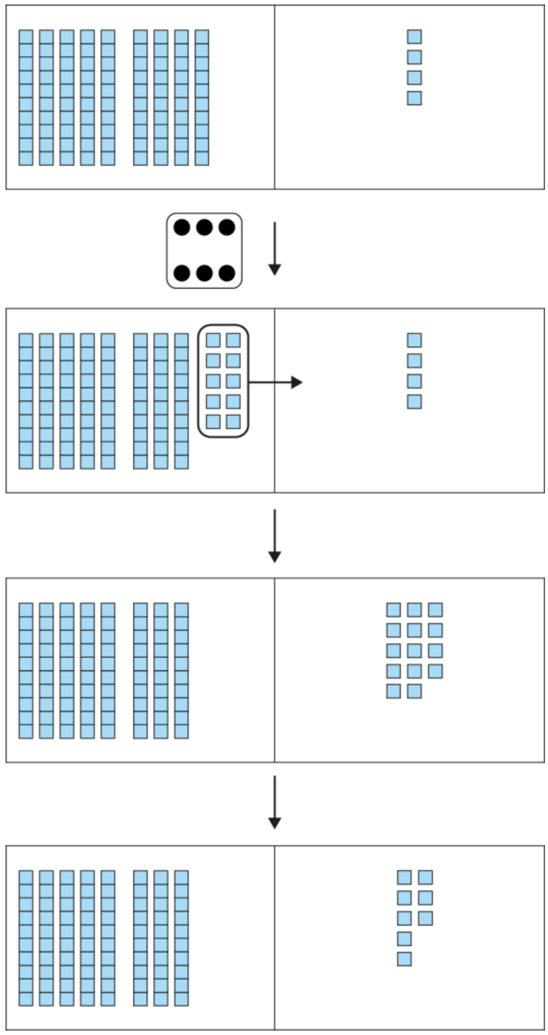
Teaching point 2:

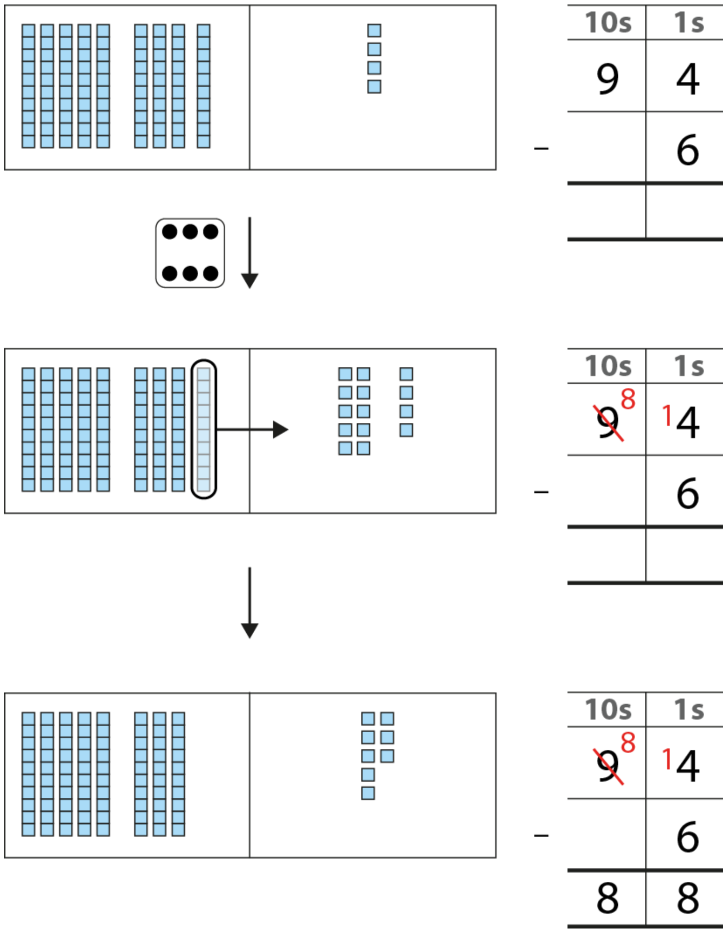
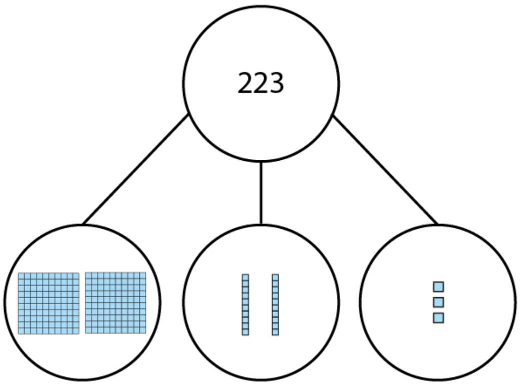
If there is an insufficient number of any unit to subtract from in a given column, we must exchange from the column to the left.

Steps in learning

	Guidance	Representations
2:1	Begin by reviewing the partitioning of two-digit numbers into tens and ones, including exchanging one of the tens for ten ones (see segment *1.14 Addition and subtraction: two-digit numbers and multiples of ten*, *Teaching point 4* for different ways of partitioning two-digit numbers). Children should understand, for example, that 23 is equivalent to two tens and three ones as well as one ten and thirteen ones. Use Dienes and part–part–whole diagrams as shown opposite, continuing to use the language of unitising.	

2:2	You could introduce the idea of exchange in column subtraction using the following game:
	• Provide each child with:
	    • a set of 99 in Dienes (nine tens and nine ones) on a chart (without place-value headings)
	    • spare ones Dienes
	• Provide each pair of children with a six-sided- or nine-sided-die.
	• Working in pairs, child A rolls the die and then subtracts the result from his/her own Dienes. Child B checks the calculation has been performed correctly.
	• Child B then takes a turn rolling the die and subtracting the value from his/her Dienes.	

	• Initially children will subtract from the ones; for example, Megan rolls a five on her first turn (*99 – 5 = 94*). However, they will eventually need to exchange a ten for ten ones in order to subtract the correct amount; for example, Megan then rolls a six on her second turn, as shown opposite (*94 – 6 = 88*). After exchanging one Dienes ten rod with ten ones, encourage children to move the ten ones over to the column with the rest of the ones. • The children continue taking turns until one child runs out of Dienes; the child who is the last left with Dienes on their chart wins.	Example turn: 94 – 6 
2:3	Now introduce the abstract representation alongside the practical or pictorial. Demonstrate some example turns of the game described in step *2:2*, working through the corresponding calculations with children. Then children can play the game again using both Dienes *and* the column subtraction representation. The Dienes should be used to help children understand the underlying structure of exchange, rather than as a tool for calculation. Make sure that children can relate the reduction of the tens value with the introduction of an additional '*1*' in the ones column of the algorithm. Children should then	

	perform the subtraction in each column using known facts. After a short period of practice in pairs, remove the Dienes scaffold to ensure that children can work with only the abstract representation; similarly, fairly quickly remove the scaffold of the place-value headings in the column algorithm.	Example turn with Dienes and column subtraction: 94 – 6 
2:4	When children have mastered exchanging tens for ones, progress to calculations with three-digit numbers, exploring exchange of one hundred with ten tens. Begin by reviewing partitioning three-digit numbers into hundreds, tens and ones, and then extend to partitioning a given number in as many different ways as possible to strengthen children's understanding of place value and to facilitate the transition to double exchange, for example: 223 = 200 + 20 + 3 223 = 100 + 120 + 3 223 = 100 + 110 + 13	

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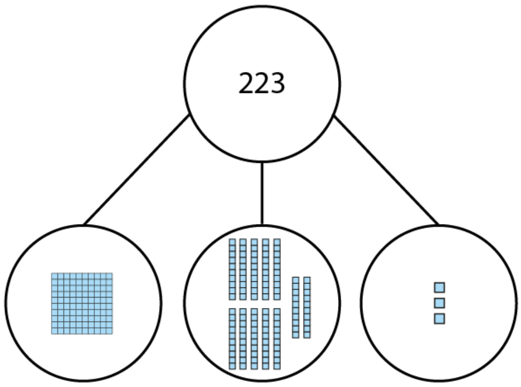
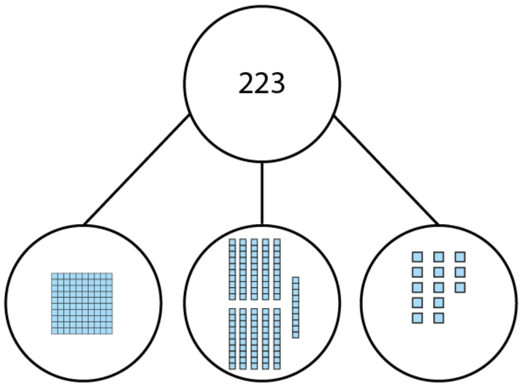
<td>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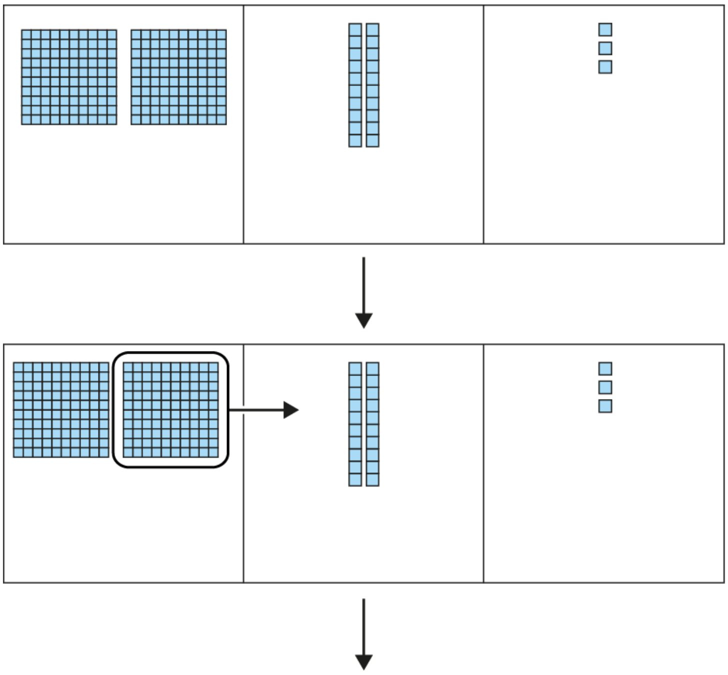
<tr>
<td>2:5</td>
<td>Now use Dienes to represent subtraction calculations that require the exchange of one hundred with ten tens. Encourage children to identify what is the same and what is different compared to exchanging one ten with ten ones.</td>
<td>223 – 142
</td>
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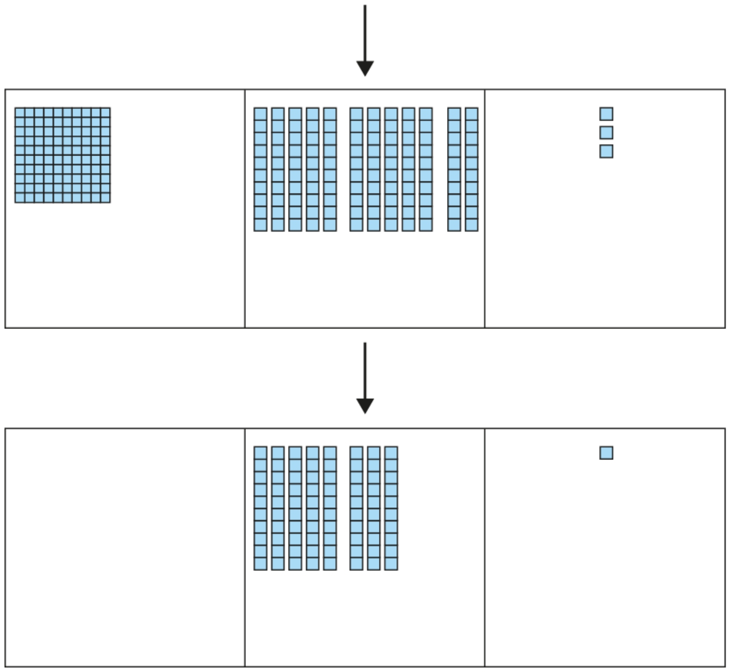
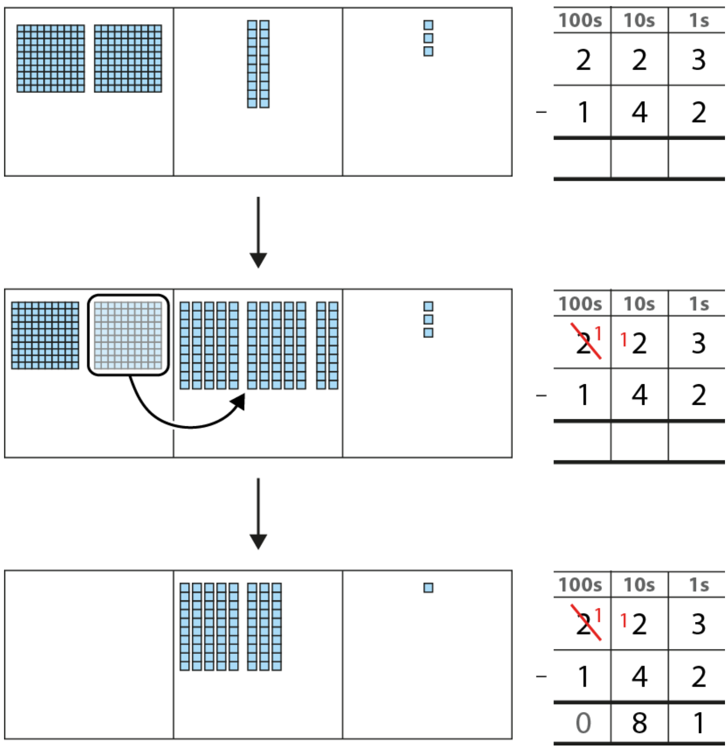
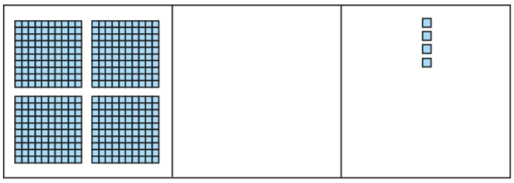
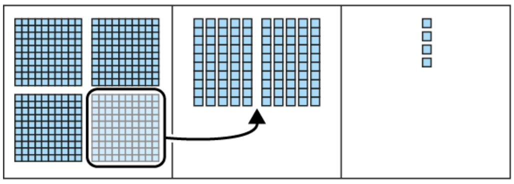
		
2:6	Then, introduce the abstract representation alongside the Dienes. After demonstrating several examples, children can again work in pairs moving between the Dienes and abstract representations, before working just in the abstract (first with place-value headings for the columns, and then without).	

2:7

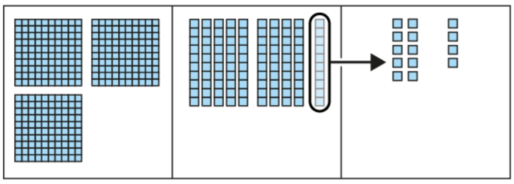
Now repeat steps *2:5* and *2:6* for calculations that require 'working through a zero' (e.g. *404 – 257*).



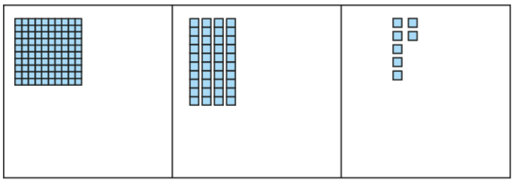
	100s	10s	1s
	4	0	4
–	2	5	7



	100s	10s	1s
	~~4~~ 3	10	4
–	2	5	7



	100s	10s	1s
	~~4~~ 3	~~10~~ 9	14
–	2	5	7



	100s	10s	1s
	~~4~~ 3	~~10~~ 9	14
–	2	5	7
	1	4	7

2:8

To promote depth, present children with a range of calculations and ask questions such as:

- *'Which calculations require exchange?'*
- *'Which calculations require exchange only once?'*
- *'Which calculations require exchange twice?'*
- *'Which calculations require exchange through a zero?'*

Encourage children to look at the numbers involved and justify their answers in that way, rather than actually performing the calculations.

563 – 213	482 – 197	824 – 319
405 – 123	316 – 103	903 – 124

2:9	Finally, present varied practice for column subtraction with exchange, including:
	• completing column subtraction calculations, including:
	  ◦ different combinations of numbers (two-digit and three-digits)
	  ◦ calculations that require exchange of one ten with ten ones
	  ◦ calculations that require exchange of one hundred with ten ones
	  ◦ calculations that require exchange of both hundreds with tens, and tens with ones.
	  ◦ calculations that require exchange 'through a zero'.
	• real-life problems, including measures contexts, for example:
	  ◦ *'There are 83 books in Year 3's book corner. 37 are fiction, and the rest are non-fiction. How many are non-fiction?'* (partitioning)
	  ◦ *'Cheryl has £135. She spends £53 on some new trainers. How much money does Cheryl have left?'* (reduction)
	  ◦ *'650 apples and 385 bananas are delivered to a school each day. How many more apples than bananas are delivered?'* (difference)
	As in step *1:6*, encourage children to:
	• draw part–part–whole diagrams, both to facilitate interpretation of word problems and to identify the inverse addition calculation for answer-checking
	• use estimation for answer-checking, when appropriate.
	To promote depth of understanding present a dòng nǎo jīn problem such as
		'Complete the calculations.'
		$\begin{array}{r} 8\ 3 \\ -\ 2\ 9 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 4\ 6\ 2 \\ -\ 2\ 5\ 6 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 3\ 4\ 5 \\ -\ 7\ 2 \\ \hline \\ \hline \end{array}$
		$\begin{array}{r} 8\ 3 \\ -\ 2\ 9 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 4\ 3\ 6 \\ -\ 2\ 5\ 4 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 3\ 0\ 5 \\ -\ 1\ 3\ 7 \\ \hline \\ \hline \end{array}$
		Dòng nǎo jīn:
		$\begin{array}{r} 5\ \square\ 7 \\ -\ \square\ 8\ \square \\ \hline 3\ 5\ 4 \\ \hline \end{array}$

	that shown opposite. For each missing number ask: • *'What could the missing number be?'* • *'What could it not be?'* • *'How do you know?'*	
2:10	As in *1.20 Algorithms: column addition*, it is worth spending some time thinking about when column methods are *not* necessary, or are *not* the most efficient method. In earlier segments, children developed a range of mental strategies for subtraction, and these should not be abandoned in favour of *always* using column subtraction. For example, to calculate *776 – 200*, it would be more efficient for children to use their understanding of place value to mentally subtract from the hundreds, than to use the column algorithm. Spend some time discussing and sorting calculations according to whether they are best suited to the column method or a mental strategy.	