Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 9

How confident are you that you understand and can support children to select appropriate and efficient strategies, depending on the context?

1 ☑ 2 ☐ 3 ☐ 4 ☐

How would you respond …?

a. Describe the different methods Ezra and Ling have used. Which is the most efficient and why?

Ezra's method:

472×30

$$\begin{array}{r} 4\ 7\ 2 \\ \times \quad\ \ 3 \\ \hline 1\ 4\ 1\ 6 \\ \hline {\scriptstyle 2}\ \ \ \ \end{array}$$

$1{,}416 \times 10 = 14{,}160$

Ling's method:

472×30

$$\begin{array}{r} 4\ 7\ 2 \\ \times \quad 3\ 0 \\ \hline 1\ 4\ 1\ 6\ 0 \\ \hline {\scriptstyle 2}\ \ \ \ \ \ \ \end{array}$$

b. What strategy can be used to solve the question?

Raksha has 2,000 ml of water, which she pours into five glasses. Nic has 800 ml of water, which she pours into four glasses. Who has more water per glass and by how much?

Responses

Note your responses to the questions here before you engage with the rest of this section:

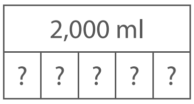
Did you notice that…?

a. Ezra has chosen to remove the zero, use short multiplication and then scale the product by a factor of 10 by replacing the zero. Ling has chosen to use short multiplication to multiply by 30 by placing a zero in the ones column to show that she is multiplying by a multiple of 10 before she starts. Children should notice that the product is the same in each case. Ask '*Whose method do you think is the most efficient?*' A generalisation can be made: *'To multiply by a multiple of 10, use short multiplication to multiply by a single-digit number and then multiply by 10.'*

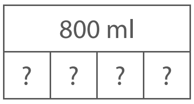
The focus here is on Ezra's method. However, it only applies when one of the factors is 10 or a multiple of 10. Ling's method is more efficient when the multiplication involves non-multiples of 10.

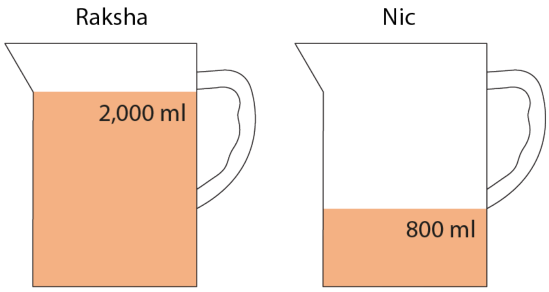
b. To help expose the structure of the problem, children might choose to use a representation to show how each child has poured their water. This might help them to 'see' the calculation that they need to do. Raksha has more by 200 ml.

Rashka $2{,}000 \div 5 = 400$



Nic $800 \div 4 = 200$





Raksha has 200 ml more per glass than Nic.

Selecting efficient methods

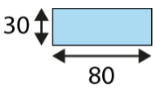
This section will consider efficient methods for multiplication and division. When children learn a written method, such as long multiplication or short division, they have a tendency to rely on this method and not draw upon the range of mental strategies and informal jottings they have used previously. Children need to be encouraged to consider the problem and then decide on the most efficient method rather than relying solely on a formal written strategy for all situations.

As children develop a range of strategies, they should be given opportunities to discuss these, along with strategies learnt previously, considering their efficiency and drawing upon knowledge of the place value system.

When children are multiplying by multiples of 10, 100 or 1,000, they can use their understanding of unitising and the base ten place value system.

Eggs come in boxes of thirty. A supermarket orders 80 boxes of eggs in one week.
How many eggs does the farmer need to supply?

80 boxes of 30 eggs:



$$30 \times 80 = 3 \times 8 \times 10 \times 10$$
$$= 3 \times 8 \times 100$$
$$= 2{,}400$$

$30 \times 80 = 2{,}400$ $80 \times 30 = 2{,}400$ *'The farmer needs to supply 2,400 eggs.'*

The understanding of adjusting the product can be applied to questions where only one factor is a multiple of 10.

A model rocket is 218mm tall. If the real rocket is three hundred times the size of the model, how tall is it in metres?

In this instance, it could be calculated using long multiplication. However, it would be more efficient to draw upon place value understanding: 218 is being multiplied by 3 hundreds. When multiplying two numbers where one number is a multiple of 10, 100 or 1,000, children could use short multiplication and adjust the product using place value.

With the question above, children might rely on mental calculation, for example doubling 218 and then adding another 218 to make it three times, before adjusting the product using place value.

By providing children with lots of opportunities to discuss calculation strategies, they will not only consolidate understanding of the different strategies they have learnt but reason as to why they may or may not be the most appropriate to use. The following questions can be used for discussion.

Water bottles come in boxes of fifteen. A school has bought 324 boxes. How many water bottles does the school have?

Oliver's method

324 boxes of 15 bottles

$$\begin{array}{rcccccl} & & \cancel{1} & \cancel{2} & & & \\ & & 3 & 2 & 4 & & \\ \times & & & 1 & 5 & & \\ \hline & 1 & 6 & 2 & 0 & & 324\times 5 \\ & 3 & 2 & 4 & 0 & & 324\times 10 \\ \hline & 4 & 8 & 6 & 0 & & \\ \hline \end{array}$$

Tanima's method

324 boxes of 15 bottles

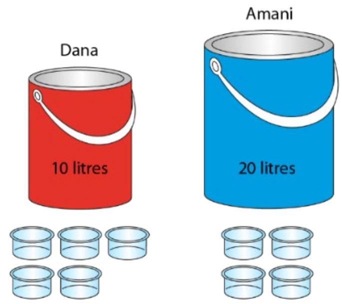
$324 \times 15 = 324 \times 3 \times 5$

Tanima has used her understanding of factors and the distributive law to calculate.

$$\begin{array}{rccc} & 3 & 2 & 4 \\ \times & & & 3 \\ \hline & 9 & 7 & 2 \\ \hline & & 1 & \end{array} \qquad \begin{array}{rcccc} & & 9 & 7 & 2 \\ \times & & & & 5 \\ \hline & 4 & 8 & 6 & 0 \\ \hline & & 3 & 1 & \end{array}$$

Children will learn how to combine operations to solve problems as they work through the curriculum. For example, division with an addition calculation may be required. Consider the following question:

Dana has ten litres of red paint, which she pours into five pots. Amani has twenty litres of blue paint, which she pours into four pots. They give a pot of red paint and a pot of blue paint to Nathan. How much paint does Nathan have in total?



$$\begin{array}{lll} & 10 \div 5 & + \; 20 \div 4 \\ = & 2 & + \; 5 \\ = & 7 & \end{array}$$

Children will need to recognise that the amount of paint will need dividing into the pots before they can add together to find how much paint there is in total. In this instance, jottings can be used to support mental calculation.

Although the problem may initially have appeared to be more complex, due to different operations being needed to solve it, the strategies required were not necessarily complex. It would not have been appropriate to use formal written strategies to calculate this problem. It can therefore be seen that written strategies should be used in conjunction with mental methods and should not replace them.

Common errors in this area may include

- children rely on written algorithms rather than selecting most efficient method.

What to look for

Can a child:

- draw upon a range of mental and written strategies to calculate fluently and efficiently?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:

- Topic 2.14: Multiplication: partitioning leading to short multiplication
- Topic 2.23: Multiplication strategies for larger numbers and long multiplication
- Topic 2.26: Mean average and equal shares
- Topic 2.28: Combining division with addition and subtraction

Notes:
Key learning from support material and self-study:
What I will focus on developing in my classroom practice: