Secondary
& Further Education
Magazine 104

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to Issue 104 of the Secondary Magazine (incorporating FE)

With the countdown to Christmas well advanced, and the summer holiday a distant memory, this issue tries to capture some of the reasons why we teach mathematics. Starting with some of the mathematics that hooks us in, touching on a possible link with the science curriculum, and considering work supporting learners who struggle to make sense of mathematics - these articles give a perspective to the work of the mathematics teacher. And don't forget to read the jokes to complete the picture. Enjoy your teaching.

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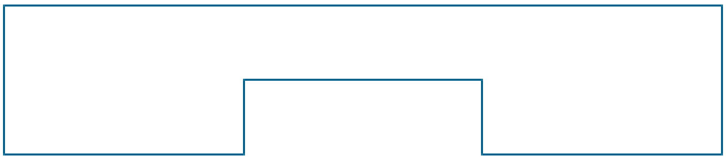
From the editor: amazing moments in mathematics

As a teacher of mathematics, there must have been many times in your life when the subject has provoked those moments of awe and wonder? Do your pupils, and other learners in your school, experience the same satisfaction from mathematics? How do you convince them that mathematics is actually a cool subject? What examples do you use? Here is a selection of mathematical episodes that might fit the bill: do you feel satisfaction or excitement as you read these suggestions, or can you suggest some alternatives?

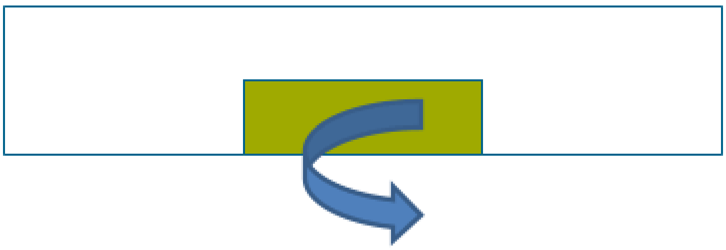
Satisfaction may come from seeing an elegant solution to a problem. For example, realising that you can find the area of this shape:



by finding the sum of three rectangles like this:

or this:

And also by subtracting the area of the green rectangle from the surrounding rectangle:



There are many number patterns that intrigue. This is one example, there are many more:

1+8x1	=	9
12x8+2	=	98
123x8+3	=	987
1234x8+4	=	9876
12345x8+5	=	98765
123456x8+6	=	987654
1234567x8+7	=	9876543
12345678x8+8	=	98765432
123456789x8+9	=	987654321

Although people regularly handle a 50p piece, the idea of a shape having a constant width (to be able to be used in a slot machine) that is not a circle is interesting. The technical name for this shape is an "equilateral curve heptagon".

A useful definition of a fractal may be found in WolframMathWorld:

> *A fractal is an object or quantity that displays self-similarity, in a somewhat technical sense, on all scales. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales.*

Understanding the concept of a fractal deepens the appreciation of the pictures of fractals available on the internet.

Benoît B Mandelbrot may be the first name that you connect with fractals, but you may also want to investigate the Koch snowflake, or the Sierpiński gasket.

Please tell us about some of your amazing mathematical moments.

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A resource for the classroom – links between mathematics and science

Following the Editorial article in Issue 103 which considered the GCSE subject content and assessment objectives document for science and the links to mathematics contained therein, the resource in this issue could be the stimulus for some joint work between mathematics and science. Explicit co-operation between the mathematics department and the science department in school could enable pupils to transfer their skills and knowledge between the two subjects, deepening their learning.

Big questions for this activity could be:

- How do butter manufacturers decide on the proportions of the cuboids that they package?
- By changing the proportions of the cuboids, would that change the melting rate of the butter?

A Google images search for butter reveals that not all countries sell butter in packs of the same proportions that we do in UK. This activity considers some of the factors involved in making these decisions.

Activity 1
Brainstorm the pupils' knowledge of the properties of butter and decide on the qualities that they want when they buy a block of butter

Activity 2
Pupils working in groups could generate a range of cuboids with a given volume. $36cm^3$ may be a suitable volume to choose. Pupils might use multilink cubes to generate the cuboids. For each of these cuboids, pupils can calculate the surface area. A spreadsheet could be used as a tool to generate these calculations. For each cuboid, pupils work out the ratio for surface area : volume

Activity 3
As a demonstration, the teacher can cut two differently proportioned blocks of butter with a volume of $36cm^3$. Fill two glass beakers (easily accessible from the Science Department) with boiling water, drop a block of butter in each beaker and watch how each melts (videoing this process may be useful to be able to review the action). Consider the melting process with respect to surface area and the surface area/volume ratio.

Activity 4
Measure an actual block of butter. Calculate the surface area, volume, and the surface area : volume ratio. Are you able to answer the original question in the light of these activities? Consider – where else in the science curriculum this concept might apply (looking for size of cells/organisms or chemical reactions in relation to particles).

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Focus on…Every Child Counts

In this article, the second in the series, Andy Tynemouth from Edge Hill University and National Adviser for Every Child Counts discusses the problems of supporting learners who struggle to make sense of mathematics

The first article in this series was published in Issue 103. These articles are intended to be read in order so, if you have not read the introduction, and are someone who likes to have the full picture - to dot the i's and cross the t's - then please have a look at it for a catch-up. If, on the other hand, you don't mind picking up part way through then please read on.

This is essentially the story of the lessons we at Every Child Counts have learned through extending our flagship programme, Numbers Count (NC)[1], from Year 1 all the way up to Year 8. It rings the changes between the strategies teachers can successfully use to support six year olds who are really struggling to learn mathematics and those used to support 12-year olds in the same boat. You may, or may not, be surprised to discover that there are very few differences.

I will start with a quick précis of the introductory article. In it I made the observation that many learners who struggle the most with mathematics can be very difficult to support. Their experience is one of failure and they lack any sense of their own mathematical skills, knowledge and understanding: what I call their mathematical resources. In some ways it may become more difficult the older and more entrenched they become. Whatever the case, it is certainly extremely problematic to move them on. But crucially it is ***not*** impossible[2]. I went on to describe the initial stages of a 1:1 intervention from a specialist mathematics teacher trained to deliver the Numbers Count programme. Based on constructivist principles the programme is more focused on supporting the learner in building on their existing resources than *'rectifying misconceptions'* or *'plugging gaps'*. This approach relies on extensive Diagnostic Assessment to give the teacher as full a picture as possible of the learner's available resources. They plan a bespoke Individual Learning Plan and then begin to teach. And here we pick up the story.

Upon completing the Diagnostic Assessment the teacher has a good idea of the learner's existing resources, how secure they are and where they need to move next. This is hard to find out from the outside. Paradoxically, from the inside the learners seem only to be aware of their deficits – they don't perceive themselves as knowing, or being able to do, any mathematics. Unsurprisingly though, even the most struggling learner has not managed to get through 7 or 8 years of schooling without picking up some mathematics, sometimes a fair bit. The main problem is that these resources are fractured and the learner sees no connection between them. They are spread about mathematics, so to speak. We can find learners who have extensive pockets of number facts at their fingertips, but no complete set of addition facts to 10. These facts are often isolated from other areas of understanding. For example, a learner can have a clutch of addition facts, a sound understanding of place value and yet be unable to derive new facts by bringing these two resources together. E.g. they may know the fact 7+5=12 but be unable to apply that understanding to 0.7+0.5. Other times you may find that a learner can count fluently in tens up to, and over, the 100 boundary. They may count *'…64, 74, 84, 94, 104…'* but ask them what 84+10 is and out come the fingers *'…85, 86, 87, 88, 89, 90, 91, 92, 93, 94…it's* ***94****!'*. This is particularly striking when they counted *'…84, 94…'* mere seconds before the question was asked.

This was a particular quandary for my development team of expert Numbers Count teachers and Teacher Leaders as we started to work with older learners for the first time. Not because the difficulties themselves were quantitatively different from those experienced by the younger learners, with whom we'd been working for some years, but because we all sensed that in some way the learners were. To be specific, we all felt that the (vast majority of) older learners were so much more self-aware, so much more capable of self-reflection. We had to find a gadget that would allow us to co-opt them as active partners in their own learning[3]. At one point, in a meeting boiling with frustration at our inability as teachers to help our learners see the way forward, somebody suggested 'it's as if they need a map of what they know'. In a split second we realised that this was exactly what they needed. These were learners in a fog of cognitive mathematical clutter, with no way to know which way to turn. The next excited half hour was spent tossing ideas to and fro until we hit on the simplest of formats. We would ***map***, with the learners, the resources they held. And more than this, we would negotiate what they could achieve with this inventory. Because we held such detailed knowledge of the resources possessed of each of the learners from our Diagnostic Assessment (described in more depth in the first article) we were in a position to provide them with a surprisingly – to them at least – extensive picture of all the things that they knew and could do. Furthermore, as expert intervention teachers, we knew what they could do and where they could go with their existent resources. We also knew what they could realistically hope to achieve.

The following excited discussion led us on a pilgrimage to formative assessment. A number of things were immediately clear. The map must be collaborative, it had to be developed ***with*** the learner. This meant that time would need to be devoted to its construction. It must be accessible: its content must be understood by the learner. And it must be dynamic, to mirror their dynamic development. This meant that whatever form its recording took it had to be easy to change and move. It was easy to see that by writing down all the things the learner could do on post-its and keeping them together we could achieve a lot of what we wanted. The statements could be concise and written with the learner to ensure accessibility – this covered our first two requirements. Making it dynamic was a little harder. Eventually we envisaged a system where we could represent the learner's resources so that they could see what they knew and could do while flagging up those things they needed further work on and some of the things they should aspire to achieve. The post-its could be moved as the learners progressed so they could see their achievements and the gradual increase in their stock of skills, knowledge and understanding. We just needed some headings that the statements could sit under.

We finally agreed upon three headings, and you won't be surprised to hear what they were:

1. What I know (I can do independently)
2. What I'm working on now (I can do with support)
3. What I would like to learn (currently out of reach)

Clearly there's nothing new in the use of terms like these. There has been a wide range of approaches to supporting learners in engaging with learning through formative assessment. Certainly since the publication of Dylan William and Paul Black's excellent and influential pamphlet 'Inside the Black Box' in 1998, schools have sought to achieve the kinds of benefits that the research seems to suggest is possible with varying levels of success. So…why should the Learning Map work? I propose three reasons. We will examine these reasons and some of the practicalities of managing a Learning Map in the next article.

Footnotes

[1]NC is one of the Every Child Counts suite of programmes developed and rolled out nationally by Edge Hill University. It is a teacher-led, one to one mathematics intervention aimed at the lowest 6% of attainers. These are learners for whom nothing else will work. Originally conceived and designed to target Year 2 its success has led to its development to address the needs of learners up to, and including, Year 8. For more information please see the Every Child Counts website.

[2]And at the very least we have good socio-economic reasons to do so – if not a moral obligation. The Every Child A Chance Trust report *The Long Term Costs of Numeracy Difficulties (2009)* estimates that the yearly cost to the exchequer of 'innumeracy' is some £2.4bn of the taxpayers' hard-earned money. We may also have more important ethical reasons to be concerned. In his report for the Government Office for Science

(2008) Professor Brian Butterworth asserts that those suffering from numeracy difficulties are more likely to be unemployed, arrested, or suffer from depression than their numerate peers.

[3]As I explained in the first article all ECC programmes, whether intervention programmes for children and learners or PD programmes for the teachers and TAs we train, are founded on constructivist principles. Ollerenshaw and Richie (cited in Bloomfield,1998: 15) observe that at the heart of a constructivist approach is the belief that: "learners have final responsibility for their learning." The great constructivist Ernst von Glasersfeld articulated two key principles that underpin constructivism:

> 1.Knowledge is not passively received but actively built up by the cognizing subject
> 2.The function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality (von Glasersfeld, 1989).

By 'ontological reality' von Glasersfeld means a mind independent reality. These two principles, if followed, lead a teacher to support learners in the construction of their own version of reality. They lead the teacher away from the view that mathematics is out there to be discovered, as it were. It leads them away from what Anna Sfard describes as a monological approach (Sfard, 2010) where the teacher passes on or transmits knowledge and towards a dialogical approach where the teacher and learner work together to establish a working model of the subject studied.

References

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Von Glasersfeld, E. (1989) Constructivism in Education In: T. Husen & T. N. Postlethwaite, (eds.) (1989) The International Encyclopedia of Education, Supplement Vol.1. Oxford/New York: Pergamon Press, pp.162–163. [accessed 25th August 2011]

Hudson, C. Price, D. and Gross, J. (2009) *The Long Term Costs of Numeracy Difficulties* London: Every Child A Chance Trust

Sfard, A. (2010) *Thinking as Communicating* Cambridge: CUP.

5 things to do

Charlie Stripp, Director of the NCETM, has started a series of occasional blogs, on topical matters of mathematics education. The blogs, with a light-hearted nod to their mathematical content, are brought together under the title Charlie's Angles.

If you are interested by the Fibonacci sequence, The life and numbers of Fibonacci is an interesting article from Plus Magazine that relates Fibonacci numbers to rabbits, bees, shells and sunflowers.

NSPCC Number Day is on Tuesday 3 December. Are you taking part?

You could apply for a Royal Institution Mathematics Enrichment and Enhancement grant. The grants scheme, administered by the Royal Institution (Ri) and supported by the Clothworkers' Foundation, is an exciting opportunity to have a mathematics enrichment and enhancement (E&E) activity at your school and help integrate E&E in school practice. The Ri is offering grants of up to £500 for eligible state-funded schools or academies in the UK to have a mathematics activity from the STEM Directories in 2013/14.

If you want to know what is the world's longest song, or what goes *pieces of seven, pieces of seven*, then here is an opportunity to explore those mathematical jokes and some others.

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Tales from the classroom: cafés and classrooms

I had a week in the Lake District during the October half term. The autumn colours were stunning – that is, the colours I could see through the rain! There is a sign in one of the shops in Ambleside that says 'It rains a lot in the Lake District – get over it!' so I got over it and had a fabulous week. One of the advantages of the heavy downpours is the opportunity to shelter in some of the local pubs and cafés. Some of these establishments were merely 'open for business' serving drinks and refreshments to a set menu in a pleasant setting, whilst others welcomed me and my muddy boots, invited me to come closer to the roaring fire and suggested the hot chocolate that really made the difference to my mood.

That got me thinking. As a mathematics adviser, I have the privilege of visiting classrooms, not cafés; there are some classrooms which are calm, purposeful where pupils make good progress; and there are others in which I want to linger, pull up my chair and bask in the flames of learning – well you get the idea. Let me describe one of these.

As pupils entered the room, it was clear that they were looking forward to spending time in that room with that teacher, learning mathematics. Pupils in this classroom had been assigned roles such as greeter, numeracy expert, literacy expert, monitor, checker so any subsequent entrants to the room were greeted and made to feel welcome. The teacher was warm and encouraging; one of the pupils came to the board to explain his solution to a problem and made a slight error in his explanation; the teacher treasured the mistake and celebrated the opportunity to learn something new. The pupil was helped (by a numeracy expert) to correct the error and returned to his seat feeling good about himself – he had given many pupils in the class a chance to correct their own errors.

At one point in the lesson, the teacher showed a fairly standard question from a book asking about the area of the walls of a squash court. He asked pupils to suggest some questions they could ask about the diagram; nearly every hand in the classroom went up with a plethora of suggestions. One pupil started to ask about the space inside the squash court - the teacher's skilful questions drew out an understanding of volume. Another pupil asked about the area of the roof which enabled some pupils to demonstrate a knowledge of Pythagoras and work on that particular problem. Another pupil brought a real context to the problem by asking about the paint that would be needed for the squash court which gave another group a focus for their subsequent work. I haven't described adequately the sheer enthusiasm of the pupils who did not shy away from potentially difficult issues and knew they would be enabled to solve problems that they suggested. The culture of this classroom was embedded in the behaviour of the teacher and the students.

I certainly didn't feel this classroom was just open for business: the warm, vibrant learning environment stimulated pupils to take risks and stay on the edge of their learning. It might be worth thinking how a visitor would describe your classroom? What makes it an exciting place where pupils look forward to their learning?

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