MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.20 Multiplication with three factors and volume

Teacher guide | Year 5

Teaching point 1:

Volume is the amount of space that something occupies.

Teaching point 2:

Volume is measured in cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).

Teaching point 3:

The volume of a cuboid can be calculated by multiplying the length, width and height.

Teaching point 4:

Both the commutative law and the associative law can be applied when multiplying three or more numbers.

Teaching point 5:

The choice of which order to multiply in can be made according to the simplest calculation.

Overview of learning

In this segment children will:

- be introduced to the concept of volume
- learn that volume is measured in cubic units
- calculate the volume of simple 3D shapes by counting unit cubes
- explore the relationship between cubic centimetres and cubic metres
- calculate the volume of cuboids by multiplying the length, width and height, and use division to find unknown dimensions when given the volume and other dimensions
- be introduced to cube numbers
- divide composite 3D shapes into smaller cuboids in order to calculate volume
- multiply three numbers in an abstract context, applying commutativity and associativity
- transform two-factor multiplication calculations into three-factor multiplication calculations to make them easier to solve.

Teaching point 1 begins by introducing the concept of volume. Initially, to draw attention to the concept, objects that are obviously very different in volume are used. Then children are encouraged to look at objects that are more similar and explore how when the shape of an object is manipulated the volume remains the same.

Teaching point 2 introduces cubic centimetres and cubic metres, and the abbreviations cm^3 and m^3 are used for the first time. Links to square units (see segment *2.16 Multiplicative contexts: area and perimeter 1*) will be useful here. Children calculate volume by counting unit cubes.

In *Teaching point 3*, children use multiplication to calculate the volume of cuboids. Layers are used initially to build up the process, calculating the volume of one layer and then multiplying this by the number of layers. Children are introduced to the rule that the volume of a cuboid can be calculated by multiplying the length, width and height. It should be emphasised that it does not matter which way the cube is oriented (i.e. which side is the length, width or height), the product will be the same. This links to commutativity, covered in *Teaching point 4*. Cube numbers are introduced briefly; a detailed understanding is not required at this stage. Division is then used to find unknown dimensions when the volume and other dimensions are given.

Teaching point 4 uses cuboids to explore the laws of commutativity (see segment *2.3 Times tables: groups of 2 and commutativity (part 1)* for commutativity in the context of *multiplication* with two factors) and associativity (see *Spine 1: Number, Addition and Subtraction*, segment *1.11* for associativity in the context of *addition* of three numbers). It should be noted that the commutative law can be applied in contexts where there is only one pair of factors and one operation, whereas the associative law is *only* applicable where there are two or more operations.

In *Teaching point 5*, children learn to multiply three numbers in an abstract context, choosing which two numbers to multiply first for the most efficient calculation. Children explore how an equation can be made easier to solve by expressing one of the factors as a multiplication of two factors. It is important that the context of volume is not used here, as two dimensions cannot be changed to three; it is recommended that arrays are used instead.

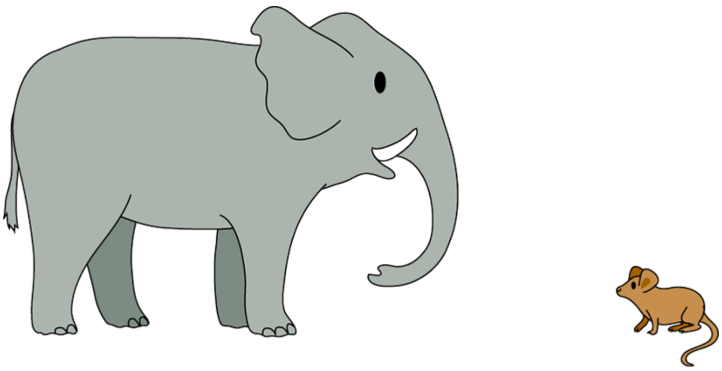
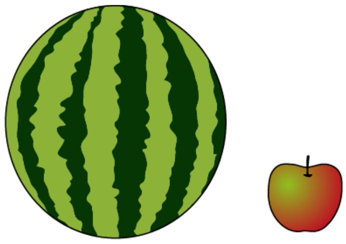
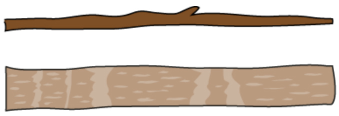
Note: in representations, measurements have been drawn proportionally correct but scaled to fit the available space.

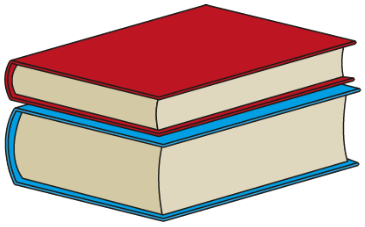
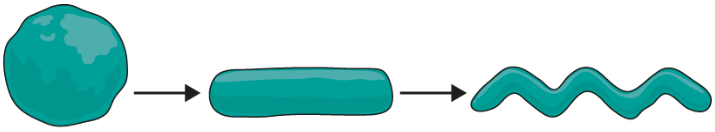
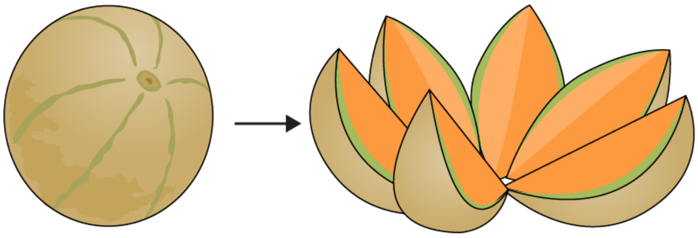
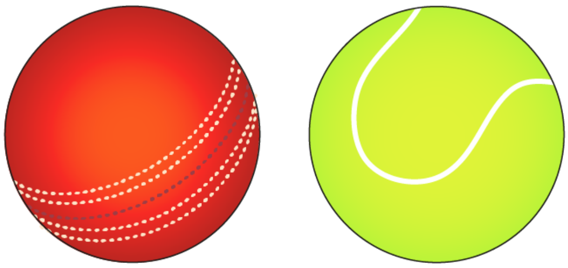
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

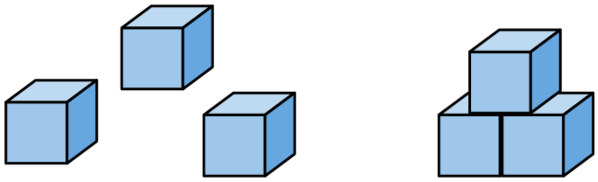
Teaching point 1:

Volume is the amount of space that something occupies.

Steps in learning

	Guidance	Representations
1:1	This teaching point looks at how to compare objects using the concept of volume. To start, encourage children to compare two things that are obviously different in the amount of space that they occupy, for example an elephant and a mouse. Explore other real-life examples in which it is easy to say which thing occupies more space. For example, you could consider different rooms, such as a cupboard and a sports hall, or different pieces of fruit. Introduce the term *'volume'* using the following stem sentence: ***'The amount of space the ___ takes up is its volume.'*** Encourage children to use the following stem sentence to compare objects: ***'The ___ has a larger volume than the ___.'***	• *'Which takes up more space, the elephant or the mouse?'*  • *'The amount of space the elephant takes up is its volume.'* • *'The amount of space the mouse takes up is its volume.'* • *'The elephant has a larger volume than the mouse.'* • *'Which has a larger volume, the melon or the apple?'*  • *'The melon has a larger volume than the apple.'*
1:2	Next look at two objects that share one dimension but have different volumes. For example, you could look at two pieces of wood that are the same length but have a different thickness. Use the following stem sentence to describe the objects: ***'The ___ has a larger volume than the ___ because it occupies more space.'***	• *'Which stick has a larger volume?'*  • *'The second stick has a larger volume than the first stick because it occupies more space.'*

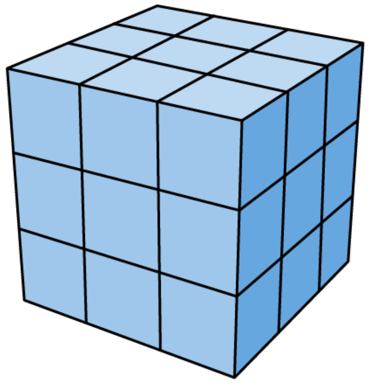
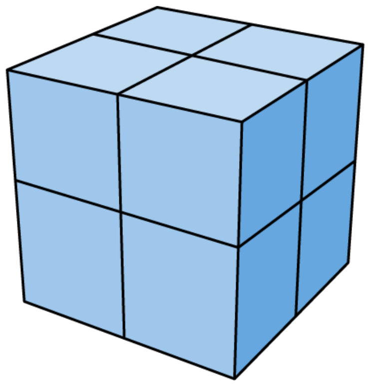
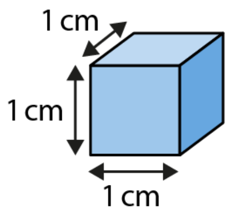
		• *'Which book has a larger volume?'*  • *'The bottom book has a larger volume than the top book because it occupies more space.'*
1:3	Next, children will learn that, although objects might look different, they can have the same volume. To demonstrate this you could: • make different shapes with the same piece of salt dough • cut up a piece of fruit and arrange the pieces into different shapes. While you are doing these demonstrations, continue to discuss the idea that the volume is remaining the same even though the shape is changing.	Making shapes from salt dough:  Cutting up fruit: 
1:4	In this step we will confirm that volume refers to the space that an object occupies, not its mass. Show children two objects that have a similar volume but a different mass, such as a cricket ball and a tennis ball. Discuss what's the same and what's different. Draw out the idea that objects with a different mass can still have the same volume, because they occupy the same amount of space. Explain that when we say something is larger, we need to be specific about what we are measuring: • Is it heavier? • Is it longer? • Does it have a larger volume?	*'What's the same and what's different?'*  • *'The shape is the same.'* • *'The volume is the same.'* • *'The mass is different.'*

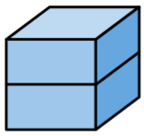
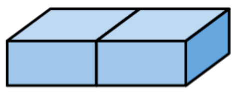
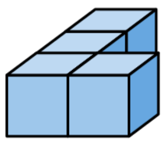
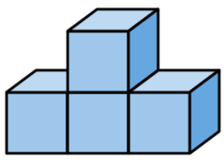
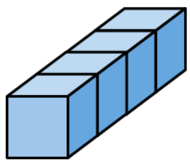
1:5	Provide children with pairs of objects to compare, and ask whether one has a larger volume than the other or whether they have the same volume. Encourage children to use specific language when referring to the volume to ensure they are not confusing volume with other measurements (see step *1:4*). You could use items such as: • a real brick and a foam brick of the same size • a beach ball and a ping pong ball • a piece of fruit whole and cut up. Finish this teaching point with a dòng nǎo jīn problem such as the one opposite.	Dòng nǎo jīn: *'Evie says that the three separate bricks have a larger volume than the stack of three bricks together, because they take up more space when they are separate. Do you agree or disagree? Explain your answer.'* 

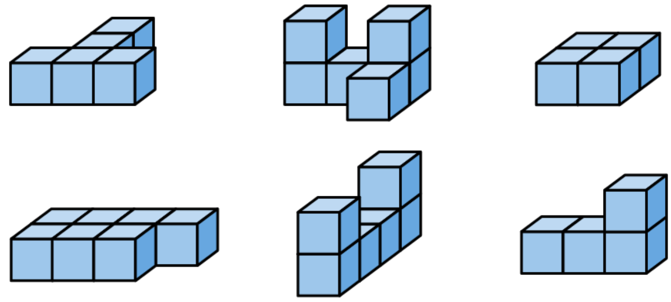
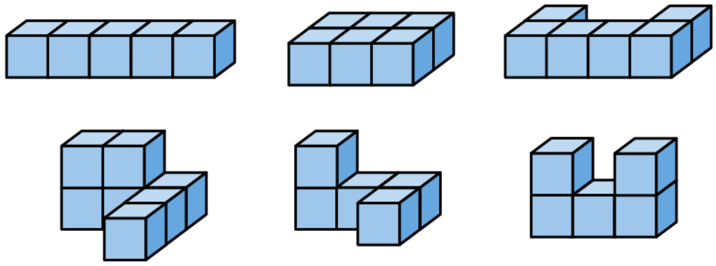
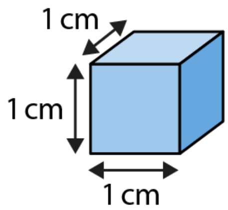
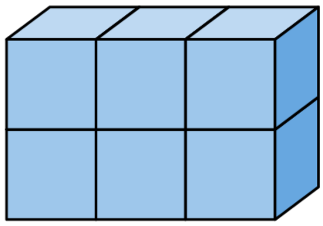
Teaching point 2:

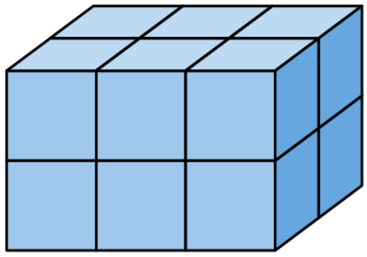
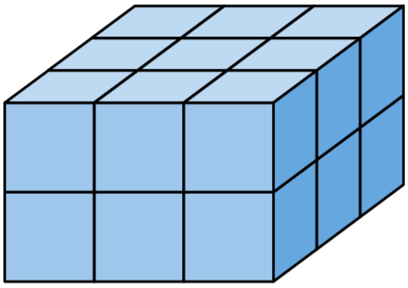
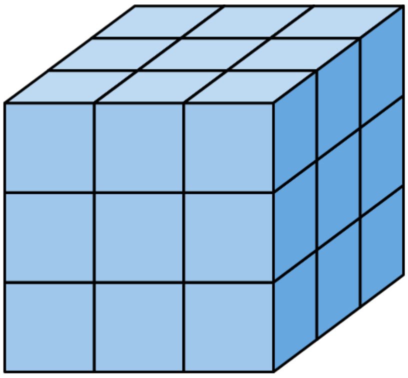
Volume is measured in cubic units, such as cubic centimetres (cm^3) and cubic metres (m^3).

Steps in learning

	Guidance	Representations
2:1	This teaching point focuses on the use of units to give value to volumes. You could introduce this topic by showing children two 3D shapes of a similar size, one comprised of larger cubes and one comprised of smaller cubes. Ask children which shape has a larger volume. Draw children's attention to the fact that in order to compare the two shapes the *'unit'* cubes must be the same size.	Comparing two shapes:  • *'Shape A has been made using twenty-seven cubes.'*  • *'Shape B has been made using eight cubes.'* • *'Shape A must have a larger volume because it is made using more cubes. True or false?'*
2:2	Next focus on the use of a standard measure that is universally recognised. In segment *2:16 Multiplicative contexts: area and perimeter 1*, children's attention was drawn to the fact that the same units need to be used when comparing areas. Children will now learn that there is also a standard unit for comparing volumes.	A 1 cm^3 cube:  volume = 1 cm^3

	To explore this, you could give children a cube that is 1 cm × 1 cm × 1 cm. A Dienes 1 cm^3 cube could be used. Use this cube to estimate the volume of a variety of real-life objects, such as books, lunchboxes or cereal boxes, so that children don't just associate volume with mathematical 3D shapes shown in books. This will develop children's estimation skills. Work towards the following generalised statement: ***'You can measure volume in cubic centimetres. You write this as "cm^3".'***	
2:3	When learning to use a cubic centimetre to measure volume, children must not overgeneralise and think that this is the only shape that has a volume of one cubic centimetre. Show children some other shapes that also have a volume of one cubic centimetre. You could cut a piece of salt dough into a cube with each side 1 cm and then roll it into a sphere. Explain that it still has the same volume, but that spheres are not as useful for measuring as cuboids because there are gaps between spheres when placed together.	Two shapes with a volume of 1 cm^3:  1 cm^3  1 cm^3
2:4	Now use three cubes, each with a volume of one cubic centimetre, and show how they can be arranged to create different shapes that have the same volume. Use the following stem sentence to describe the shapes: ***'This shape has a volume of ___ cm^3.'*** Repeat the same exercise with a different number of cubes, continuing to use the stem sentence.	*'What is the volume of each shape?'*  4 cm^3  4 cm^3  4 cm^3 • *'This shape has a volume of 4 cm^3.'*

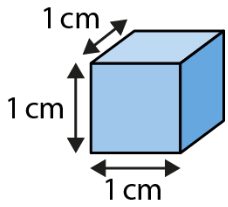
2:5	Next show children shapes that have been created using various numbers of 1 cm × 1 cm × 1 cm cubes. Ask the children to work out what the volume of each shape is. Continue to use the stem sentence from the previous step. You could also use alternative question styles, such as the second example opposite.	• *'What is the volume of each shape?'*  • *'Which of these shapes have a volume of 6 cm³?'* 
2:6	To reinforce the learning in step *2:2* on the universal unit of volume, ask children to compare shapes that have been created from one cubic centimetre cubes. Show children a series of shapes in ascending volume, as seen in the example opposite. Ask children to count the small cubes to find the volume of each shape. Then, compared to the previous shape, ask children: • *'What's the same?'* (the width of each cuboid, and the size of each small cube) • *'What's different?'* (the volume) Remind children that you can compare the shapes because you have used the same unit of volume. As intelligent practice, you could ask children to create cuboids using a given number of 1 cm³ cubes and then sketch the cuboids using isometric paper, recording the volumes. Children could then swap their drawings with a	*'Count the small cubes to find the volume of each shape.'*  volume = 1 cm³  6 cm³ • *'This shape has a volume of 6 cm³.'*

	partner and ask the partner to find the cuboid that has a given volume.	 12 cm³ • *'This shape has a volume of 12 cm³.'*  18 cm³ • *'This shape has a volume of 18 cm³.'*  27 cm³ • *'This shape has a volume of 27 cm³.'*
2:7	Remind children that a cubic centimetre is a unit of volume but that other units are also used. Discuss the idea that when you count cubes, people need to know which size cube you are using, so you put the unit after the number to show this. Introduce the use of *'V'* to mean *'volume'*. Remind children that putting cm³ after a number means that we are measuring using cubes that are 1 cm × 1 cm × 1 cm.	

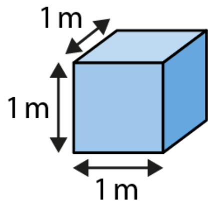
You could return to the generalisation used in step *2:2*.

Extend this step to consider a unit of 1 m × 1 m × 1 m. Ask the children how they might measure the volume of something very large, such as a classroom or a swimming pool. Identify that using centimetres would be a very inefficient way to do this. Work towards the following generalised statement: ***'You can measure volume in cubic metres. You write this as "m³".'***

Units of volume:



$V = 1\ \text{cm}^3$



$V = 1\ \text{m}^3$

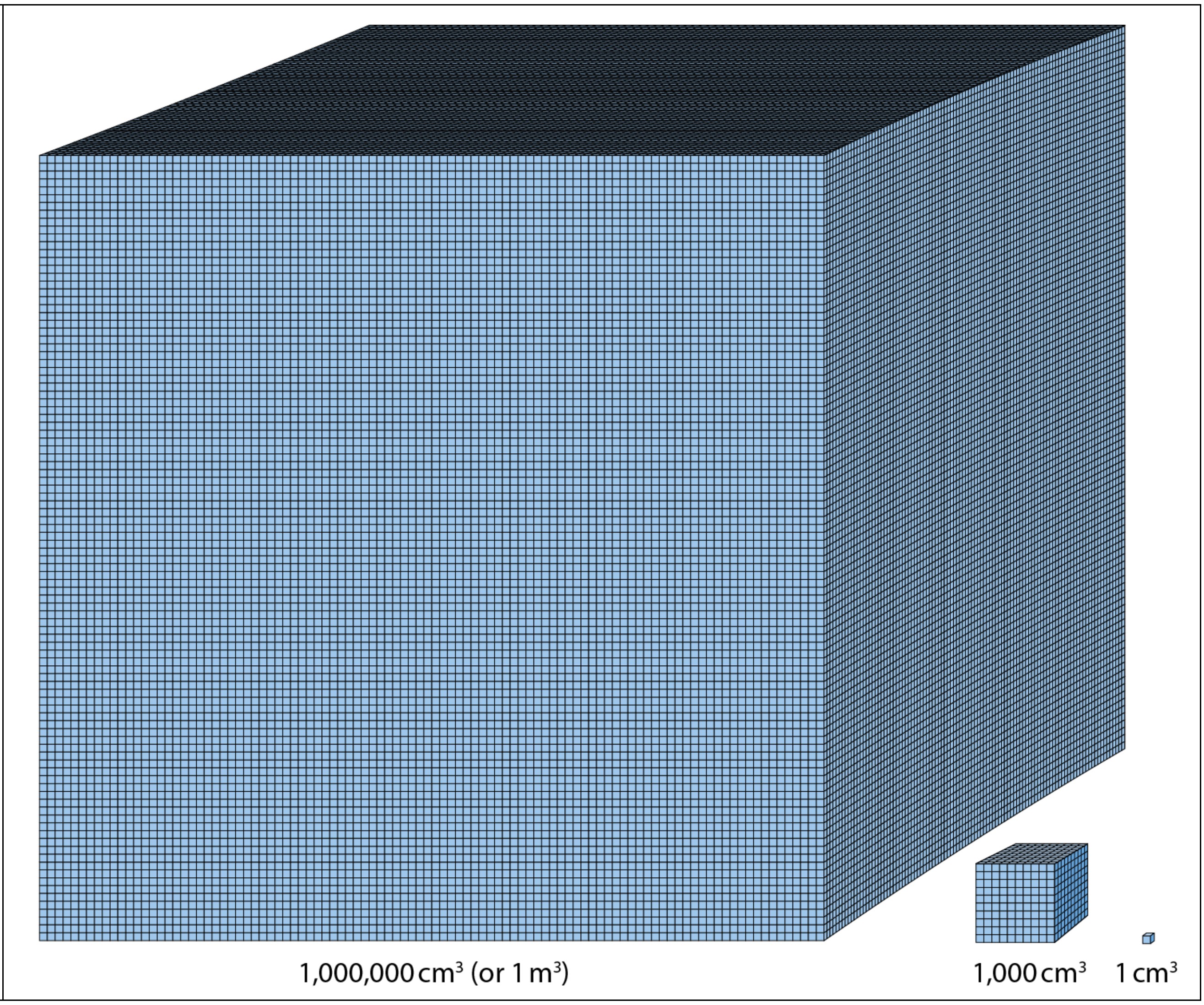
2:8

Finish this teaching point by encouraging children to consider which unit of volume they would choose to use for different 3D objects. Show children a variety of objects and ask them which unit of volume they would use. Choose a mixture of small and large objects. Draw out the idea that different units are suitable for different sized objects. You could ask children to record their findings in a table like the one below.

To explore the relationship between cm^3 and m^3 further, explain that there are a million cubic centimetres in one cubic metre. Work as a class to understand how this can be true. You may want to use a Dienes 10 cm × 10 cm × 10 cm cube to show what 1,000 cm^3 looks like, and then explain that you would need to repeat this 1,000 times to make 1 m^3. Show children the written equation: $1\ \text{m}^3 = 1{,}000{,}000\ \text{cm}^3$

'Which unit of volume would you use to measure these objects? Tick one column.'

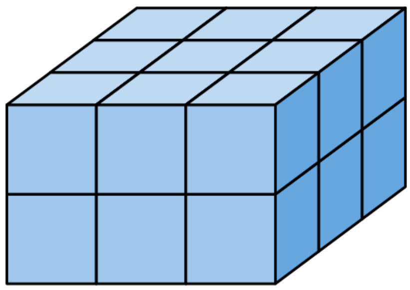
Object	Use cm^3	Use m^3
pencil case		
textbook		
cupboard		
bookcase		

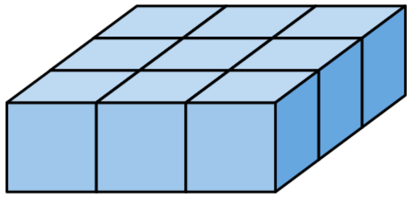
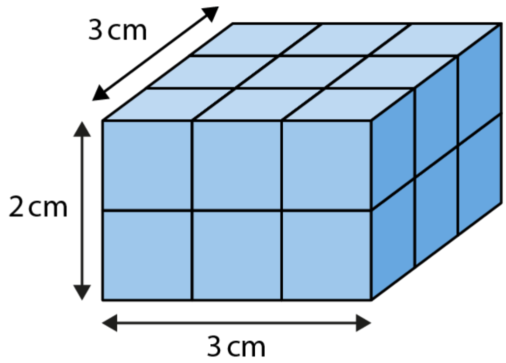

1,000,000 cm³ (or 1 m³)
1,000 cm³
1 cm³

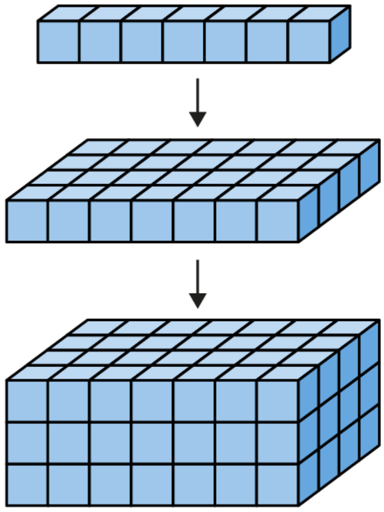
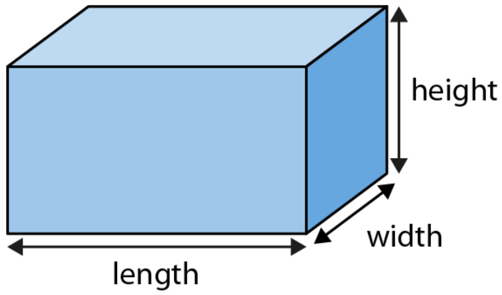
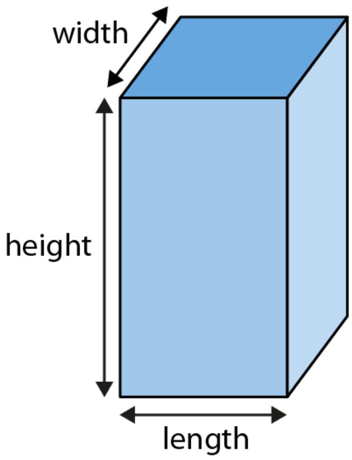
Teaching point 3:

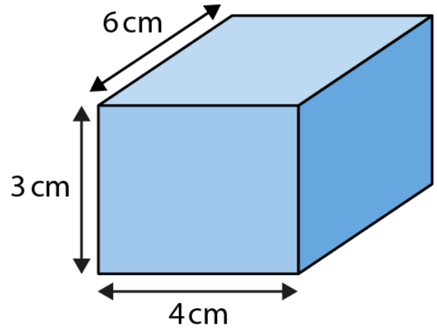
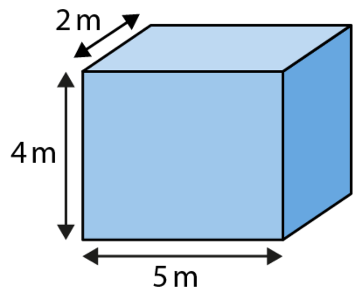
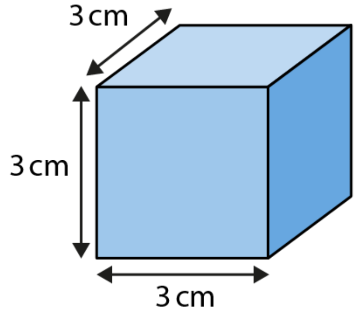
The volume of a cuboid can be calculated by multiplying the length, width and height.

Steps in learning

	Guidance	Representations
3:1	Begin this teaching point by reminding children of the features of a cuboid. You might want to ask them to bring in different cuboids from home such as cereal boxes, books or bricks. As a class, discuss the features of a cuboid, drawing out the following points about a cuboid: • It has six rectangular faces. • It has twelve edges. • It has eight vertices. Ensure that some of the cuboids are also cubes, such as dice or Dienes. Discuss how the features of a cube are different to, or the same as, those of other cuboids: • It has six *square* faces. • It has twelve edges. • It has eight vertices.	
3:2	This step covers how to work out the volume of a cuboid if you know its dimensions, i.e. the lengths of its edges. Show children a cuboid made up of 1 cm^3 cubes, such as the one opposite, and ask them how they could use the lengths of the edges to help them to work out the volume.	Finding the volume of a cuboid made up of smaller cubes: *'What is the volume of this cuboid?'* 

	To begin with, focus on one layer at a time to show children how the cuboid has been built up. Use the following stem sentences: • ***'This layer has ___ rows of cubes.'*** • ***'There are ___ 1 cm³ cubes in this layer.'*** • ***'This layer has a volume of ___ cm³.'*** • ***'There are ___ layers of ___ cm³.'*** • ***'The volume of the cuboid is ___ cm³.'*** Then work with the complete cuboid. Identify the lengths of the three edges and then multiply them together. Now repeat for another cuboid, building up layers to help children visualise the cubes that are contained within the larger cuboid.	Step 1 – working out the volume of one layer:  • *'This layer has three rows of three cubes.'* $3 \times 3 = 9$ • *'So there are nine 1 cm³ cubes in this layer.'* • *'This layer has a volume of 9 cm³.'* Step 2 – adding the layers together: • *'There are two layers of 9 cm³.'* $9 \times 2 = 18$ • *'The volume of the cuboid is 18 cm³.'* Finding the volume of a complete cuboid: *'What is the volume of this cuboid?'*  $3\text{ cm} \times 3\text{ cm} \times 2\text{ cm} = 18\text{ cm}^3$ • *'The volume of the cuboid is 18 cm³.'*

		Building up layers to find the volume: *'What is the volume of this cuboid?'*  $7\text{ cm} \times 4\text{ cm} \times 3\text{ cm} = 84\text{ cm}^3$ • *'The volume of the cuboid is 84 cm*^{3}*.'*
3:3	Show children a cuboid with the edges labelled with length, width and height. Use the following generalised statement: ***'The volume of a cuboid can be found by multiplying the length by the width by the height.'***	Finding the volume of a cuboid: 
3:4	So that children do not focus on which edge is labelled as which, point out that the orientation of the cuboid does not matter; the volume will still be the same.	

3:5	Now provide children with practice finding the volume of a range of cuboids with all the dimensions labelled. Use both cm^3 and m^3.	*'Find the volume of these cuboids.'*  
3:6	Now show children a cube and ask them to work out the volume. Draw attention to the fact that all the edges are the same length. This is a good opportunity to draw attention to cube numbers. You might like to use a table such as the one opposite to build up a list of cube numbers, guiding children to the understanding that a cube number is the product of three numbers that are the same.	*'Find the volume of this cube.'*  $3\text{ cm} \times 3\text{ cm} \times 3\text{ cm} = 27\text{ cm}^3$ • *'The volume of this cube is 27 cm³.'*

Cube numbers:

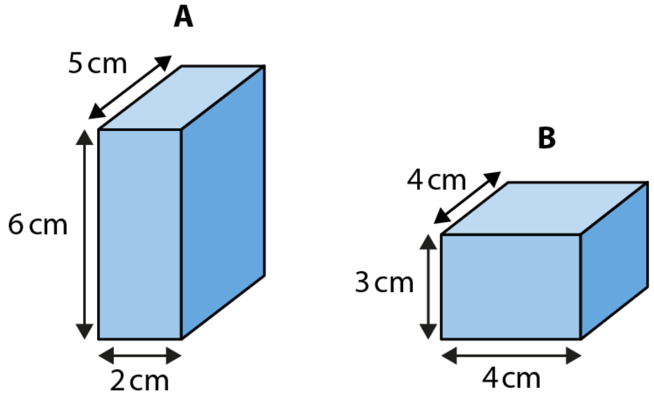
'Fill in the missing information.'

		Cube number
1^3	$1 \times 1 \times 1$	1
2^3		8
3^3	$3 \times 3 \times \square$	
	$\square \times 4 \times 4$	64
5^3		

3:7

Now provide reasoning tasks that allow children to explore the common misconception around volume: that a taller, wider or deeper cuboid will always have a larger volume. Children should explore questions similar to the one opposite in order to learn that all three dimensions must be considered when finding the volume, because all three are used in the calculation. The example opposite requires children to use their reasoning skills because Amy is *right* that A is bigger, but the reason given is *wrong*.

'Amy says that cuboid A is bigger than cuboid B because it is taller. Is she right? Explain your answer.'



3:8

In this step, explore how to calculate an unknown dimension if the volume and two of the dimensions are known.

To develop children's understanding, provide them with information about a box with one unknown dimension, and ask them to draw the box and label the sides. Ask:

- *'What do we know?'*
- *'What do we need to find out?'*

Make sure children know what information needs to be found.

Write the information as a missing number problem, then try adding different numbers into the gap, working upwards from 1:

- $3 \times 4 \times ? = 36$
- $3 \times 4 \times 1 = 12$
- $3 \times 4 \times 2 = 24$
- $3 \times 4 \times 3 = 36$

Look at the sequence and ask children '*What do you notice about the products?*' Help them to notice that they are products in the 12 times table. From here, make the link that *$3 \times 4 = 12$* and so we can replace the '*3×4*' in the missing number problem with '12'. We can then find the unknown dimension by using multiplication facts:

$3 \times 4 \times ? = 36$

- *'is the same as'*

$12 \times ? = 36$

- *'And we know that* $12 \times 3 = 36$*'*
- *'So the height is three.'*

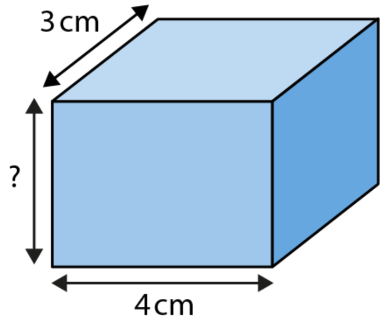
We can also write this using division:

- $36 \div 12 = ?$
- $36 \div 12 = 3$

Work through further examples, such as those opposite, using both multiplication and division methods.

Missing dimension – Example 1:

'A box has a volume of 36 cm^3. The width is 3 cm and the length is 4 cm. What is the height?'



- *'We know the width is 3 cm.'*
- *'We know the length is 4 cm.'*
- *'We know the volume is 36 cm^3.'*
- *'We need to find the height.'*
- *'We can write this as'*

$3 \times 4 \times ? = 36$

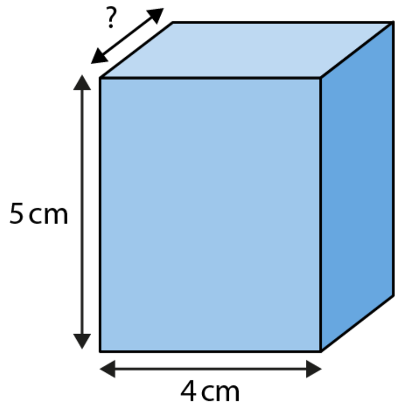
- *'which is the same as'*

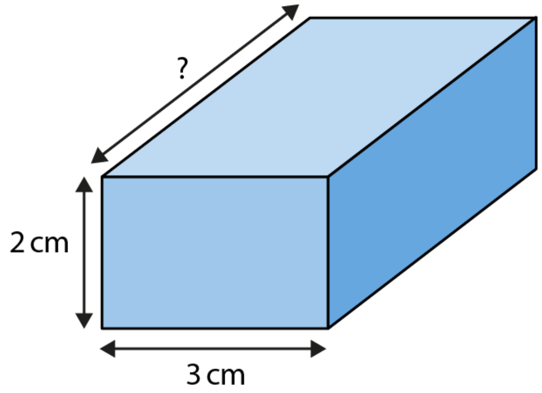
$12 \times ? = 36$

- *'And we know that'*

$12 \times 3 = 36$

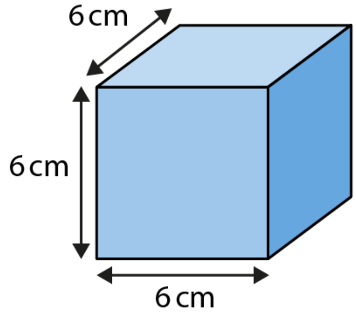
- *'So the height is 3 cm.'*

		Missing dimension – Example 2: *'A box has a volume of 40 cm³. The length is 4 cm and the height is 5 cm. What is the width?'*  • *'We know the length is 4 cm.'* • *'We know the height is 5 cm.'* • *'We know the volume is 40 cm³.'* • *'We need to find the width.'* $4 \times 5 \times ? = 40$ $20 \times ? = 40$ $20 \times 2 = 40$ • *'So the width is 2 cm.'*

		Missing dimension – Example 3: *'A box has a volume of 30 cm³. The length is 3 cm and the height is 2 cm. What is the width?'*  • *'We know the length is 3 cm.'* • *'We know the height is 2 cm.'* • *'We know the volume is 30 cm³.'* • *'We need to find the width.'* $3 \times 2 \times ? = 30$ $6 \times ? = 30$ $30 \div 6 = ?$ $30 \div 6 = 5$ • *'So the width is 5 cm.'*
3:9	Provide children with varied practice of working out volume using given side-lengths, and working out unknown side-lengths when volume is provided. Children should be encouraged to apply their knowledge of factors to solve such problems. Use dòng nǎo jīn problems such as the one on the next page to build towards problems that have several possible solutions.	Working out the volume of a cuboid: *'A cereal box has a height of 20 cm, a length of 10 cm and a width of 3 cm. What is its volume?'* 

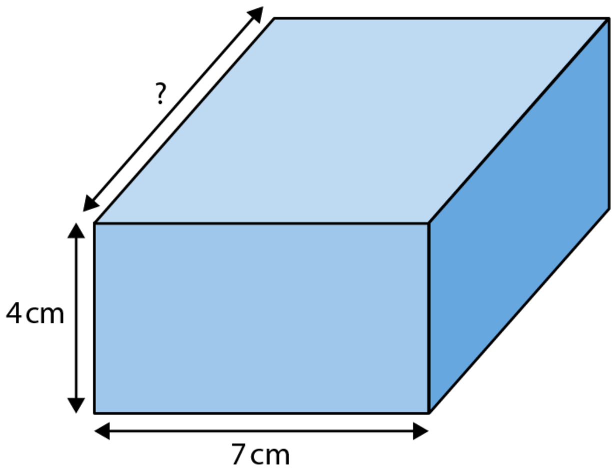
Working out the volume of a cube:

'The width, height and length of a box are all 6 cm. What is the volume of the box?'



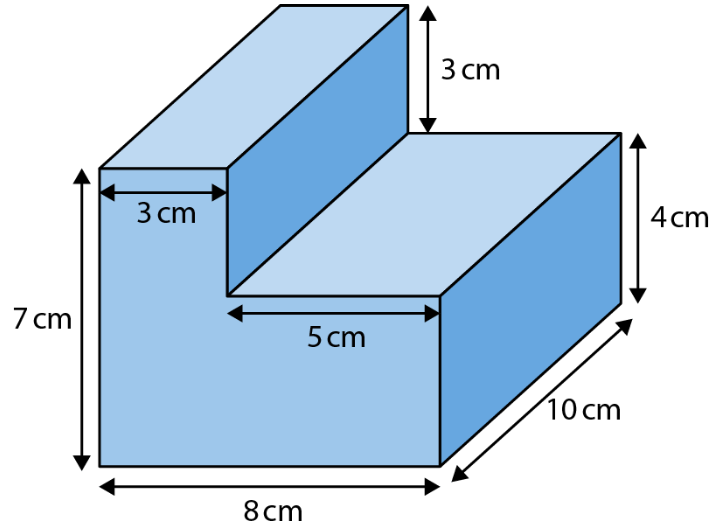
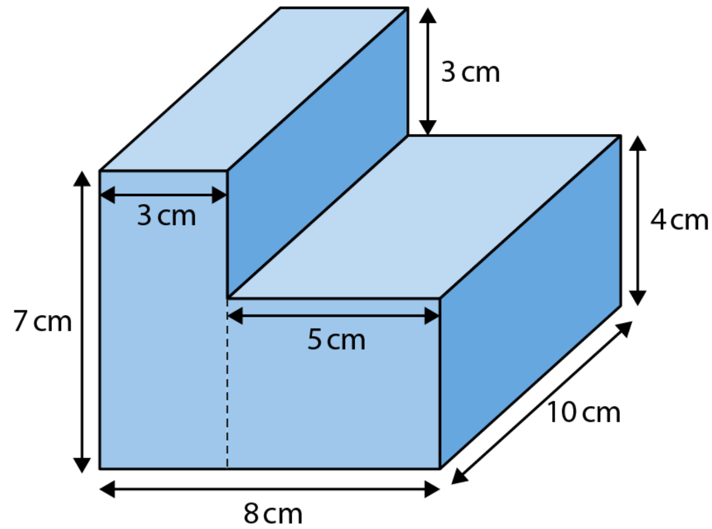
Working out a missing dimension:

'We know that a box has a height of 4 cm and a length of 7 cm. The volume of the box is 280 cm^3. What is the width of the box?'

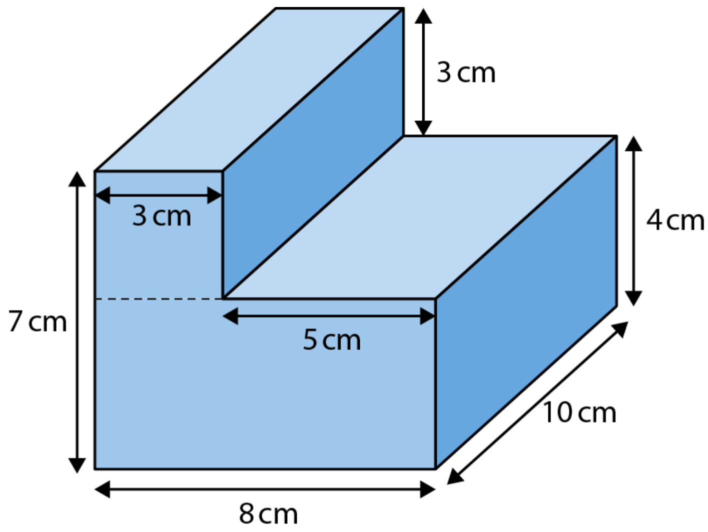


Dòng nǎo jīn:

- *'Danny has 24 m^3 of sand to fill a sandpit. He knows that the sandpit is 2 m deep. What might the width and the length of the sandpit be?'*
- *'A swimming pool has 300 m^3 of water in it. What might the dimensions of the pool be?'*

3:10	In this step compound shapes are divided into cuboids to work out the volume. To start, show children a compound shape such as the one opposite, and ask them how the shape could be divided into cuboids. Ask children to split the shape in two different ways. Take one of the options and write an equation for each cuboid. Show children that to work out the total volume of the shape, you add these two equations together. Repeat these steps with the shape split in a different way. Develop this using a variety of shapes, and then extend it to use shapes that can be split into three or more cuboids.	• *'What is the volume of this shape?'*  Split 1:  *Cuboid 1:* 7 cm × 3 cm × 10 cm *Cuboid 2:* 5 cm × 4 cm × 10 cm $V = 7\text{ cm} \times 3\text{ cm} \times 10\text{ cm} + 5\text{ cm} \times 4\text{ cm} \times 10\text{ cm}$ $V = 410\text{ cm}^3$

Split 2:



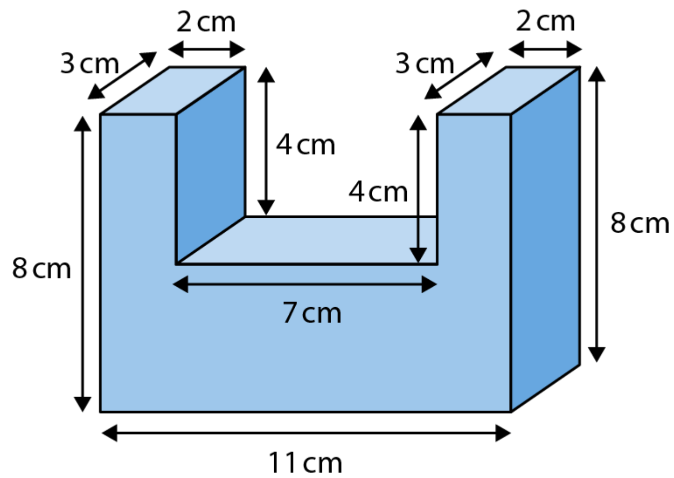
Cuboid 1: 3 cm × 3 cm × 10 cm

Cuboid 2: 8 cm × 4 cm × 10 cm

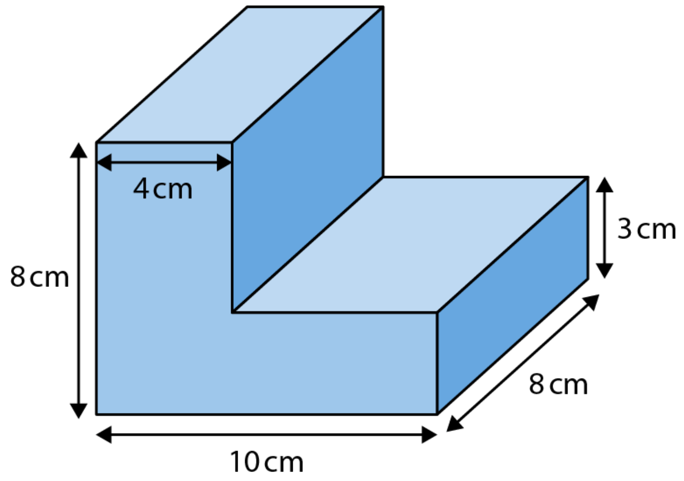
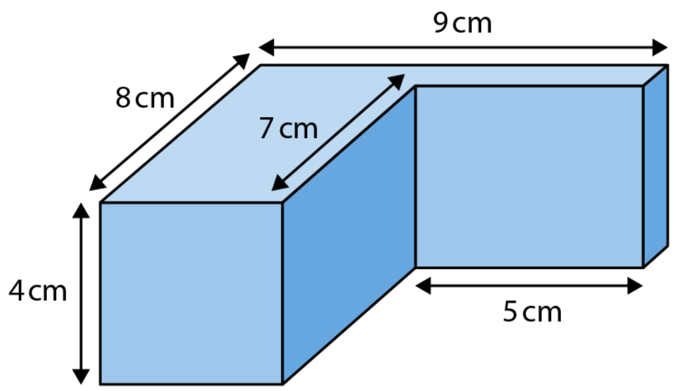
V = 3 cm × 3 cm × 10 cm + 8 cm × 4 cm × 10 cm

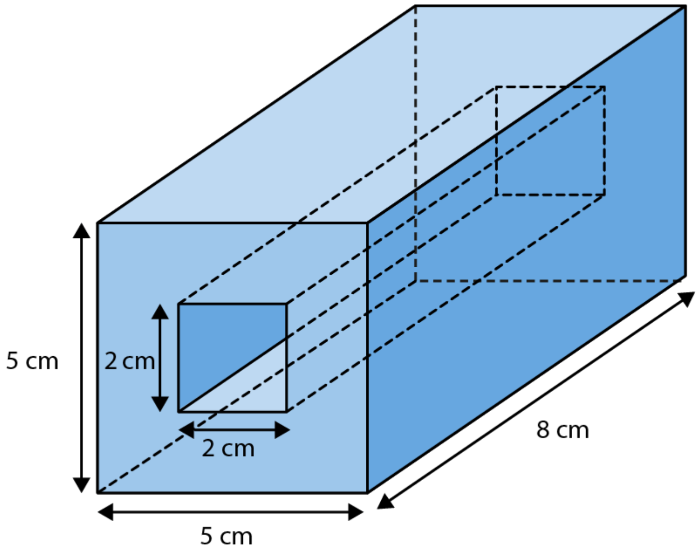
V = 410 cm^3

- *'What is the correct volume of this shape? Tick or cross each answer.'*



Volume (cm^3)	✓ or ✗
4 × 2 × 3 + 11 × 3 × 4 + 4 × 3 × 2	
8 × 7 × 3 + 7 × 4 × 3 + 8 × 7 × 2	
11 × 8 × 2 + 2 × 3 × 4 + 11 × 7 × 2	
2 × 8 × 3 + 7 × 4 × 3 + 2 × 8 × 3	

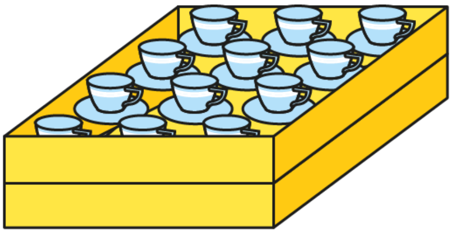
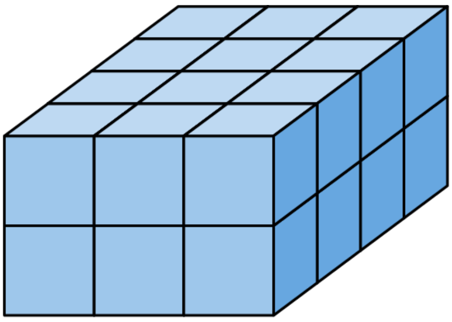
		Dòng nǎo jīn: • *'Lisa says you can find the volume of the shape on the previous pages using this expression:'* $11 \times 8 \times 3 - 7 \times 3 \times 4$ • *'Chang says you can't use subtraction to find the volume of this shape.'* *'Who is right? Explain your answer.'*
3:11	Complete this teaching point by asking children to find the volume of various compound shapes with one or more dimension that must be deduced.	*'Find the volume of these shapes.'*  

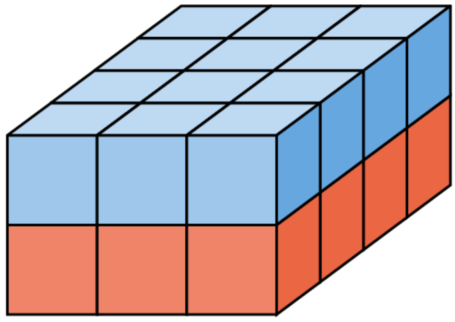
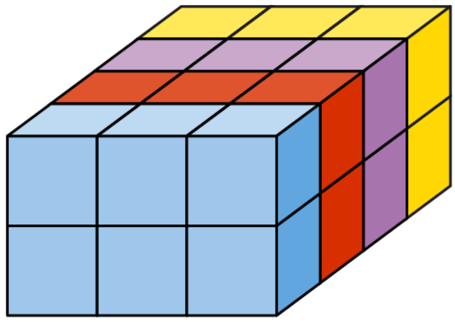
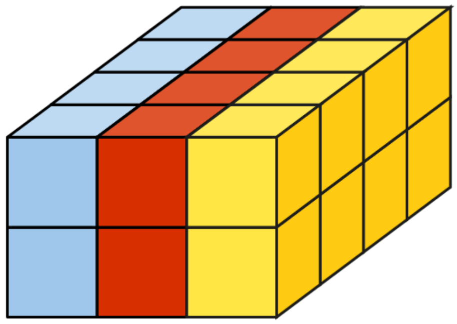
		Dòng nǎo jīn: *'This cuboid has a hole in the middle. Find the volume of the surrounding shape.'* 

Teaching point 4:

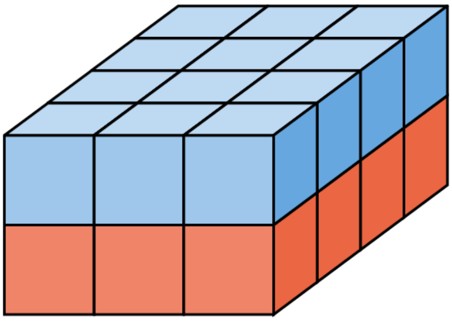
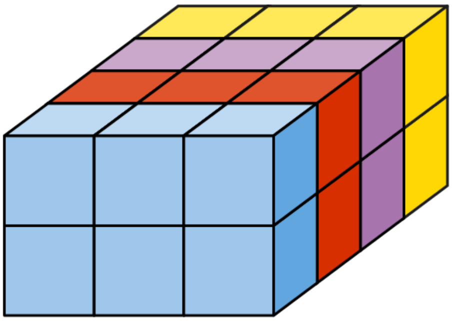
Both the commutative law and the associative law can be applied when multiplying three or more numbers.

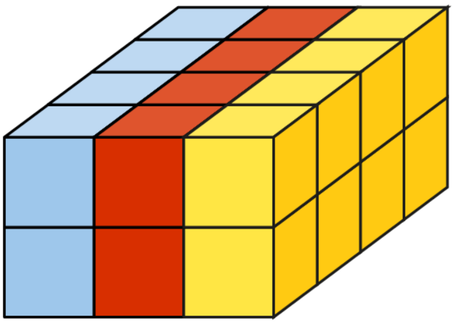
Steps in learning

	Guidance	Representations
4:1	In this teaching point, the laws of commutativity and associativity are applied to multiplication problems with three factors. Start by showing children a real-life context of stacked arrays, such as a tray of 12 teacups in a *4* × *3* arrangement. Ask children *'If we stack up two trays, how many teacups will there be in total?'* Write the problem as an expression. Ask children to describe what the expression shows. Draw out the understanding that the number of cups in one tray is calculated first. Then the product is multiplied to calculate the total number of cups in two trays. Make sure children understand what each number represents. Use the stem sentence: ***'The ___ refers to the ___.'*** Repeat this step using another real-life example, such as eggs in stacked egg boxes. Continue to use the stem sentence above.	*'If we stack up two trays, how many teacups will there be in total?'*  • *'One tray has three columns and four rows. There are two trays. We can write this as* 3 × 4 × 2.*'* • *'The "3" refers to the number of columns.'* • *'The "4" refers to the number of rows.'* • *'The "2" refers to the number of trays.'* $3 \times 4 \times 2 = 12 \times 2$ $= 24$
4:2	Next, deepen understanding by making links to volume. Use cubes to represent the teacup problem from step *4:1*, with each cube representing one teacup. Recap that you can use multiplication to find the total number of cubes. Explore how you can slice up the cuboid in different ways to work out the total number of cubes. Create an equation to represent each different way of slicing the cuboid, pointing out that the volume is always the same whichever way you choose.	Dividing a cuboid in different ways: *'How many cubes are there?'* 

	Continue to use the stem sentence from the previous step to ensure children know what each number refers to in this context.	Split 1:  $3 \times 4 \times 2 = 12 \times 2$ $= 24$ Split 2:  $3 \times 2 \times 4 = 6 \times 4$ $= 24$ Split 3:  $4 \times 2 \times 3 = 8 \times 3$ $= 24$

4:3	Now make links to the law of commutativity. For multiplication equations with two factors, children learnt that the factors can be written in either order and that the product remains the same (see segment *2.3 Times tables: groups of 2 and commutativity (part 1)*). We can now apply commutativity to cases where there are three factors. Refer to the cuboid in step *4:2*, recalling that we can divide the cuboid in three ways: • 3×4, two times (horizontal slice) • 3×2, four times (vertical slice one way) • 4×2, three times (vertical slice other way) Ask children to write the equations for each of these contexts, and then ask *'What do you notice?'* • $3 \times 4 \times 2 = 24$ • $3 \times 2 \times 4 = 24$ • $4 \times 2 \times 3 = 24$ Children should notice that: • the products are the same for each equation • each equation has the same factors, but they are written in a different order. By the end of this step, children should be confident with the generalised statement: ***'If you change the order of the factors, the product remains the same.'***	

4:4	Next, explore the associative law, which has been applied in the context of addition (see *Spine 1: Number, Addition and Subtraction*, segment *1.11*) but not yet in the context of multiplication. Using the representations from step *4:2*, explain that we can choose to multiply any two factors first, and the product will remain the same. Introduce the idea that choosing to multiply two factors first can make the overall calculation easier. We will build on this in the next teaching point. Explain to children that the brackets are there to show which calculation is being performed first. This is a significant step for children to not calculate from left to right, and they should be given practice to become fluent in the process. Work towards this generalised statement: ***'When we multiply three numbers, the product will be the same whichever pair we multiply first.'***	Multiplying any two factors first: *'How many cubes are there?'* Version 1:  $3 \times 4 \times 2 = 24$ $(3 \times 4) \times 2 = 24$ $3 \times (4 \times 2) = 24$ Version 2:  $3 \times 2 \times 4 = 24$ $(3 \times 2) \times 4 = 24$ $3 \times (2 \times 4) = 24$

		Version 3:  $4 \times 2 \times 3 = 24$ $(4 \times 2) \times 3 = 24$ $4 \times (2 \times 3) = 24$

Teaching point 5:

The choice of which order to multiply in can be made according to the simplest calculation.

Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>5:1</td><td>Now children can practise multiplying three numbers, choosing which two numbers to multiply first.

Explore a range of multiplication problems, highlighting the effectiveness of finding the easiest way to calculate the product in each case.

Draw attention to those factors that are easier to multiply by, such as two (doubling) or multiples of ten and one hundred. Encourage children to use existing knowledge of times tables to make the problems easier to solve.

You could use problems such as the ones shown below and opposite.

'Which do you find the simplest way to solve 6 × 2 × 5? Explain your answer.'

• $6 \times 2 \times 5$
• $2 \times 5 \times 6$
• $6 \times 5 \times 2$</td><td>• 'Fill in the missing numbers to solve $3 \times 5 \times 2$ in two different ways.'

$3 \times 5 \times 2 = \boxed{} \times 2$
$= \boxed{}$

$3 \times 5 \times 2 = 3 \times \boxed{}$
$= \boxed{}$

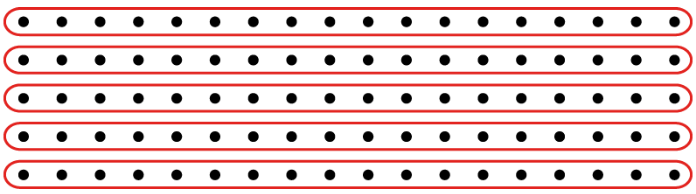
• 'Which calculation do you find easier? Choose either A or B in each case, and explain your answer.'

<table><tr><th>A</th><th>B</th></tr><tr><td>$(25 \times 4) \times 8$</td><td>$25 \times (4 \times 8)$</td></tr><tr><td>$(182 \times 2) \times 2$</td><td>$182 \times (2 \times 2)$</td></tr><tr><td>$(3 \times 3) \times 3$</td><td>$3 \times (3 \times 3)$</td></tr><tr><td>$(16 \times 50) \times 2$</td><td>$16 \times (50 \times 2)$</td></tr></table></td></tr>
<tr><td>5:2</td><td>Now look at how an equation can be made easier to solve by expressing one of the factors as a multiplication of two factors.

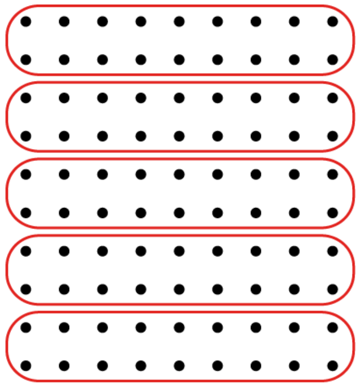
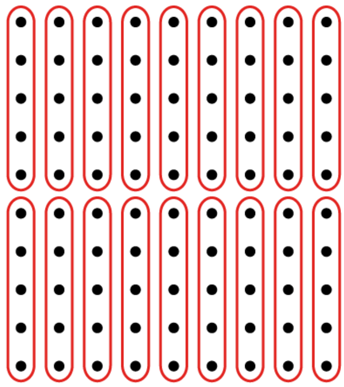
It is important that this step is not linked to area and volume, as two dimensions cannot be changed to three. Instead, you could use arrays.

Begin with an array of 18 × 5. Circle the groups of 18 across each row, to show five groups of 18.

Then explain that we could think of 18 as 9 × 2, and we could change the</td><td>Using arrays to show multiplication:



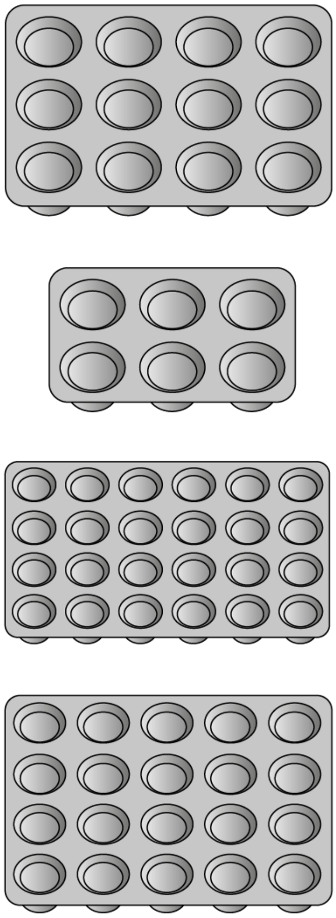
18×5</td></tr>
</table>

	calculation to $9 \times 2 \times 5$. Use an array to show this new interpretation. Circle groups of 18 in the new array, to show five groups of 9×2. To work out the solution you could choose to multiply 2×5 first, to get 10, and then work out $9 \times 10 = 90$. Again, show this step by circling groups on the array. This method uses associativity (see step *4:4*): the '2' is associated with the '5' rather than the '9', to make the calculation easier.	 $9 \times 2 \times 5$  $5 \times 2 \times 9 = 10 \times 9$ $= 90$
5:3	Provide children with practice changing two-factor multiplication calculations into three-factor calculations to make them easier to solve. Work through an example together before asking children to practise on their own. Explore the idea that some factors will have several options for how they can be expressed as two factors, and we need to identify the easiest option by looking for features such as doubling or multiplying by ten.	• *'Solve the following calculation.'* 24×5 $24 = 24 \times 1$ **$24 = 12 \times 2$** $24 = 8 \times 3$ $24 = 6 \times 4$ $24 = 12 \times 2$ so $24 \times 5 = 12 \times 2 \times 5$ $= 12 \times 10$ $= 120$ • *'Solve the following calculations by changing them into three-factor calculations.'* 22×5 16×4 8×50

5:4

Complete this segment by providing children with varied practice of working with volume, and also multiplying three numbers in other contexts. You could use questions such as those below and opposite.

- *'A rabbit digs a hole that is 30 cm deep, 18 cm wide and 12 cm long. How much soil will a gardener need to fill this hole?'*
- *'A school sets out chairs in the school hall to show a film. There are 12 rows and 8 chairs in each row. If all the chairs are full and each person pays £5, how much money is collected?'*
- *'Muffins are baked in the following trays. If each muffin is sold for £2.50, how much money is made per tray?'*



Dòng nǎo jīn:

'A shop sells boxes that each contain 12 cans of fizzy drink. The boxes of cans are arranged in a stack. There are 240 cans in total.'

- *'Sophie thinks the stack could be 10 boxes high and 2 boxes deep.'*
- *'Anda thinks the stack could be 5 boxes high and 4 boxes deep.'*
- *'Micah thinks the stack could be 12 boxes high and 2 boxes deep.'*

'Who is correct? Explain your answer.'